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THEORETICAL INVESTIGATION OF ACOUSTIC SURFACE WAVES ON PIEZOELECTRIC CRYSTALS

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Contract No. F19628-69-C-0132
Project No. 5635
Task No. 563503
Work Unit No. 56350301

FINAL REPORT

Period Covered: 1 December 1968 through 30 November 1969

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Contract Monitor: Andrew J. Slobodnik, Jr., 1/Lt, USAF
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Prepared for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
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ABSTRACT

This report describes the analyses of several piezoelectric and pure elastic surface wave propagation problems and computer programs which implement their numerical study. In addition, the formal analysis of an electric current line source located above a piezoelectric crystal half space is presented in some detail.

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I. INTRODUCTION

This report describes the analyses of several piezoelectric and pure elastic surface wave propagation problems and computer programs which implement their numerical study. In addition, the formal analysis of an electric current line source located above a piezoelectric crystal half space is presented in some detail.

The physical configurations of the propagation problems considered in the sequel are shown in Figure 1 and are enumerated below:

(1) Surface wave propagation on a piezoelectric half space in the presence of an infinitesimal electric or "magnetic" conductor located at an arbitrary but fixed distance h above the crystal surface.

(2) Surface wave propagation on a piezoelectric or pure elastic half space contiguous to a perfect isotropic elastic conductor (e.g. gold or aluminum) of arbitrary thickness h.

(3) Surface wave propagation on a piezoelectric or pure elastic half space contiguous to a perfect fluid half space.

(4) Surface wave propagation on a piezoelectric or pure elastic half space contiguous to an isotropic elastic layer of arbitrary thickness h.

The following section contains the details of the analyses of the propagation problems described above including special degenerate cases which are encountered. These cases correspond to conditions of surface wave propagation wherein one or more components of displacement vanish or the electric and mechanical fields become decoupled (in the general piezoelectric case the surface wave contains all components of displacement and is coupled via the piezoelectric constants to the electric field). In practice degenerate cases have been found to occur when the sagital plane lies either in a plane of symmetry of the crystal under consideration, in the basal plane of crystals of class 6 mm, or in the principal plane(s) of cubic crystals.

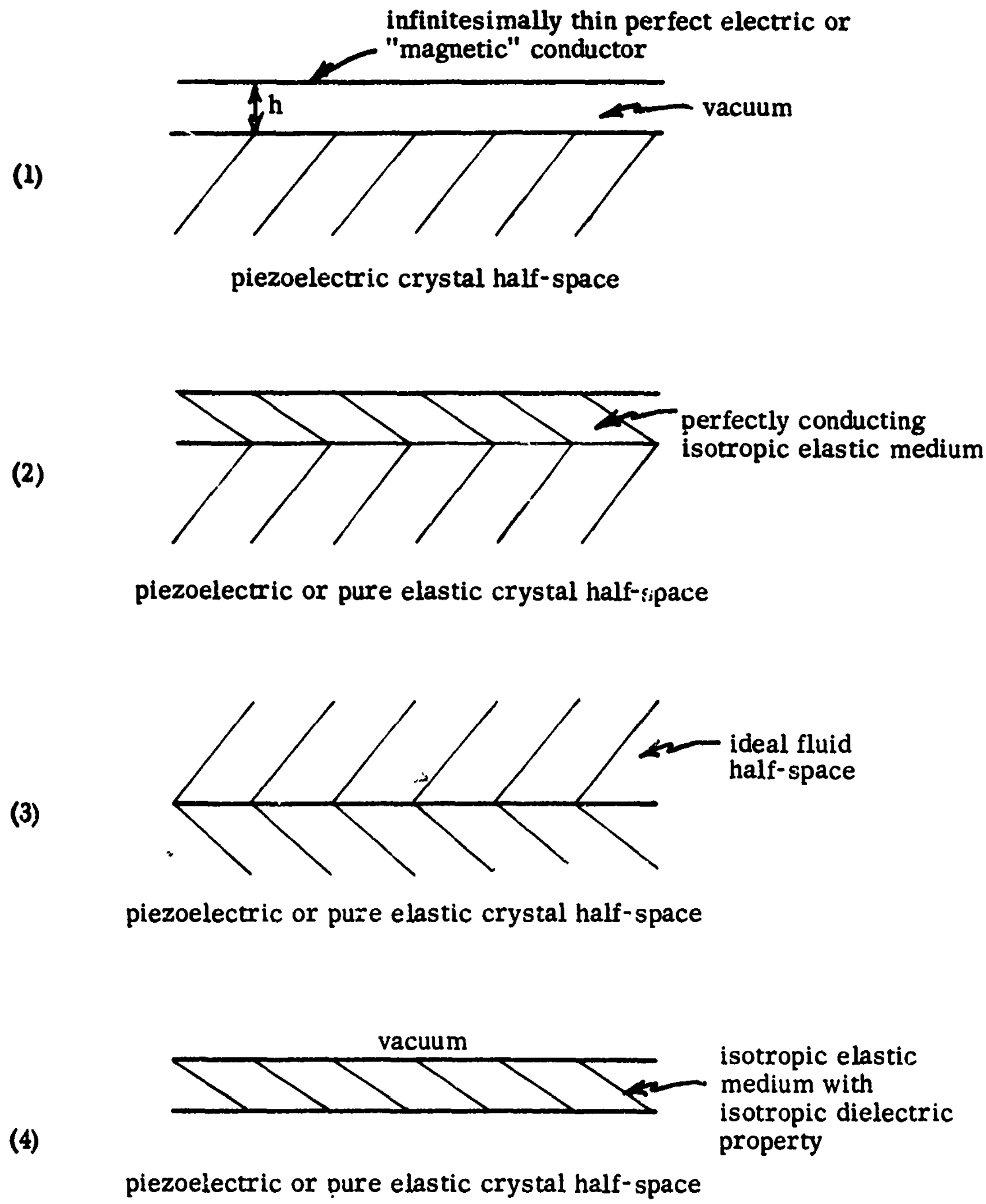

infinitesimally thin perfect electric or
"magnetic" conductor
h
vacuum
(1)
piezoelectric crystal half-space
perfectly conducting
isotropic elastic medium
(2)
piezoelectric or pure elastic crystal half-space
ideal fluid
half-space
(3)
piezoelectric or pure elastic crystal half-space
vacuum
isotropic elastic
medium with
isotropic dielectric
property
(4)
piezoelectric or pure elastic crystal half-space

Figure 1

The third section presents an outline of the analysis pertaining to the excitation of surface and bulk wave by an electric current line source located above a piezoelectric half space. This problem was originally undertaken as an approach to the study of interdigital electric transducers but was ultimately abandoned due to lack of funds and the fact that inordinant amounts of computation would be required to extract useful data regarding the efficiency of excitation of surface waves.

Finally, Section II provides detailed descriptions of the computer programs written to implement the numerical analyses of the surface wave propagation problems described in Section II. The material presented in this section provides the reader with sufficient information for the use of the computer programs and the comprehension of the programming methods employed therein. Source deck listings for the various programs are also provided.

II. PROPAGATION OF SURFACE WAVES ON PIEZOELECTRIC SUBSTRATES

1. Surface Waves on Piezoelectric Crystals in the Presence of Infinitesimally Thin Electric and "Magnetic" Conductors

In this section the formal analysis is presented for the propagation characteristics of surface waves on a general piezoelectric crystal surface in the presence of perfect electric and "magnetic" conductors. The geometries under consideration are depicted in Figure 2.

A rectangular coordinate system is chosen with the x_3 axis normal to the crystal surface and the x_1 axis in the direction of propagation. Arbitrary orientations of the crystal surface with respect to the crystal axes are considered. This is accomplished by means of a coordinate rotation through the Euler angles from the crystal axes to the desired x_1, x_2, x_3 coordinate system. The matrix defining such a rotation is given by

$$V = \begin{pmatrix} \cos\alpha\cos\gamma - \sin\alpha\cos\beta\sin\gamma & \sin\alpha\cos\gamma + \cos\alpha\cos\beta\sin\gamma & \sin\beta\sin\gamma \\ -\cos\alpha\sin\gamma - \sin\alpha\cos\beta\cos\gamma & -\sin\alpha\sin\gamma + \cos\alpha\cos\beta\cos\gamma & \sin\beta\cos\gamma \\ \sin\alpha\sin\beta & -\cos\alpha\sin\beta & \cos\beta \end{pmatrix}$$

where α, β, and γ are the Euler Angles. Since the x_1, x_2, x_3 coordinate system is relative to the crystal surface and direction of propagation, the form of the differential equations for the mechanical displacements and electric potentials in this coordinate system is independent of the surface under consideration. Only the values of the coefficients change with the surface orientation relative to the crystal axes. This is also true of the boundary conditions. Different cuts are thus distinguished only through the transformed tensor quantities involved in coefficients of the differential operators.

In terms of the Euler transformation matrix V, the tensor quantities of interest; viz. the elastic constants ($C_{ijk\ell}$), the piezoelectric constants (e_{ijk}), and the dielectric constants (ϵ_{ij}) are transformed as follows:

$$C'_{ijk\ell} = \sum_{r,s,t,u=1}^{3} V_{ir} V_{js} V_{kt} V_{\ell u} C_{rstu} \quad , \tag{1}$$

$$e'_{ijk} = \sum_{r,s,t=1}^{3} V_{ir} V_{js} V_{kt} e_{rst} \quad , \tag{2}$$

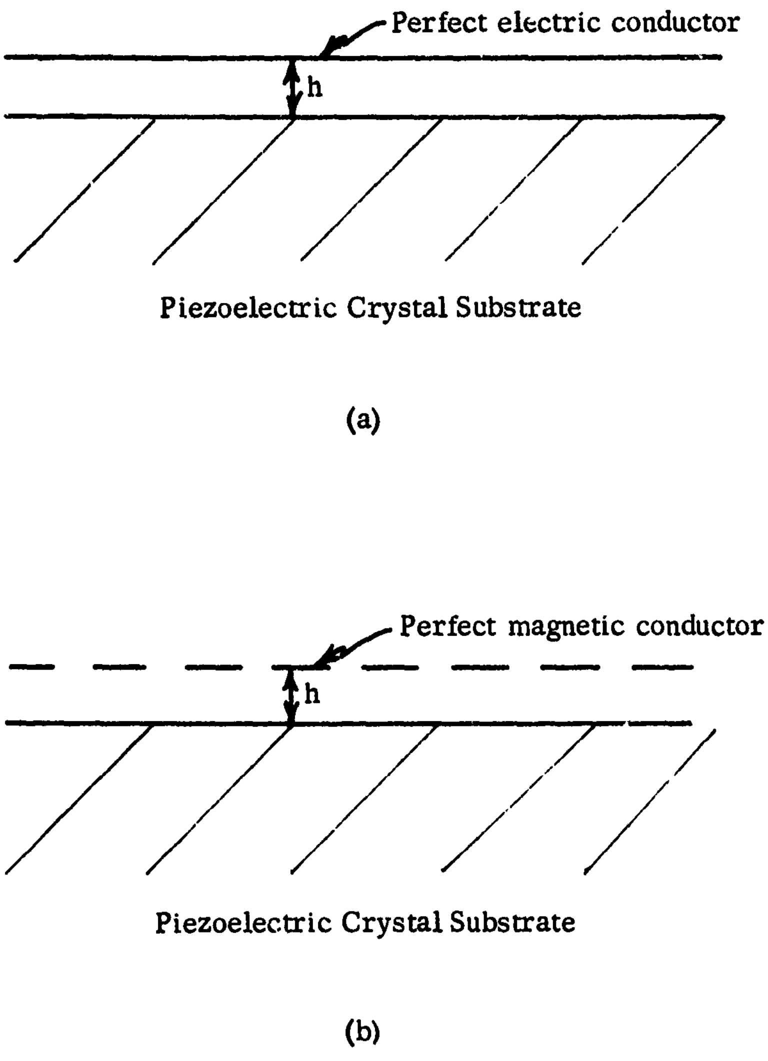

Perfect electric conductor
h
Piezoelectric Crystal Substrate
(a)
Perfect magnetic conductor
h
Piezoelectric Crystal Substrate
(b)

Figure 2

$$\epsilon'_{ij} = \sum_{r,s=1}^{3} V_{ir} V_{js} \epsilon_{rs} \quad , \tag{3}$$

where the primed quantities refer to a rotated coordinate system and the unprimed correspond to the coincidence of the $x_1x_2x_3$ coordinate system and the crystal axes, i.e., when V is the identity matrix. The elastic and piezoelectric constants can be reduced to 2 index symbols in the usual fashion, viz. $C'_{ijk\ell} \rightarrow C_{pq}$ and $e'_{ijk} \rightarrow e_{ip}$ where p or q = 1, 2, 3, 4, 5, 6 are equivalent to 11, 22, 33, 23, or 32, 13 or 31, and 12 or 21 respectively.

The differential equations for the components U_i, i = 1, 2, 3, of the mechanical displacement and electric potential φ are given by

$$\left.\begin{array}{ll} C'_{ijk\ell} U_{k,\ell i} + e'_{kij} \varphi_{,ki} = \rho \ddot{U}_j & j = 1, 2, 3 \\ e'_{ik\ell} U_{k,\ell i} - \epsilon'_{ik} \varphi_{,ki} = 0 & \end{array}\right\} x_3 > 0 \tag{4}$$

$$\nabla^2 \varphi = 0 \qquad -h \le x_3 \le 0 \quad . \tag{5}$$

In the above equations, indices preceded by a comma denote differentiation with respect to space coordinates. The summation convention for repeated indices is employed as is the dot notation for differentiation with respect to time.

As indicated above, the surface waves under consideration are assumed to be traveling in the x_1 direction along a surface whose normal is in the x_3 direction. The displacements and potentials are considered to be independent of the x_2 coordinate. Consequently, traveling wave solutions of the form

$$U_i = \beta_i \, e^{-\alpha\omega x_3/v_s} \, e^{j\omega(t - x_1/v_s)}$$

and

$$\varphi = \beta_4 \, e^{-\alpha\omega x_3/v_s} \, e^{i\omega(t - x_1/v_s)}$$

are sought. When these surface waveforms (as identified by an exponential decay into the crystal) are substituted into the differential equations for $x_3 > 0$ a linear homogeneous system of four equations in the unknowns $\beta_1, \beta_2, \beta_3, \beta_4$

results. The determinant of the coefficients of the unknowns in these equations must be zero in order that a non-trivial solution exist, that is

$$\det \begin{pmatrix} C_{55}\alpha^2 + 2C_{15}j\alpha - C_{11} + \rho v_s^2 & C_{45}\alpha^2 + [C_{14} + C_{56}]j\alpha - C_{16} & C_{35}\alpha^2 + [C_{13} + C_{55}]j\alpha - C_{15} & e_{35}\alpha^2 + [e_{15} + e_{31}]j\alpha - e_{11} \\ C_{45}\alpha^2 + [C_{14} + C_{56}]j\alpha - C_{16} & C_{44}\alpha^2 + 2C_{46}j\alpha - C_{66} + \rho v_s^2 & C_{34}\alpha^2 + [C_{36} + C_{45}]j\alpha - C_{56} & e_{34}\alpha^2 + [e_{14} + e_{36}]j\alpha - e_{16} \\ C_{35}\alpha^2 + [C_{13} + C_{55}]j\alpha - C_{15} & C_{34}\alpha^2 + [C_{36} + C_{45}]j\alpha - C_{56} & C_{33}\alpha^2 + 2C_{35}j\alpha - C_{55} + \rho v_s^2 & e_{33}\alpha^2 + [e_{13} + e_{35}]j\alpha - e_{15} \\ e_{35}\alpha^2 + [e_{15} + e_{31}]j\alpha - e_{11} & e_{34}\alpha^2 + [e_{14} + e_{36}]j\alpha - e_{16} & e_{33}\alpha^2 + [e_{13} + e_{35}]j\alpha - e_{15} & -\epsilon_{33}\alpha^2 - 2\epsilon_{13}j\alpha + \epsilon_{11} \end{pmatrix} = 0 \quad (6)$$

Evaluation of the above determinant results in an eighth order polynomial in α of the form

$$A_8\alpha^8 + jA_7\alpha^7 + A_6\alpha^6 + jA_5\alpha^5 + A_4\alpha^4 + jA_3\alpha^3 + A_2\alpha^2 + jA_1\alpha + A_o = 0 \quad , \qquad (7)$$

with the coefficients A_n, $n = 0, 1, \ldots, 8$, purely real. Since the fields must be bounded, or go to zero as $x_3 \to \infty$, only the roots with non-negative real parts are allowed. If the unknown in equation (7) is considered to be $j\alpha$ instead of α then the polynomial in $j\alpha$ has purely real coefficients. Thus, either the roots $j\alpha$ are real or occur in conjugate pairs, e.g.

$$j\alpha_1 = a + jb$$

$$j\alpha_2 = a - jb$$

whence

$$\alpha_1 = b - ja$$

$$\alpha_2 = -b - ja \quad .$$

Therefore, the roots α are either pure imaginary or occur in pairs with positive and negative real parts.

In the range of velocities where generally surface waves can exist (i.e. velocities below the lowest bulk wave velocity in the direction of propagation under consideration) the roots occur such that four with positive real parts can be selected. However, if for a given velocity four such roots are not found the possibility of the existence of a degenerate* surface wave remains and must be considered. These special waves are discussed in detail in the section on degenerate cases. Upon obtaining the admissable values of α, corresponding values of β_i (to within a constant factor) can be found for each α.

In addition to the equations for $x_3 > 0$, the differential equation (5) for $-h \le x_3 \le 0$ must be satisfied together with appropriate boundary conditions at $x_3 = 0$ and $x_3 = -h$. Assuming that the crystal surface is stress free ($T_{3j} = 0$ at $x_3 = 0$), the mechanical boundary conditions at each point of the surface of the crystal are

*The term degenerate is used to signify that certain components of displacement and/or the electric potential vanish identically.

$$T_{3j}\Big|_{x_3=0} = C'_{3jk\ell}U_{k,\ell} + e'_{k3j}\varphi_{,k}\Big|_{x_3=0} = 0 \quad , \tag{8}$$

$$j = 1, 2, 3 \quad .$$

For the electric wall case (Figure 2a) the boundary conditions on the electric potential are the continuity of φ at $x_3 = 0$ and, without loss of generality, $\varphi(-h) = 0$. Also the normal component of electrical displacement must be continuous across the surface of the crystal.

The total fields (mechanical displacement and potential) may be expressed as a linear combination of the fields associated with the admissible values of α for $x_3 > 0$, namely,

$$U_i = \sum_{\ell=1}^{4} B^{(\ell)} \beta_i^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)} , \quad i = 1, 2, 3 , \tag{9}$$

$$\varphi = \sum_{\ell=1}^{4} B^{(\ell)} \beta_4^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \quad . \tag{10}$$

In the region $-h \le x_3 \le 0$ the potential is a solution of Laplace's equation (5). A solution satisfying the continuity condition at $x_3 = 0$ and vanishing at $x_3 = -h$ is

$$\varphi = \sum_{\ell=1}^{4} B^{(\ell)} \beta_4^{(\ell)} \operatorname{csch}\left(\frac{\omega h}{v_s}\right) \sinh\left(\frac{\omega}{v_s}(x_3 + h)\right) e^{j\omega(t-x_1/v_s)} , \tag{11}$$

$$-h \le x_3 \le 0 \quad .$$

Finally, the normal component of $\vec{D}$ (viz. D_3) must be continuous across the surface $x_3 = 0$. Inside the crystal the electrical displacement is given by $D_i = e_{ik\ell}U_{k,\ell} - \epsilon_{ik}\varphi_{,k}$, $(i = 1, 2, 3)$, while in the region $-h \le x_3 \le 0$, $\vec{D} = -\epsilon_0 \nabla\varphi$.

Substituting the waveforms (9), (10) in equation (8) and expressing the continuity of D_3 at $x_3 = 0$ in terms of equations (9), (10), and (11) yields the following set of homogeneous equations for the amplitudes $B^{(\ell)}$, namely,

$$\sum_{\ell=1}^{4}\Bigg[\beta_1^{(\ell)}[jC_{15}+\alpha^{(\ell)}C_{55}]+\beta_2^{(\ell)}[jC_{56}+\alpha^{(\ell)}C_{45}]+\beta_3^{(\ell)}[jC_{55}+\alpha^{(\ell)}C_{35}]$$

$$+\beta_4^{(\ell)}[je_{15}+\alpha^{(\ell)}e_{35}]\Bigg]B^{(\ell)}=0 \quad , \tag{12}$$

$$\sum_{\ell=1}^{4}\Bigg[\beta_1^{(\ell)}[jC_{14}+\alpha^{(\ell)}C_{45}]+\beta_2^{(\ell)}[jC_{46}+\alpha^{(\ell)}C_{44}]+\beta_3^{(\ell)}[jC_{45}+\alpha^{(\ell)}C_{34}]$$

$$+\beta_4^{(\ell)}[je_{14}+\alpha^{(\ell)}e_{34}]\Bigg]B^{(\ell)}=0 \quad , \tag{13}$$

$$\sum_{\ell=1}^{4}\Bigg[\beta_1^{(\ell)}[jC_{13}+\alpha^{(\ell)}C_{35}]+\beta_2^{(\ell)}[jC_{36}+\alpha^{(\ell)}C_{34}]+\beta_3^{(\ell)}[jC_{35}+\alpha^{(\ell)}C_{33}]$$

$$+\beta_4^{(\ell)}[je_{13}+\alpha^{(\ell)}e_{33}]\Bigg]B^{(\ell)}=0 \quad , \tag{14}$$

$$\sum_{\ell=1}^{4}\Bigg[\beta_1^{(\ell)}[je_{31}+\alpha^{(\ell)}e_{35}]+\beta_2^{(\ell)}[je_{36}+\alpha^{(\ell)}e_{34}]+\beta_3^{(\ell)}[je_{35}+\alpha^{(\ell)}e_{33}]$$

$$-\beta_4^{(\ell)}\left[j\epsilon_{13}+\alpha^{(\ell)}\epsilon_{33}+\epsilon_o\coth\left(\frac{\omega h}{v_s}\right)\right]\Bigg]B^{(\ell)}=0 \quad . \tag{15}$$

The transcendental equation obtained by setting the determinant of the matrix ($\hat{L}$) of coefficients of this system equal to zero determines the surface wave velocities.

In the limiting case ($h \to 0$) the region $-h \le x_3 \le 0$ disappears and the boundary conditions on the electric potential and normal component of displacement in the crystal are replaced by $\varphi(0) = 0$. In this case equation (15) above reduces to

$$\sum_{\ell=1}^{4}\beta_4^{(\ell)}B^{(\ell)}=0 \quad . \tag{16}$$

In addition to the electric wall problem, the magnetic wall case (Figure 2b) also has been considered. The only change in the formulation of this problem is the boundary condition at $x_3 = -h$. In this case the solution for the potential in the region $-h \le x_3 \le 0$ assumes the form

$$\phi = \sum_{\ell=1}^{4} B^{(\ell)} \beta_4^{(\ell)} \operatorname{sech}\left(\frac{\omega h}{v_s}\right) \cosh\left(\frac{\omega}{v_s}(x_3 + h)\right) e^{j\omega(t - x_1/v_s)} \quad . \tag{17}$$

This function satisfies the condition that the normal component of electrical displacement (D_3) vanish at the magnetic wall $x_3 = -h$.

Equation (15), the continuity of D_3 at $x_3 = 0$, is modified for the magnetic wall case by replacing $\coth(\omega h/v_s)$ with $\tanh(\omega h/v_s)$. Otherwise the equations (12)-(15) remain unchanged. The limiting case $h \to 0$ requires no special change as it did in the electric wall case since the continuity condition on D_3 at $x_3 = 0$ now simply becomes $D_3 = 0$ and $D_3\big|_{x_3=0} \sim \tanh(\omega h/v_s)$ (outside the crystal), which goes to zero as $h \to 0$. Thus equation (15) with the $\tanh(\omega h/v_s)$ term needs no modification for the limiting case.

Once a surface wave velocity has been found the partial field amplitudes $B^{(j)}$, $j=1,2,3,4$ may be calculated to within a constant factor. Consequently, $B^{(4)}$ is chosen as unity (except for certain degenerate cases described later) and $B^{(1)}$, $B^{(2)}$, $B^{(3)}$ are found from equations (12)-(14). These amplitudes are used to evaluate the displacement components (eq. (9)), electric potential (eq. (10)), the components of stress, strain, electric displacement, electric field, and the time average power flow as functions of ωx_3. The explicit forms of the components of the aforementioned physical quantities are:

<u>Stress</u>

$$\frac{T_{11}}{\omega} = \sum_{\ell=1}^{4} B^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \left\{ \beta_1^{(\ell)}\left[\frac{-j}{v_s} C_{11} - \frac{\alpha^{(\ell)}}{v_s} C_{15}\right] + \beta_2^{(\ell)}\left[\frac{-j}{v_s} C_{16} - \frac{\alpha^{(\ell)}}{v_s} C_{14}\right] + \beta_3^{(\ell)}\left[\frac{-j}{v_s} C_{15} - \frac{\alpha^{(\ell)}}{v_s} C_{13}\right] + \beta_4^{(\ell)}\left[\frac{-j}{v_s} e_{11} - \frac{\alpha^{(\ell)}}{v_s} e_{31}\right] \right\}$$

$$\frac{T_{12}}{\omega} = \frac{T_{21}}{\omega} = \sum_{\ell=1}^{4} B^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \left\{ \beta_1^{(\ell)}\left[\frac{-j}{v_s} C_{16} - \frac{\alpha^{(\ell)}}{v_s} C_{56}\right] + \beta_2^{(\ell)}\left[\frac{-j}{v_s} C_{66} - \frac{\alpha^{(\ell)}}{v_s} C_{46}\right] + \beta_3^{(\ell)}\left[\frac{-j}{v_s} C_{56} - \frac{\alpha^{(\ell)}}{v_s} C_{36}\right] + \beta_4^{(\ell)}\left[\frac{-j}{v_s} e_{16} - \frac{\alpha^{(\ell)}}{v_s} e_{36}\right] \right\}$$

$$\frac{T_{13}}{\omega} = \frac{T_{31}}{\omega} = \sum_{\ell=1}^{4} B^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \Big\{ \beta_1^{(\ell)} \left[\frac{-j}{v_s} C_{15} - \frac{\alpha^{(\ell)}}{v_s} C_{55}\right]$$

$$+ \beta_2^{(\ell)} \left[\frac{-j}{v_s} C_{56} - \frac{\alpha^{(\ell)}}{v_s} C_{45}\right] + \beta_3^{(\ell)} \left[\frac{-j}{v_s} C_{55} - \frac{\alpha^{(\ell)}}{v_s} C_{35}\right] + \beta_4^{(\ell)} \left[\frac{-j}{v_s} e_{15} - \frac{\alpha^{(\ell)}}{v_s} e_{35}\right] \Big\}$$

$$\frac{T_{22}}{\omega} = \sum_{\ell=1}^{4} B^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \Big\{ \beta_1^{(\ell)} \left[\frac{-j}{v_s} C_{12} - \frac{\alpha^{(\ell)}}{v_s} C_{25}\right]$$

$$+ \beta_2^{(\ell)} \left[\frac{-j}{v_s} C_{26} - \frac{\alpha^{(\ell)}}{v_s} C_{24}\right] + \beta_3^{(\ell)} \left[\frac{-j}{v_s} C_{25} - \frac{\alpha^{(\ell)}}{v_s} C_{23}\right] + \beta_4^{(\ell)} \left[\frac{-j}{v_s} e_{12} - \frac{\alpha^{(\ell)}}{v_s} e_{32}\right] \Big\}$$

$$\frac{T_{23}}{\omega} = \frac{T_{32}}{\omega} = \sum_{\ell=1}^{4} B^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \Big\{ \beta_1^{(\ell)} \left[\frac{-j}{v_s} C_{14} - \frac{\alpha^{(\ell)}}{v_s} C_{45}\right]$$

$$+ \beta_2^{(\ell)} \left[\frac{-j}{v_s} C_{46} - \frac{\alpha^{(\ell)}}{v_s} C_{44}\right] + \beta_3^{(\ell)} \left[\frac{-j}{v_s} C_{45} \frac{-\alpha^{(\ell)}}{v_s} C_{34}\right] + \beta_4^{(\ell)} \left[\frac{-j}{v_s} e_{14} - \frac{\alpha^{(\ell)}}{v_s} e_{34}\right] \Big\}$$

$$\frac{T_{33}}{\omega} = \sum_{\ell=1}^{4} B^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \Big\{ \beta_1^{(\ell)} \left[\frac{-j}{v_s} C_{13} - \frac{\alpha^{(\ell)}}{v_s} C_{35}\right]$$

$$+ \beta_2^{(\ell)} \left[\frac{-j}{v_s} C_{36} - \frac{\alpha^{(\ell)}}{v_s} C_{34}\right] + \beta_3^{(\ell)} \left[\frac{-j}{v_s} C_{35} - \frac{\alpha^{(\ell)}}{v_s} C_{33}\right] + \beta_4^{(\ell)} \left[\frac{-j}{v_s} e_{13} - \frac{\alpha^{(\ell)}}{v_s} e_{33}\right] \Big\}$$

<u>Strain</u>

$$\frac{S_{11}}{\omega} = \sum_{\ell=1}^{4} \frac{-j}{v_s} B^{(\ell)} \beta_1^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)}$$

$$\frac{S_{22}}{\omega} = 0$$

$$\frac{S_{33}}{\omega} = \sum_{\ell=1}^{4} \frac{-\alpha^{(\ell)}}{v_s} B^{(\ell)} \beta_3^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)}$$

$$\frac{S_{12}}{\omega} = \frac{S_{21}}{\omega} = \frac{1}{2}\sum_{\ell=1}^{4} \frac{-j}{v_s} B^{(\ell)} \beta_2^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)}$$

$$\frac{S_{13}}{\omega} = \frac{S_{31}}{\omega} = \frac{1}{2}\sum_{\ell=1}^{4} B^{(\ell)} \left[\frac{-\alpha^{(\ell)}}{v_s}\beta_1^{(\ell)} - \frac{j}{v_s}\beta_3^{(\ell)}\right] e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)}$$

$$\frac{S_{23}}{\omega} = \frac{S_{32}}{\omega} = \frac{1}{2}\sum_{\ell=1}^{4} \frac{-\alpha^{(\ell)}}{v_s} B^{(\ell)} \beta_2^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)}$$

<u>Electric Field</u>

$$\frac{E_1}{\omega} = \frac{j}{v_s}\sum_{\ell=1}^{4} B^{(\ell)} \beta_4^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)} = \frac{j}{v_s}\varphi$$

$$\frac{E_3}{\omega} = \frac{1}{v_s}\sum_{\ell=1}^{4} \alpha^{(\ell)} B^{(\ell)} \beta_4^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)}$$

<u>Electric Displacement</u>

$$\frac{D_1}{\omega} = \sum_{\ell=1}^{4} B^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \Bigg\{\beta_1^{(\ell)}\left[\frac{-j}{v_s}e_{11} - \frac{\alpha^{(\ell)}}{v_s}e_{15}\right]$$
$$+ \beta_2^{(\ell)}\left[\frac{-j}{v_s}e_{16} - \frac{\alpha^{(\ell)}}{v}e_{14}\right] + \beta_3^{(\ell)}\left[\frac{-j}{v_s}e_{15} - \frac{\alpha^{(\ell)}}{v_s}e_{13}\right] - \beta_4^{(j)}\left[\frac{-j}{v_s}\varepsilon_{11} - \frac{\alpha^{(\ell)}}{v_s}\varepsilon_{13}\right]\Bigg\}$$

$$\frac{D_2}{\omega} = \sum_{\ell=1}^{4} B^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \Bigg\{\beta_1^{(\ell)}\left[\frac{-j}{v_s}e_{21} - \frac{\alpha^{(\ell)}}{v_s}e_{25}\right]$$
$$+ \beta_2^{(\ell)}\left[\frac{-j}{v_s}e_{26} - \frac{\alpha^{(\ell)}}{v_s}e_{24}\right] + \beta_3^{(\ell)}\left[\frac{-j}{v_s}e_{25} - \frac{\alpha^{(\ell)}}{v_s}e_{23}\right] - \beta_4^{(\ell)}\left[\frac{-\ell}{v_s}\varepsilon_{21} - \frac{\alpha^{(\ell)}}{v_s}\varepsilon_{23}\right]\Bigg\}$$

$$\frac{D_3}{\omega} = \sum_{\ell=1}^{4} B^{(\ell)} e^{-\alpha^{(\ell)}\omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \Bigg\{\beta_1^{(\ell)}\left[\frac{-j}{v_s}e_{31} - \frac{\alpha^{(\ell)}}{v_s}e_{35}\right]$$
$$+ \beta_2^{(\ell)}\left[\frac{-j}{v_s}e_{36} - \frac{\alpha^{(\ell)}}{v_s}e_{34}\right] + \beta_3^{(\ell)}\left[\frac{-j}{v_3}e_{35} - \frac{\alpha^{(\ell)}}{v_s}e_{33}\right] - \beta_4^{(\ell)}\left[\frac{-j}{v_s}\varepsilon_{13} - \frac{\alpha^{(\ell)}}{v_s}\varepsilon_{33}\right]\Bigg\}$$

Power Flow

The flow of complex mechanical power at any point in the piezoelectric medium is given in component form as follows:

$$P_i = -\frac{1}{2}\sum_{\ell=1}^{3} T_{i\ell}\dot{\mathrm{U}}_\ell^*$$

The real part of this expression represents the time average power flow at a point.

Since all fields decay exponentially in the x_3 direction there is no net flow of real power in this direction. Thus, only P_1 and P_2 need be considered.

The components of the total time-average power flow are as follows:

$$p_1^t = \int_0^\infty \mathrm{Re}\,[P_1]\,dx_3 = \mathrm{Re}\,[P_1^t]$$

$$p_2^t = \int_0^\infty \mathrm{Re}\,[P_2]\,dx_3 = \mathrm{Re}\,[P_2^t]$$

where Re $[P_{1,2}]$ means the real part of $P_{1,2}$. The final expressions for the complex mechanical power flow P_1^t and P_2^t are

$$\frac{P_1^t}{\omega} = \frac{1}{2}\sum_{\ell=1}^{4}\sum_{k=1}^{4}\frac{1}{[\alpha^{(\ell)}+\overset{*}{\alpha}^{(k)}]}\,B^{(\ell)}\overset{*}{B}^{(k)}\Big\{\overset{*}{\beta}_1^{(k)}\,[\beta_1^{(\ell)}(C_{11}-j\alpha^{(\ell)}C_{15})$$

$$+\beta_2^{(\ell)}(C_{16}-j\alpha^{(\ell)}C_{14})+\beta_3^{(\ell)}(C_{15}-j\alpha^{(\ell)}C_{13})+\beta_4^{(\ell)}(e_{11}-j\alpha^{(\ell)}e_{31})]$$

$$+\overset{*}{\beta}_2^{(k)}\,[\beta_1^{(\ell)}(C_{16}-j\alpha^{(\ell)}C_{56})+\beta_2^{(\ell)}(C_{66}-j\alpha^{(\ell)}C_{46})+\beta_3^{(\ell)}(C_{56}-j\alpha^{(\ell)}C_{36})$$

$$+\beta_4^{(\ell)}(e_{16}-j\alpha^{(\ell)}e_{36})]$$

$$+\overset{*}{\beta}_3^{(k)}\,[\beta_1^{(\ell)}(C_{15}-j\alpha^{(\ell)}C_{55})+\beta_2^{(\ell)}(C_{56}-j\alpha^{(\ell)}C_{45})+\beta_3^{(\ell)}(C_{55}-j\alpha^{(\ell)}C_{35})$$

$$+\beta_4^{(\ell)}(e_{15}-j\alpha^{(\ell)}e_{35})]\Big\}$$

$$\frac{P_2^t}{\omega} = \frac{1}{2} \sum_{\ell=1}^{4} \sum_{k=1}^{4} \frac{1}{[\alpha^{(\ell)} + \overset{*}{\alpha}^{(k)}]} B^{(\ell)} \overset{*}{B}(k) \Big\{ \overset{*}{\beta}_1^{(k)} [\beta_1^{(\ell)}(C_{16} - j\alpha^{(\ell)} C_{56})$$

$$+ \beta_2^{(\ell)}(C_{66} - j\alpha^{(\ell)} C_{46}) + \beta_3^{(\ell)}(C_{56} - j\alpha^{(\ell)} C_{36}) + \beta_4^{(\ell)}(e_{16} - j\alpha^{(\ell)} e_{36})]$$

$$+ \overset{*}{\beta}_2^{(k)} [\beta_1^{(\ell)}(C_{12} - \alpha^{(\ell)} C_{25}) + \beta_2^{(\ell)}(C_{26} - j\alpha^{(\ell)} C_{24}) + \beta_3^{(\ell)}(C_{25} - j\alpha^{(\ell)} C_{23})$$

$$+ \beta_4^{(\ell)}(e_{12} - j\alpha^{(\ell)} e_{32})]$$

$$+ \overset{*}{\beta}_3^{(k)} [\beta_1^{(\ell)}(C_{14} - j\alpha^{(\ell)} C_{45}) + \beta_2^{(\ell)}(C_{46} - j\alpha^{(\ell)} C_{44}) + \beta_3^{(\ell)}(C_{45} - j\alpha^{(\ell)} C_{34})$$

$$+ \beta_4^{(\ell)} (e_{14} - j\alpha^{(\ell)} e_{34})] \Big\} .$$

The flow of electromagnetic power (Poynting Vector) in a piezoelectric medium requires a knowledge of the magnetic field as well as the electric field. It is a common belief (although an incorrect one) that the complex Poynting vector $E \times H^*$ reduces to $\varphi\dot{D}^*$ when the electric field is approximately derivable from a scalar potential function φ. This mistaken notion is based upon the following derivation for energy flow out of a closed surface.

Maxwell's equations are as follows for fields derivable approximately from a scalar potential function (i.e. where $\vec{E} \cong -\nabla\varphi$) in a non-conducting medium.

$$\nabla \times E = \frac{-\partial B}{\partial t} \cong 0 \qquad \nabla \cdot B = 0$$

$$\nabla \times H = \frac{\partial D}{\partial t} = \dot{D} \qquad \nabla \cdot D = 0$$

Now the flow of power out of any closed surface with a surface normal element $d\vec{s}$ is

$$P = \frac{1}{2} \iint_S (E \times H^*) \cdot d\vec{s}$$

but

$$\iint_S (E \times H^*) \cdot d\vec{s} = \iiint_V \nabla \cdot (E \times H^*)\, dv$$

where the second integral is over the volume enclosed by the surface S. Using

$$\nabla \cdot (E \times H^*) = H^* \cdot \nabla \times E - E \cdot \nabla \times H^* \cong -E \cdot \nabla \times H^*$$

it follows that

$$\frac{1}{2} \iiint_V \nabla \cdot (E \times H^*)\, dv \cong -\frac{1}{2} \iiint_V E \cdot \nabla \times H^*\, dv = \frac{1}{2} \iiint_V (\nabla\varphi \cdot \dot{D}^*) \quad .$$

Furthermore

$$\nabla \cdot (\varphi\dot{D}^*) = \nabla\varphi \cdot \dot{D}^* + \varphi\nabla \cdot (\dot{D}^*) = \nabla\varphi \cdot \dot{D}^*$$

may be used to infer that

$$\frac{1}{2} \iiint_V \nabla \cdot (E \times H^*)\, dv \cong \frac{1}{2} \iiint_V \nabla \cdot (\varphi\dot{D}^*)\, dv$$

$$= \frac{1}{2} \iint_S \varphi\dot{D}^* \cdot d\vec{a} \quad .$$

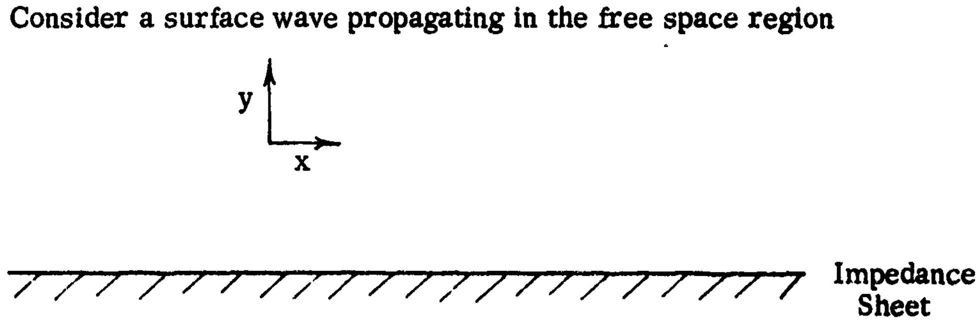
Therefore

$$P = \frac{1}{2} \iint_S (E \times H^*) \cdot d\vec{s} \cong \frac{1}{2} \iint_S \varphi\dot{D}^* \cdot d\vec{a} \quad .$$

This equation states that the surface integral over a closed surface of $E \times H^*$ is equal to that of $\varphi\dot{D}^*$ or alternately that $\nabla \cdot (E \times H^*) = \nabla \cdot (\varphi\dot{D}^*)$, consequently $E \times H^* = \varphi\dot{D}^* + \vec{A}$ where $\vec{A}$ is some vector whose divergence is zero $(\nabla \cdot A = 0)$.

It seems a natural assumption to assume that $\vec{A} = 0$ and that $E \times H^* = \varphi\dot{D}^*$. However, this is not necessarily so as an illustrative example from electromagnetic theory shows.

Consider a surface wave propagating in the free space region



A solution of the following form will exist.

$$H_z = e^{-j\beta x} \, e^{-\sqrt{\beta^2 - k_o^2}\, y}$$

$$E_x = \frac{-\sqrt{\beta^2 - k_o^2}}{j\omega\epsilon} \, Hz$$

$$E_y = \frac{\beta}{\omega\epsilon} \, H_z$$

The wave impedance of this wave is

$$Z = \frac{E_x}{H_z} = \frac{+j\sqrt{\beta^2 - k_o^2}}{\omega\epsilon}$$

Assume an impedance sheet with surface impedance jX_s. Then we find

$$\frac{\sqrt{\beta^2 - k_o^2}}{\omega\epsilon} = X_s$$

determining the value of β. If $k_o << \beta$ (low frequency limit) then we have

$$\beta \simeq \omega\epsilon \, X_s$$

and the field quantities assume the form

$$H_z \simeq e^{-j\beta x} \, e^{-\beta y} \quad ,$$

$$E_x \cong j\frac{\beta}{\omega\epsilon} H_z \quad , \quad \text{and} \quad E_y = \frac{\beta}{\omega\epsilon} H_z \quad .$$

This is the quasi-static case ($k_o << \beta$) and an approximate expression for the electric field is derivable from a scalar potential. Indeed, the scalar potential may be taken to be

$$\varphi = \frac{H_z}{\omega\epsilon} \cong \frac{e^{-j\beta x} \, e^{-\beta y}}{\omega\epsilon}$$

with the result $\vec{E} \cong -\nabla\varphi$.

For this approximation the x component of the Poynting vector (E x H*)

reduces to $N_x = E_y H_z^* = \beta/\omega\epsilon\, e^{-2\beta y}$ while

$$\varphi \dot{D}_x^* = \frac{H_z}{\omega\epsilon}\, \epsilon\, \dot{E}_x^*$$

$$= -\frac{\beta}{\omega\epsilon}\, e^{-2\beta y} \qquad ,$$

and clearly $\vec{N} \neq \varphi\dot{D}^*$.

Summing up, the electromagnetic power flow (E x H*) at any point requires a full solution of Maxwell's equations and cannot be obtained from the scalar potential method. Only the total power flow out of a closed surface can be obtained from the scalar potential method since

$$\iint_S E \times H^* \cdot d\vec{a} = \iint_S \varphi\dot{D}^* \cdot d\vec{a}$$

where S is closed. A more thorough investigation into the electromagnetic power flow will be taken up during the transducer study part of the study. For now it can be said that the electromagnetic power flow is expected to be much smaller than the mechanical power flow based on computer runs where $\varphi\dot{D}^*$ was used as an order of magnitude estimate of the Poynting vector.

2. Surface Waves on Non-Piezoelectric (Pure Elastic) Media

Surface wave propagation on a free surface of a non-piezoelectric elastic medium can be accounted for by appropriate modifications of the foregoing analysis. In this case the piezoelectric constants e_{ijk} are identically zero and the electric field and mechanical displacements are decoupled. Consequently, the fourth order matrix appearing in equation (6) reduces to the third order matrix obtained by deleting the last row and column. Subject to this modification equation (7) reduces to a sixth order equation in α which in general will have three roots with positive real parts. The boundary conditions at the free surface are given by equation (8) with the coefficients $e'_{k3j} \equiv 0$. Inasmuch as there is no coupling to an electric field the additional boundary conditions applicable thereto are unnecessary.

The relative amplitudes of the component of displacement $\beta_k^{(\ell)}$, $k = 1, 2, 3$ are evaluated for each $\alpha^{(\ell)}$ with positive real part in a manner identical to that used in the general piezoelectric case. Upon evaluation of these quantities the boundary conditions are invoked and the equations (12-14) result with the piezoelectric constants equal to zero and the sums only over the indices 1, 2, and 3. The characteristic equation for the determination of surface wave velocities again obtains from the condition that the determinant of the matrix coefficients associated with the linear system (12-14) vanishes.

The stresses, strains, power flow, etc. may be calculated as before by using the appropriate evaluations of $\beta_k^{(\ell)}$, $k = 1, 2, 3$, $B^{(k)}$, $k = 1, 2, 3$ and setting the β_4's, $A^{(4)}$, and $e_{\ell p}$ equal to zero in the equations for these quantities given previously.

3. Degenerate Cases – Piezoelectric Medium

The modes of surface wave propagation described as degenerate cases arise when the four coupled partial differential equations (4) which govern the mechanical components u_1, u_2, and u_3 and the electric field (via a scalar potential φ) reduce to two independent sets of coupled equations, assuming traveling wave motions independent of the coordinate normal to the sagital plane.

With the coordinate system chosen assuming no variation in the x_2 direction, the equations of motion (4) for the displacement components and potential may be written in the operator form (assuming $e^{j\omega t}$ time dependence)

$$L_{11}U_1 + L_{12}U_2 + L_{13}U_3 + L_{14}\varphi = 0$$

$$L_{21}U_1 + L_{22}U_2 + L_{23}U_3 + L_{24}\varphi = 0$$

$$L_{31}U_1 + L_{32}U_2 + L_{33}U_3 + L_{34}\varphi = 0$$

$$L_{41}U_1 + L_{42}U_2 + L_{43}U_3 + L_{44}\varphi = 0 \quad ,$$

where

$$L_{11} = C_{55}\frac{\partial^2}{\partial x_3^2} + 2C_{15}\frac{\partial^2}{\partial x_3 \partial x_1} + C_{11}\frac{\partial^2}{\partial x_1^2} + \omega^2\rho$$

$$L_{12} = L_{21} = C_{45}\frac{\partial^2}{\partial x_3^2} + (C_{14} + C_{56})\frac{\partial^2}{\partial x_3 \partial x_1} + C_{16}\frac{\partial^2}{\partial x_1^2}$$

$$L_{13} = L_{31} = C_{35}\frac{\partial^2}{\partial x_3^2} + (C_{13} + C_{55})\frac{\partial^2}{\partial x_3 \partial x_1} + C_{15}\frac{\partial^2}{\partial x_1^2}$$

$$L_{14} = L_{41} = e_{35}\frac{\partial^2}{\partial x_3^2} + (e_{15} + e_{31})\frac{\partial^2}{\partial x_3 \partial x_1} + e_{11}\frac{\partial^2}{\partial x_1^2}$$

$$L_{22} = C_{44}\frac{\partial^2}{\partial x_3^2} + 2C_{46}\frac{\partial^2}{\partial x_3 \partial x_1} + C_{66}\frac{\partial^2}{\partial x_1^2} + \omega^2\rho$$

$$L_{23} = L_{32} = C_{34}\frac{\partial^2}{\partial x_3^2} + (C_{36} + C_{45})\frac{\partial^2}{\partial x_3 \partial x_1} + C_{56}\frac{\partial^2}{\partial x_1^2}$$

$$L_{24} = L_{42} = e_{34}\frac{\partial^2}{\partial x_3^2} + (e_{14} + e_{36})\frac{\partial^2}{\partial x_3 \partial x_1} + e_{16}\frac{\partial^2}{\partial x_1^2}$$

$$L_{33} = C_{33}\frac{\partial^2}{\partial x_3^2} + 2C_{35}\frac{\partial^2}{\partial x_3 \partial x_1} + C_{55}\frac{\partial^2}{\partial x_1^2} + \omega^2\rho$$

$$L_{34} = L_{43} = e_{33}\frac{\partial^2}{\partial x_3^2} + (e_{13} + e_{35})\frac{\partial^2}{\partial x_3 \partial x_1} + e_{15}\frac{\partial^2}{\partial x_1^2}$$

$$L_{44} = -\varepsilon_{33}\frac{\partial^2}{\partial x_3^2} - 2\varepsilon_{13}\frac{\partial^2}{\partial x_3 \partial x_1} - \varepsilon_{11}\frac{\partial^2}{\partial x_1^2}$$

The elastic c_{ij}, piezoelectric ϵ_{ij}, and dielectric $\mathcal{E}_{ij}$ constants refer to the transformed (from the crystal coordinate system) quantities and are represented in terms of the abbreviated double subscript notation.

If the elastic and piezoelectric constants are such that the operator matrix $[L_{ij}]_{i,j=1,2,3,4}$ is appropriately sparse, the equations (1) decouple and the possibility of degenerate cases is encountered.

For example, in the case reported by Bleustein[4] the elastic and piezoelectric constants are such that $L_{12} = L_{14} = L_{23} = L_{34} \equiv 0$ and the equations of motion decouple into two independent systems; one system governing u_2 and φ and the other characterizing the behaviors of u_1 and u_3. This is an example of one of the two general degenerate cases which has been found to exist for a number of crystals on particular cuts and directions of propagation.

A second degenerate case which also has been reported and which appears with some regularity is manifest by the conditions $L_{12} = L_{23} = L_{24} \equiv 0$. In this case the equations of motion decouple into a coupled system of partial differential equations for the displacement components u_1, u_3 and the potential φ, and a single partial differential equation for the displacement component u_2. This particular degenerate case has been studied extensively for surface wave propagation on the basal plane of hexagonal crystals[7] and it has been shown that a surface wave solution with the displacement component u_2 alone cannot exist. It should be noted that the latter observation carries over to the general case, independent of the crystal class, surface cut, and direction of propagation.

There are other degenerate cases that can be considered. For example,

$$L_{12} = L_{13} = L_{14} \equiv 0$$

or

$$L_{31} = L_{32} = L_{34} \equiv 0 \quad .$$

These cases were considered in the analysis leading to the present study but numerical examples of these cases have not been found.

The conditions $L_{41} = L_{42} = L_{43} \equiv 0$ lead to a complete decoupling of the electric and mechanical fields and has not been found to occur for surface wave propagation on specific surfaces of any piezoelectric crystal considered thus far.

The occurrence of degenerate cases can also be described in terms of the linear equations for the relative amplitudes, β_ℓ, $\ell = 1, 2, 3, 4$, of the mechanical displacement and electric potential. The determinant of the coefficients of this system of equations is given by equation (6), wherein the elements of the determinant correspond to evaluations of the operators L_{ij} with

$$\frac{\partial}{\partial x_1} = -j\frac{\omega}{v_s} \quad \text{and} \quad \frac{\partial}{\partial x_3} = -\frac{\alpha\omega}{v_s} \quad .$$

For the sake of the following discussions let the linear system of equations for the determination of the β's be denoted

$$\sum_{\ell=1}^{4} A_{i\ell}\beta_\ell = 0 \quad , \qquad i = 1, 2, 3, 4 \quad . \tag{18}$$

The possible combinations of the elastic and piezoelectric constants which caused the equations of motion to decouple lead to the decoupling of the linear equations in the exact same fashion. Inasmuch as the linear equations (18) are employed in the numerical analyses, the various cases of decoupling are discussed again in more detail below.

The following degenerate cases have been found to exist and are accounted for in the computer program which implements the numerical analysis. They are denoted by representing the matrix $\hat{A} = \{A_{i\ell}\}_{i,\ell=1,2,3,4}$ with its appropriate zeros displayed, viz.,

$$\text{Case (1)} \quad \begin{vmatrix} A_{11} & 0 & A_{13} & A_{14} \\ 0 & A_{22} & 0 & 0 \\ A_{13} & 0 & A_{33} & A_{34} \\ A_{14} & 0 & A_{34} & A_{44} \end{vmatrix} \qquad \text{Case (2)} \quad \begin{vmatrix} A_{11} & 0 & A_{13} & 0 \\ 0 & A_{22} & 0 & A_{24} \\ A_{13} & 0 & A_{33} & 0 \\ 0 & A_{24} & 0 & A_{44} \end{vmatrix}$$

In Case (1) we note that β_2 decouples from β_1, β_3, and β_4. The determinant of $\hat{A}$ is zero if $A_{22} = 0$ (as a function of α) or if the determinant

$$A_1 = \begin{vmatrix} A_{11} & A_{13} & A_{14} \\ A_{13} & A_{33} & A_{34} \\ A_{14} & A_{34} & A_{44} \end{vmatrix} = 0 \quad .$$

The condition $A_{22} = 0$ leads to a quadratic equation in α. If $A_{22} = 0$ the system of equations yields a solution $\beta_1 = \beta_3 = \beta_4 = 0$ while β_2 can be chosen as an arbitrary constant.

If $A_1 = 0$, giving a sixth order equation in α, the system of equations requires a solution $\beta_2 = 0$ while either β_1, β_3, or β_4 may be chosen arbitrarily and the remaining two β's calculated from any two of the three equations not involving A_{22}.

In case (2) we note that β_1 and β_3 decouple from β_2 and β_4. The determinant of $\hat{A}$ goes to zero if the determinant

$$A_2 = \begin{vmatrix} A_{11} & A_{13} \\ A_{13} & A_{33} \end{vmatrix} = 0$$

or the determinant

$$A_3 = \begin{vmatrix} A_{22} & A_{24} \\ A_{24} & A_{44} \end{vmatrix} = 0 \quad .$$

Both the equations $A_2 = 0$ and $A_3 = 0$ lead to quartic equations in α. If $A_2 = 0$ the system yields the solution $\beta_2 = \beta_4 = 0$ while β_1 or β_3 may be arbitrarily chosen and the remaining β calculated from either the first or third equation of the system. If $A_3 = 0$ the system yields the solution $\beta_1 = \beta_3 = 0$ while β_2 or β_4 may be arbitrarily chosen and the remaining β calculated from either the second or fourth equation of the system.

Let the coefficients of the amplitudes $B^{(l)}$ (equations (12)-(16)) be considered elements of the matrix $\hat{L}$. In case (1) $\hat{L}$ takes the form

$$\begin{vmatrix} 0 & L_{12} & L_{13} & L_{14} \\ L_{21} & 0 & 0 & 0 \\ 0 & L_{32} & L_{33} & L_{34} \\ 0 & L_{42} & L_{43} & L_{44} \end{vmatrix} .$$

If the determinant

$$L_1 = \begin{vmatrix} L_{12} & L_{13} & L_{14} \\ L_{32} & L_{33} & L_{34} \\ L_{42} & L_{43} & L_{44} \end{vmatrix} = 0$$

(considered as a function of the velocity v_s) then the following solution is found: $B^{(1)} = 0$ while $B^{(2)}$, $B^{(3)}$, or $B^{(4)}$ can be chosen arbitrarily and the remaining B's calculated from any two of the three equations not involving L_{21}. This situation corresponds to a wave with displacement components U_1, U_3 and potential φ and U_2 is identically zero.

If L_{21} is zero we would be led to a solution where U_1, U_3, and φ are zero while only U_2 would be present in the wave. However, it can be shown that L_{21} can not be equal to zero and therefore such a mode does not exist.*

In case (2) $\hat{L}$ takes the form

$$\begin{vmatrix} L_{11} & L_{12} & 0 & 0 \\ 0 & 0 & L_{23} & L_{24} \\ L_{31} & L_{32} & 0 & 0 \\ 0 & 0 & L_{43} & L_{44} \end{vmatrix} .$$

If the determinant

$$L_2 = \begin{vmatrix} L_{11} & L_{12} \\ L_{31} & L_{32} \end{vmatrix} = 0$$

then the following solution is found: $B^{(3)} = B^{(4)} = 0$ while $B^{(1)}$ or $B^{(2)}$ can be arbitrarily chosen and the remaining B can be calculated from the first or third equation of the system (equations (12) or (14)). This situation corresponds to a wave with displacement components U_1 and U_3 while U_2 and φ are identically zero.

If the determinant

$$L_3 = \begin{vmatrix} L_{23} & L_{24} \\ L_{43} & L_{44} \end{vmatrix} = 0$$

*This case was considered in detail for surface wave propagation in the basal plane of hexagonal piezoelectric crystals by Tseng and White[7].

then the following solution is found: $B^{(1)} = B^{(2)} = 0$ while $B^{(3)}$ or $B^{(4)}$ can be arbitrarily chosen and the remaining B can be calculated from the second or fourth equation of the system (equations (13) or (15)). This situation corresponds to a wave with displacement component U_2 and potential φ while U_1 and U_3 are identically zero.

It should be noted that in paragraph 1.1 it was stated that four α's with positive real parts can be found when in the range of velocities below the slowest bulk wave velocity in the direction of propagation being considered. However, in the degenerate cases surface waves may exist when there are less than four such α's provided the appropriate α's have positive real parts. For example, in case (1) only three roots of A_1 must have positive real parts for the existence of a surface wave. The roots of $A_{22} = 0$ are not required to assume any particular form. That is, both roots corresponding to $A_{22} = 0$ may be purely imaginary but if three of the six roots corresponding to $A_1 = 0$ have positive real parts a surface wave may still exist. Similarly, for case (2) it is possible to have a solution with only two α's with positive real parts provided that both of the α's come from the same equation (i.e. both come from $A_2 = 0$ or both from $A_3 = 0$). An example of the latter situation has been reported in the literature[4] and corresponds to a wave with displacement component U_2 and potential φ. It may be noted that the (necessarily) degenerate waveforms that occur with higher velocities than the bulk waves alluded to above appear to be the non-attenuated limits of leaky or pseudo-waves and have been described for non-piezoelectric crystals by Lim and Farnell.[5]

One further case of a peculiar nature will be mentioned here although it does not fit into the category of degenerate cases. On a surface of a hexagonal crystal, solutions of the algebraic equations for the decay coefficients $\alpha^{(k)}$ exist which cause the minors of the last row and column of $\hat{A}$ to vanish. Since all the minors of the elements of $\hat{A}$ are not new the rank of the determinant is not reduced but the component β_4 is forced to be zero. Thus only β_1, β_2, and β_3 exist for such an α. The total wave however is not an example of a degenerate case since the other α's are needed for a surface wave solution. This behavior is a manifestation of the fact that one of the general bulk wave solutions is decoupled from the electric field for all directions of propagation in a piezoelectric hexagonal crystal.

4. Degenerate Cases — Non-piezoelectric (Pure Elastic) Medium

For surface wave propagation in a pure elastic medium the system of equations discussed in the proceding paragraph takes the form

$$\sum_{\ell=1}^{3} A_{i\ell}\beta_{\ell} = 0 \qquad i = 1, 2, 3 \tag{19}$$

where the $A_{i\ell}$ are the same as in the piezoelectric case but involve only the elastic constants (the piezoelectric constants are zero and the dielectric constants do not enter the problem). The equation for α is now sixth order. Generally three roots with positive real parts are required for a surface wave solution.

Only one degenerate case occurs for a pure elastic medium. In this case the $\hat{A}$ matrix assumes the form

$$\begin{pmatrix} A_{11} & 0 & A_{13} \\ 0 & A_{22} & 0 \\ A_{13} & 0 & A_{33} \end{pmatrix} .$$

As before the condition $A_{22} = 0$ leads to a quadratic equation in α but this case is of no interest inasmuch as it would lead to a solution with U_2 the only component of displacement and this is impossible for the same reason as in the piezoelectric case (viz. L_{21} cannot equal zero for the form of the decay coefficient α required for a surface wave solution). If

$$A_1 = \begin{vmatrix} A_{11} & A_{13} \\ A_{13} & A_{33} \end{vmatrix} = 0$$

we obtain the solution $\beta_2 = 0$, β_1, $\beta_3 \neq 0$, and either β_1 or β_3 can be chosen arbitrarily with the other β being calculated from one of the two remaining equations of the system. In this case the $\hat{L}$ matrix assumes the form

$$\begin{vmatrix} 0 & L_{12} & L_{13} \\ L_{21} & 0 & 0 \\ 0 & L_{32} & L_{33} \end{vmatrix} ,$$

and the boundary conditions can be satisfied if

$$L_1 = \begin{vmatrix} L_{12} & L_{13} \\ L_{32} & L_{33} \end{vmatrix} = 0 \quad .$$

The above form of the linear system requires that $B^{(1)} = 0$ while either $B^{(2)}$ or $B^{(3)}$ can be chosen arbitrarily and the remaining amplitude calculated from either of the equations

$$L_{12}B^{(2)} + L_{13}B^{(3)} = 0 \quad ,$$

or

$$L_{32}B^{(2)} + L_{33}B^{(3)} = 0 \quad .$$

This solution corresponds to a wave with displacement components U_1 and U_3 while U_2 is identically zero, i.e., a Rayleigh wave.

5. Surface Wave Propagation in an Isotropic Elastic, Perfectly Conducting Film on a Piezoelectric Substrate

An additional problem that has been considered is that of a finite thickness layer of isotropic elastic conductor on a piezoelectric substrate. When the displacement component waveforms

$$U_i = \beta_i \, e^{-\alpha\omega x_3/v_s} \, e^{j\omega(t - x_1/v_s)} \quad , \qquad i = 1, 2, 3 \ ,$$

are substituted into the equations of motion for an isotropic elastic medium, a linear system of equations for the relative amplitudes β_1, β_2, β_3, of the displacement components is obtained. The determinant of the system, set equal to zero, yields the equation for the determination of the exponents α, namely

$$\det \begin{pmatrix} \mu\alpha^2-(2\mu+\lambda)+\rho v_s^2 & 0 & j\alpha[\lambda+\mu] \\ 0 & \mu\alpha^2-\mu+\rho v_s^2 & 0 \\ j\alpha[\lambda+\mu] & 0 & (2\mu+\lambda)\alpha^2-\mu+\rho v_s^2 \end{pmatrix} = 0 \qquad (20)$$

where λ, μ are the Lame constants of the medium. The polynomial form of (18) is of order six and, inasmuch as the medium is of finite thickness, the solution corresponding to all six roots is needed to satisfy the boundary conditions.

The assumed forms of the solutions in the piezoelectric medium with those employed in Section II.1, namely,

where λ, μ are the Lame constants of the medium. The polynomial form of (18) is of order six and inasmuch as the medium is of finite thickness, the solutions corresponding to all six roots are needed to satisfy the boundary conditions.

The assumed forms of the solutions in the piezoelectric medium are identical with those employed in Section II.1, namely,

$$U_i^p = \sum_{\ell=1}^{4} A^{(\ell)} \beta_{pi}^{(\ell)} e^{-\alpha_p^{(\ell)} \omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \qquad (21)$$

$$, \quad i = 1,2,3 \ ,$$

$$\varphi^p = \sum_{\ell=1}^{4} A^{(\ell)} \beta_{pi}^{(\ell)} e^{-\alpha_p^{(\ell)} \omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \ . \qquad (22)$$

In the elastic conductor the total displacements assume the form

$$U_i^c = \sum_{k=1}^{6} B^{(k)} \beta_{ci}^{(k)} e^{-\alpha_c^{(k)} \omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \ , \quad i = 1,2,3 \ , \qquad (23)$$

and the potential function is identically zero. Thus there are 10 unknown amplitude coefficients $A^{(\ell)}$, $B^{(k)}$, $\ell = 1, 2, 3, 4$, $k = 1, \ldots, 6$ to be determined.

The boundary conditions applicable in this problem are as follows:

Continuity of displacement at $x_3 = 0$	$U_i^p(x_1,0) = U_i^c(x_1,0)$	$i = 1,2,3$
Continuity of stress components at $x_3 = 0$	$T_{3j}^p(x_1,0) = T_{3j}^c(x_1,0)$	$j = 1,2,3$
Vanishing of stress components at the free surface at $x_3 = -h$	$T_{3j}^c(x_1,-h) = 0$	$j = 1,2,3$
Vanishing of potential at $x_3 = 0$	$\varphi^p(x_1,0) = 0$	

Applying these ten conditions to the above solutions yields a system of ten homogeneous algebraic equations in the ten unknown amplitude coefficients $A^{(i)}$, $B^{(j)}$. From this point the solution for surface wave velocities and field distributions proceeds as before except that there are now ten equations instead of (4). The explicit form of the determinant of the system stemming from the boundary conditions is given in Appendix I.

6. Surface Wave Propagation at the Interface between a Piezoelectric Substrate and a Semi-Infinite Fluid Medium

The physical problem considered in this section is that of a surface wave propagating along the interface between a piezoelectric crystal and a semi-infinite fluid. Again a rectangular coordinate system is chosen with the x_3 axis normal to the crystal surface and the x_1 axis in the direction of propagation as in the preceding problems. The fields in the crystal and fluid media are assumed to be independent of the x_2 direction. Arbitrary orientations of the crystal surface with respect to the crystal axes are handled by means of an Euler Transformation as before and the differential equations in the crystal medium are the same.

The elastic properties of the fluid medium are described in terms of a single elastic constant λ (modulus of compression); the effect of viscosity is ignored. Consequently, the differential equations in the fluid are

$$U_{1,11} + U_{3,13} = \rho_f \ddot{U}_1/\lambda$$

$$U_{1,13} + U_{3,33} = \rho_f \ddot{U}_3/\lambda \qquad , x_3 < 0 , \qquad (24)$$

$$\nabla^2 \varphi = 0 \qquad ,$$

wherein the derivatives with respect to x_2 are taken to be zero in keeping with the uniformity of the field in this direction and ρ_f is the density of the fluid medium.

In the crystal medium ($x_3 > 0$) traveling wave solutions of the form

$$U_i = \beta_i \, e^{-\alpha_c \omega x_3/v_s} \, e^{j\omega(t-x_1/v_s)} \quad , \qquad i = 1,2,3 \quad ,$$
$$\varphi = \beta_4 \, e^{-\alpha_c \omega x_3/v_s} \, e^{j\omega(t-x_1/v_s)} \tag{25}$$

are sought and all analytical considerations pertaining to the crystal medium are identical with those discussed in Sections II.1 and II.2.

In the fluid ($x_3 < 0$) the particle displacements and electric potential are decoupled and are assumed to have the forms

$$U_1 = \gamma_1 \, e^{-\alpha_f \omega x_3/v_s} \, e^{j\omega(t-x_1/v_s)} \quad ,$$
$$U_3 = \gamma_3 \, e^{-\alpha_f \omega x_3/v_s} \, e^{j\omega(t-x_1/v_s)} \quad , \tag{26}$$
$$\varphi = C e^{\omega x_3/v_s} \, e^{j\omega(t-x_1/v_s)} \quad .$$

Substitution of these displacement and potential waveforms into the differential equations (24) leads to the following equation for α_f in terms of the velocity v_s, namely

$$\det \begin{pmatrix} \rho_f v_s^2 - \lambda & j\alpha_f \lambda \\ j\alpha_f \lambda & \lambda\alpha_f^2 + \rho_f v_s^2 \end{pmatrix} = 0 \quad , \tag{27}$$

from which it follows that

$$\alpha_f = \pm\sqrt{\frac{\lambda - \rho_f v_s^2}{\lambda}} \quad . \tag{28}$$

The relative amplitudes γ_1 and γ_3 are obtained from the homogeneous linear system of equations whose coefficient matrix appears in equation (7), viz.,

$$\gamma_1 = \frac{j\alpha_f \lambda}{\lambda - \rho_f v_s^2}\,\gamma_3 \quad . \tag{29}$$

The sign of α_f in equation (28) is determined by the condition that the surface wave is bounded as $x_1 \rightarrow +\infty$.

The total field in the crystal is expressed as a linear combination of the "partial" fields associated with the allowed values of α_c, namely

$$\begin{aligned} U_i &= \sum_{\ell=1}^{4} B^{(\ell)} \beta_i^{(\ell)} e^{-\alpha_c^{(\ell)} \omega x_3/v_s} e^{j\omega(t - x_1/v_s)} \\ \varphi &= \sum_{\ell=1}^{4} B^{(\ell)} \beta_4^{(\ell)} e^{-\alpha_c^{(\ell)} \omega x_3/v_s} e^{j\omega(t - x_1/v_s)} \quad . \end{aligned} \tag{30}$$

In the fluid medium the total displacements and potential are given by equation (26).

The amplitude coefficients $B^{(1)}$, $B^{(2)}$, $B^{(3)}$, $B^{(4)}$, C and γ_3 are determined by the boundary conditions:

U_3 continuous at $x_3 = 0$

φ continuous at $x_3 = 0$

D_3 continuous at $x_3 = 0$ (electric displacement)

T_{33} continuous at $x_3 = 0$

$T_{31} = 0$ at $x_3 = 0$

$T_{32} = 0$ at $x_3 = 0$

The components of the electric displacement vector $\vec{D}$ in the crystal are given by

$$D_i = e_{ik\ell} U_{k,\ell} - \varepsilon_{ik} \varphi_{,k} \quad , \qquad i = 1, 2, 3 \quad ;$$

in the fluid medium, $\vec{D} = -\epsilon_f \nabla\varphi$, where ϵ_f is the dielectric constant of the fluid.

Application of the boundary conditions to the total field solutions leads to a set of six homogeneous equations in the unknown amplitudes $B^{(\ell)}$ $\ell = 1, 2, 3, 4$, C and γ_3. The coefficient matrix of this system of equations, $M = [M_{ik}]_{j,k=1,\ldots,6}$, assumes the form*

*The explicit equations for the elements of M are given in Appendix II.

$$M = \left(\begin{array}{ccccc|c} M_{11} & M_{12} & M_{13} & M_{14} & 0 & M_{16} \\ \hline M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & 0 \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} & 0 \\ M_{41} & M_{42} & M_{43} & M_{44} & 0 & M_{46} \\ M_{51} & M_{52} & M_{53} & M_{54} & 0 & 0 \\ M_{61} & M_{62} & M_{63} & M_{64} & 0 & 0 \end{array}\right) . \qquad (31)$$

The characteristic equation for the surface wave velocity v_s is obtained from the condition for the existence of a non-trivial solution of the aforementioned homogeneous system, namely, $\det M = 0$.

The complex solutions to the equation $\det M = 0$ can be obtained in a straightforward fashion using the iterative scheme described in the programming sections and such a procedure has been built into the computer program for the fluid problems. However such a procedure is time consuming and does not fully exploit prior work on piezoelectric surface wave propagation problems. Inasmuch as a computer program exists to calculate piezoelectric surface wave characteristics under a variety of conditions, in particular, when the surface of the crystal is traction free and the adjacent half space is a massless, non elastic dielectric, it is desirable to make maximum use of this program. This can be done for a wide range of parameter values for the fluid medium by making use of a perturbation scheme for obtaining the roots of $\det M = 0$ which utilizes the results of this program and requires, in addition, only the evaluation of a few determinants at specified velocities.

The implementation of the perturbation procedure is based on the fact that a particular sub-matrix N of the matrix M (as indicated by the partitioned matrix in equation (31)) is the coefficient matrix of the linear system corresponding to the boundary conditions at the surface of a crystal in contact with a medium whose elastic properties are those of a vacuum but whose dielectric constant is that of the fluid medium. Consequently, the equation $\det N = 0$ is the characteristic equation for the velocity of the surface waves

which can propagate in this configuration, and the roots of this equation can be found using the computer program for the first problem with $\omega h = \infty$ and the dielectric constant of a vacuum ϵ_o replaced by the dielectric constant of the fluid (ϵ_f).

The perturbation procedure is based on the assumption that the complex velocity which satisfies $\det M = 0$ corresponds to a small perturbation on the real velocity solution of $\det N = 0$, i.e. that the mechanical loading of the substrate by the fluid medium is quite small.

Formally the perturbation scheme is derived as follows. Let v_{s_o} be the velocity such that $\det N(v_{s_o}) = 0$ and assume that there exists a complex perturbation Δv_s such that the $\det M(v_s) = \det M(v_{s_o} + \Delta v_s) = 0$ and $|\Delta v_s / v_{s_o}| \ll 1$. For $|\Delta v_s / v_{s_o}| \ll 1$

$$\det M(v_s) = \det M(v_{s_o}) + \frac{d}{dv_s}\left[\det M(v_s)\right]\Big|_{v_s = v_{s_o}} \cdot \Delta v_s + O[(\Delta v_s)^2] = 0 \quad , \tag{32}$$

whereupon neglecting terms $O[(\Delta v_s)^2]$ yields

$$\Delta v_s = - \frac{\det M(v_{s_o})}{\frac{d}{dv_s}(\det M(v_{s_o}))} \quad . \tag{33}$$

Expanding $\det M(v_{s_o})$ about the last column (which has only two elements) gives

$$\Delta v_s = \frac{M_{46}K}{M_{16}(\det N)' - M_{46}K' - M'_{46}K} \tag{34}$$

where the quantity K in (34) is the minor of the element M_{46} in the matrix M, the primes denote differentiation with respect to v_s, and all quantities are evaluated at v_{s_o}. In obtaining (34) explicit use has been made of the fact that $\det N(v_{s_o}) = 0$.

The derivatives of the determinants employed in the perturbation procedure are calculated numerically. The derivative of the matrix element M_{46} was obtained analytically.

In both methods of obtaining the complex velocity v_s the functions involved (matrix elements and minors of the matrix M) contain α_f as an independent variable which in turn is a dependent variable with argument v_s. Equation (28) shows that there is an ambiguity in the sign of α_f. The resolution of this ambiguity leads to the particular character of the piezoelectric surface wave.

The variation of the surface wave in the direction of propagation is assumed to be bounded in the positive ($x_1 \to +\infty$) direction of propagation. This assumption imposes the requirement that Im $[v_s] \geq 0$. Consequently, the sign of α_f must be chosen such that this condition is satisfied.

Since λ and ρ_f are positive real it can be shown that

$$\mathrm{Re}\left[-\frac{\alpha_f^{(\pm)}}{v_s}\right] \gtrless 0 \quad ,$$

where $\alpha_f^{(\pm)}$ denotes to the values of α_f from equation (28) corresponding to the positive and negative signs of the radical. Consequently, if $\alpha_f^{(-)}$ is required to obtain a root v_s such that Im $[v_s] \geq 0$ the corresponding surface wave is of the leaky type. On the other hand, if $\alpha_f^{(+)}$ is required to obtain a solution of the determental equation the surface wave is evanescent in character. In all numerical cases considered the surface wave was a leaky wave.

In the program described in the following section, two values of input velocity are required depending on the program option used. If the perturbation scheme is used,a very accurate value (at least 6 place accuracy) of velocity must be input. This value is to be computed from the existing surface wave program wherein the dielectric constant of the fluid medium is substituted for that of free space (outside the crystal medium). On the other hand, if the root finding scheme is employed only a reasonable estimate of the complex velocity is required.

A final word of caution is in order regarding the use of the computer program. In checking out the various options available with the program, it was found that if the leaky wave velocity is a small perturbation on the surface wave velocity in the absence of the fluid medium, then the use of the perturbation scheme led to more reliable results than the root finding option. On the other hand, if the fluid medium significantly loaded the substrate material, the root

finding scheme gave good results whereas the perturbation scheme (as would be expected) gave erratic results in some cases. The former differences stem from the fact that the change in the velocity due to the air loading is on the order of the errors incurred in the root finding scheme while the latter are due to the approximations inherent to the perturbation procedure.

7. Surface Wave Propagation in an Isotropic Elastic Film on a Piezoelectric Substrate

This section gives a brief description of the theoretical analysis of surface wave propagation on a semi-infinite piezoelectric substrate with a contiguous isotropic dielectric-elastic layer, as shown in Figure 3.

The substrate is assumed to be a completely general piezoelectric (or non-piezoelectric) crystal medium with arbitrary surface normal direction relative to the crystal axes of the medium. The material layer adjacent to the substrate is assumed to be a general isotropic elastic medium with isotropic dielectric properties. Only pure modes of propagation are considered, that is, leaky surface waves or evanescent (or cut off) modes of propagation have not been accounted for in the computer program.

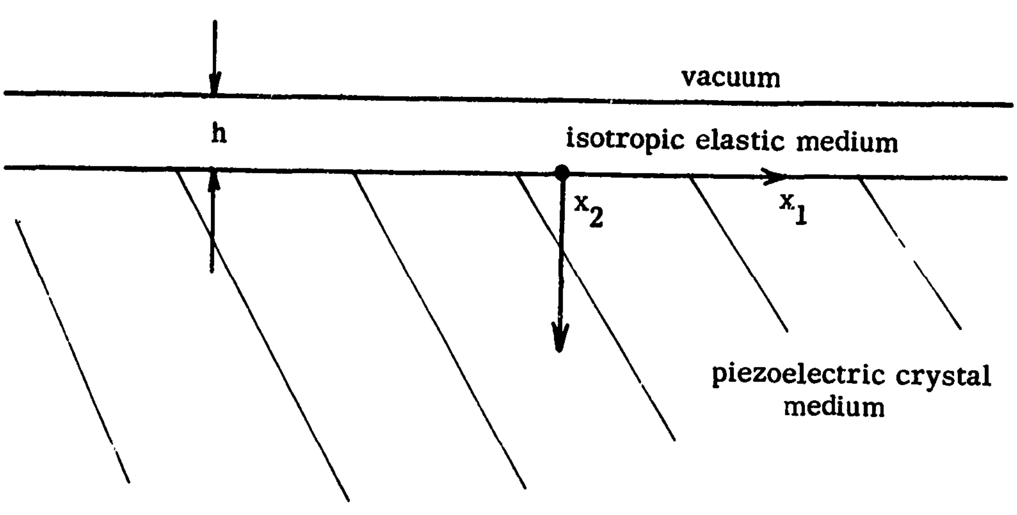


Figure 3. Semi-infinite Piezoelectric Substrate with a Contiguous Isotropic Elastic Layer.

The coordinate system employed in the analysis is illustrated in Figure 3. The piezoelectric crystal medium occupies the region $x_3 > 0$ and the direction of propagation is assumed to be in the x_1 direction. The fields in both the crystal and layer are assumed to be independent of the x_2 coordinate. Arbitrary orientations of the crystal surface with respect to the crystal axes are considered as before by an Euler Transformation. Inasmuch as the dielectric layer is isotropic, both from an elastic and an electromagnetic point of view, the quantities characterizing the medium are invarient under coordinate transformations.

The analysis pertaining to the crystal or "substrate" medium is identical to that described in Sections II.1 and II.2.

The elastic properties of the dielectric layer are described in terms of two elastic constants, λ_d, the modulus of compression or Lame's constant, and μ_d the shear modulus. Inasmuch as the layer is non-piezoelectric the differential equations for the mechanical displacements and electric potential decouple and assume the form

$$\mu_d \nabla^2 \vec{U} + (\lambda_d + \mu_d) \nabla (\nabla \cdot \vec{U}) = \rho_d \ddot{\vec{U}} \quad , \qquad -h < x_3 < 0 \quad , \tag{35}$$

and

$$\nabla^2 \varphi = 0 \quad , \tag{36}$$

where $\vec{U} = (U_1, U_2, U_3)$ and ρ_d is the density of the dielectric medium.

In the dielectric medium the assumed displacement waveforms may be expressed as

$$U_i = \beta_{d_i} e^{-\alpha_d \omega x_3 / v_s} e^{j\omega(t - x_1/v_s)} \quad , \qquad i = 1, 2, 3 \quad . \tag{37}$$

Substitution of these waveforms into the differential equations (35) yields a linear system of homogeneous equations in the unknowns β_{d1}, β_{d2} and β_{d3}. The existence of non-trivial solutions requires that the determinant of the coefficients of the system vanish thus leading to the following classical equations for normalized transverse wave numbers α_d in terms of the velocity v_s, namely,

$$\alpha_d^{(1,2)} = \alpha_d^{\pm}\,(\text{shear}) = \pm\sqrt{\frac{\mu_d - \rho_d v_s^2}{\mu_d}} \tag{38}$$

$$\alpha_d^{(3,4)} = \alpha_d^{\pm}\,(\text{compressional}) = \pm\sqrt{\frac{\lambda_d + 2\mu_d - \rho_d v_s^2}{\lambda_d + 2\mu_d}} \tag{39}$$

and

$$\alpha_d^{(5,6)} = \alpha_d^{(1,2)} \quad ,$$

since the shear mode is degenerate for an isotropic elastic medium.*

Finally, in the dielectric medium, the two independent solutions of (36), assuming $e^{-j\omega x_1/v_s}$ variation in the x_1 direction, are

$$\varphi_{1,2} = C_{1,2}\; e^{\pm \omega x_3/v_s}\; e^{j\omega(t - x_1/v_s)} \quad . \tag{40}$$

In the "free space" region $-\infty < x_3 < -h$ there are no mechanical displacements but a potential function exists and must satisfy the differential equation (36). In addition, the requirement that the potential be bounded as $x \to -\infty$ is imposed. Therefore, the form of the potential is taken to be

$$\varphi_s = C_3\; e^{\omega x_3/v_3}\; e^{j\omega(t - x_1/v_s)} \quad . \tag{41}$$

The total displacement and potential waveforms in the piezoelectric crystal are expressed as linear combinations of the "partial" fields associated with the allowed values of α_c. Denoting these values $\alpha_c^{(\ell)}$, $\ell = 1, 2, 3, 4$, the displacement components and potential may be expressed as

---

*The term degenerate is used here in the sense that in the characteristic equation for the normalized transverse wave numbers α (for example, corresponding to the determental equation (3) for the general piezoelectric crystal) the roots $\alpha^{(1)}$ and $\alpha^{(2)}$ are double roots and hence two linearly independent eigenvectors can be defined for each distinct value.

$$U_i^{(c)} = \sum_{\ell=1}^{4} B^{(\ell)} \beta_{Ci}^{(\ell)} e^{-\alpha_c^{(\ell)} \omega x_3/v_s} e^{j\omega(t-x_1/v_s)} , \quad i = 1,2,3 ,$$

$$\varphi = \sum_{\ell=1}^{4} B^{(\ell)} \beta_{Ci}^{(\ell)} e^{-\alpha_c^{(\ell)} \omega x_3/v_s} e^{j\omega(t-x_1/v_s)} . \tag{42}$$

In the dielectric medium the total displacement components assume the form

$$U_i^{(d)} = \sum_{\ell=1}^{6} D^{(\ell)} \beta_{d_i}^{(\ell)} e^{-\alpha_d^{(\ell)} \omega x_3/v_s} e^{j\omega(t-x_1/v_s)} , \quad i = 1,2,3 , \tag{43}$$

while the total potential is given by

$$\varphi^d = \left(C_1 e^{\omega x_3/v_s} + C_2 e^{-\omega x_3/v_s} \right) e^{j\omega(t-x_1/v_s)} \tag{44}$$

In the free space region the total potential is given by equation (41).

The as yet unspecified amplitude coefficients $B^{(\ell)}$, $\ell = 1, 2, 3, 4$, ;$D^{(\ell)}$, $\ell = 1, 2, \ldots, 6$; C_1; C_2; and C_3 are determined, to within a constant, together with the surface wave velocity v_s, by the following continuity and boundary conditions:

(i) U_1, U_2, and U_3 continuous at $x_3 = 0$

(ii) φ continuous at $x_3 = 0$ and $x_3 = -h$

(iii) Continuity of the normal component of electric displacement at $x_3 = 0$ and $x_3 = -h$

(iv) Continuity of shear and normal stresses (T_{31}, T_{32}, T_{33}) at $x_3 = 0$

(v) The surface $x_3 = -h$ is stress free ($T_{31} = T_{32} = T_{33} = 0$).

The components of the electric displacement vector $\vec{D}$ are given by

$$D_i = e_{ik\ell} U_{k,\ell} - \epsilon_{ik} \varphi_{,k} , \qquad i = 1,2,3 \qquad x_3 > 0 ,$$

$$\vec{D} = \epsilon_d \nabla\varphi \qquad -h < x_3 < 0 ,$$

and

$$\vec{D} = \epsilon_o \nabla\varphi \qquad x_3 < -h \quad .$$

The components of stress T_{31}, T_{32}, and T_{33} are given by

$$T_{3j} = C_{3jk\ell}U_{k,\ell} + e_{k3j}\varphi_{,k} \quad , \qquad j = 1,2,3, \qquad x_3 > 0 \quad .$$

$$\left.\begin{aligned} T_{31} &= \mu_d(U_{1,3} + U_{3,1}) \\ T_{32} &= \mu_d U_{2,3} \\ T_{33} &= (\lambda_d + 2\mu_d)\, U_{3,3} + \lambda_d U_{1,1} \end{aligned}\right\} \; -h < x_3 < 0 \quad .$$

Application of the continuity and boundary conditions (i)....(v) to the total displacements and potentials (40), (41), (42), (43), and (44) leads to a system of thirteen linear homogeneous equations in the thirteen unknown amplitudes $B^{(\ell)}$, $\ell = 1,2,3,4$, $D^{(\ell)}$, $\ell = 1,\ldots 6$, C_1, C_2 and C_3. The equation for the surface wave velocity v_s is obtained from the condition for the existence of a non-trivial solution of this system of equations, namely, that the determinant of the system vanish. The explicit forms of the coefficients L_{ij}, $i,j = 1,\ldots,13$, of this system are contained in Appendix III where the appropriate boundary conditions represented by each row of the matrix are indicated.

If the substrate is non-piezoelectric, modifications of the foregoing analyses identical tot hose described in Section II.1 are required. In this case, the characteristic equation for the surface wave velocity is the determinant of a (9 x 9) matrix comprised of the coefficients of the amplitudes $D^{(\ell)}$, $\ell = 1,\ldots,6$, and $B^{(\ell)}$, $\ell = 1,2,3$ in the homogeneous system of 9 equations in 9 unknowns derived from the boundary conditions given above upon neglecting the electric field and setting the piezoelectric constants equal to zero.

Degenerate Cases (Piezoelectric Substrate)

The same degenerate cases arise as those considered in the preceeding sections and the selection of β's proceeds as before.

For case (1) (Section II.2) solutions are sought wherein U_1, U_3 and φ only exist in the crystal. This type of solution uses only the α values which

lead to $\beta_2 = 0$ and β_1, β_3, and β_4 non-zero. Also in this problem solutions where U_2 only exists in the crystal must be considered (e.g. Love waves or the piezoelectric perturbations thereof). This solution stems from the root α which leads to non-zero β_2 and zero β_1, β_3, and β_4.

As in the previously considered degenerate cases the determinant of the boundary condition matrix $\hat{L}$ factors into the product of two determinants. The determinant which corresponds to the U_1, U_3, φ solutions is denoted M and assumes the form,

$$M = \begin{vmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{18} & L_{19} & L_{1,10} & L_{1,11} & L_{1,12} & L_{1,13} \\ L_{31} & - & - & - & - & - & - & - & - & - \\ L_{41} & - & - & - & - & - & - & - & - & - \\ L_{61} & - & - & - & - & - & - & - & - & - \\ L_{71} & - & - & - & - & - & - & - & - & - \\ L_{91} & - & - & - & - & - & - & - & - & - \\ L_{10,1} & - & - & - & - & - & - & - & - & - \\ L_{11,1} & - & - & - & - & - & - & - & - & - \\ L_{12,1} & - & - & - & - & - & - & - & - & - \\ L_{13,1} & - & - & - & - & - & - & - & - & L_{13,13} \end{vmatrix}$$

The solution for the U_2 case depends upon existence of zeros of a determinant N where N is a (3 x 3) determinant.

$$N = \begin{vmatrix} L_{25} & L_{26} & L_{27} \\ L_{55} & L_{56} & L_{57} \\ L_{85} & L_{86} & L_{87} \end{vmatrix} .$$

For case (2) (Section II.2) solutions wherein only U_1 and U_3 exist are considered. This type of solution uses only those α's which lead to zero β_2 and β_4 and non-zero β_1 and β_3. Solutions where only U_2 and φ exist also must be considered. This solution employs the α's which give non-zero β_2 and β_4 but zero β_1 and β_3.

The solution for the U_1, U_3 case depends upon existence of zeros of a determinant P (6 x 6), where,

$$P = \begin{vmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{17} & L_{18} & \\ L_{31} & - & - & - & - & - & - \\ L_{41} & - & - & - & - & - & - \\ L_{61} & - & - & - & - & - & - \\ L_{71} & - & - & - & - & - & - \\ L_{81} & - & - & - & - & - & - \end{vmatrix}$$

The solution for the U_2, U_4 case depends upon existence of zeros of a determinant Q (7 x 7), which assumes the form

$$Q = \begin{vmatrix} L_{25} & L_{26} & L_{29} & L_{2,10} & L_{2,11} & L_{2,12} & L_{2,13} & & & \\ L_{55} & - & - & - & - & - & - & - & - & - \\ L_{85} & - & - & - & - & - & - & - & - & - \\ L_{10,5} & - & - & - & - & - & - & - & - & - \\ L_{11,5} & - & - & - & - & - & - & - & - & - \\ L_{12,5} & - & - & - & - & - & - & - & - & - \\ L_{13,5} & - & - & - & - & - & - & - & - & - \end{vmatrix}$$

Degenerate Cases (Non-Piezoelectric Substrate)

The degenerate cases U_1, U_3, only or U_2 only involve one α with zero β_1, β_3, and non-zero β_2 and two α's with zero β_2 and non-zero β_1, β_3. The U_1, U_3 case requires the investigation of the roots of a determinant of the same form as P except for relabeling the columns due to a relabeling of the α's. This determinant is designated R and has the form,

$$R = \begin{vmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{18} & L_{19} \\ L_{31} & - & - & - & - & - & - \\ L_{41} & - & - & - & - & - & - \\ L_{61} & - & - & - & - & - & - \\ L_{71} & - & - & - & - & - & - \\ L_{91} & - & - & - & - & - & - \end{vmatrix}$$

Solutions with U_2 only (Love waves) lead to the consideration of the roots of a determinant which is identical to N.

III. ANALYSIS OF AN ELECTRIC CURRENT LINE SOURCE ABOVE A PIEZOELECTRIC HALF-SPACE

In this section a stuuy is made of the excitation of piezoelectric waves by means of an interdigital electrode transducer. Arbitrary crystals and crystal orientations are considered as in the preceeding chapter. The problem is treated from a field theory point of view and is case in the formalism of a Green's function solution. Due to the complexity of the problem it is expediant to make some simplifying assumptions before the analysis is attempted. Consequently, it is assumed that the coupling between the individual strips can be neglected and that the current on the strips can be approximated by an assumed current distribution. Furthermore it is assumed that only current flow normal to the array is of importance in exciting the piezoelectric waves and that the strips of the array can be considered to be of infinite extent thus reducing the problem to a two dimensional one.

With these assumptions in mind the Green's function sought is one for an infinitesimal two dimensional electric dipole above a piezoelectric substrate as illustrated in Figure 4.

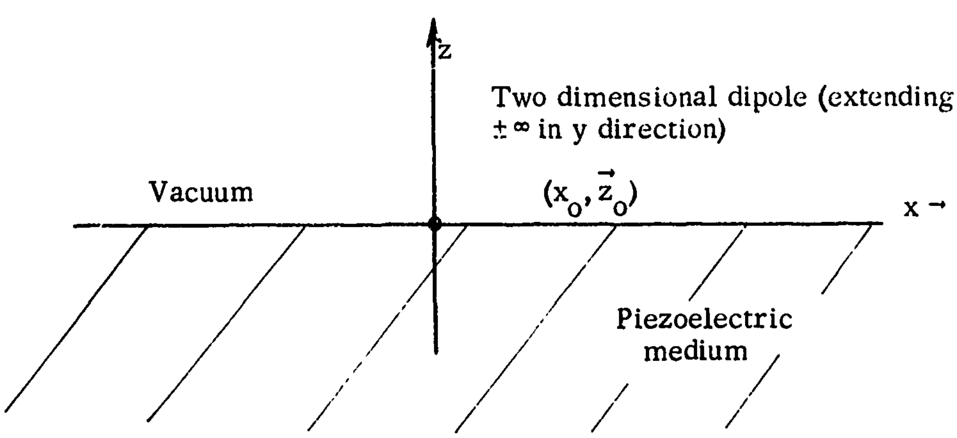


Figure 4

The dipole is located at x_0, z_0 and extends to $\pm\infty$ in the y direction. It is oriented in the x direction. The crystal fills the region $z \leq 0$ while a vacuum exists in the region $z > 0$. Assuming an $e^{j\omega t}$ time dependence, the equation for the electric field in the vacuum is as follows:

$$\nabla \times \nabla \times \vec{E} = j\omega\mu_0 \vec{J} + \omega^2 \mu_0 \vec{D} \quad . \tag{45}$$

For the two dimensional dipole $\vec{J} = \vec{U}_x\, \delta(z-z_o)\, \delta(x-x_o)$ and equation (45) reduces to

$$\nabla^2\vec{E} + k_o^2\vec{E} = -j\omega\mu_o \left[1 + \frac{\nabla\nabla}{k_o^2} \right] \cdot \vec{J}$$

where 1 is the unit dyadic, $k_o = 2\pi/\lambda_o$, and λ_o is the free space wavelength. Setting $\vec{E} = (1 + \nabla\nabla/k_o^2) \cdot \vec{G}$ it is easily seen that

$$\nabla^2\vec{G} + k_o^2\vec{G} = -j\omega\mu_o\, \delta(z-z_o)\, \delta(x-x_o)\, \vec{U}_x \quad . \tag{46}$$

A particular solution to equation (46) is

$$\vec{G} = -\vec{U}_x \frac{\omega\mu_o}{4\pi} \int_{-\infty}^{\infty} \frac{e^{j\sqrt{k_o^2 - k_x^2}\,|z-z_o|}}{\sqrt{k_o^2 - k_x^2}}\, e^{jk_x(x-x_o)}\, dk_x \quad . \tag{47}$$

A particular solution for $\vec{E}$ (viz. $\vec{E}_p$) is derivable from $\vec{G}$ and in the region $0 < z < z_o$ the following expression results, namely,

$$\begin{aligned} \frac{E_p}{k_o} = \frac{1}{4\pi}\sqrt{\frac{\mu_o}{\epsilon_o}} \Bigg\{ &-\vec{U}_x \int_{-\infty}^{\infty} \sqrt{1-K_x^2}\; e^{-j\sqrt{1-K_x^2}(\bar{z}-\bar{z}_o)}\; e^{jK_x(\bar{x}-\bar{x}_o)}\, dK_x \\ &-\vec{U}_z \int_{-\infty}^{\infty} K_x\, e^{-j\sqrt{1-K_x^2}(\bar{z}-\bar{z}_o)}\; e^{jK_x(\bar{x}-\bar{x}_o)}\, dK_x \Bigg\} \\ = {} & \frac{E_{px}}{k_o}\vec{U}_x + \frac{E_{pz}}{k_o}\vec{U}_z \end{aligned} \tag{48}$$

where $K_x = k_x/k_o$, $\bar{x} = k_o x$, and $\bar{z} = k_o z$. A solution for the total electric field may be obtained as a superposition of $\vec{E}_p$ and a general solution of the homogeneous equation $\nabla^2\vec{E} + k_o^2\vec{E} = 0$. That is, in the region $0 < z < z_o$ E may be written in the form

$$\frac{E_x}{k_o} = \frac{E_{px}}{k_o} + \int_{-\infty}^{\infty} B_o(K_x)\, e^{j\sqrt{1-K_x^2}(\bar{z}-\bar{z}_o)}\, e^{jK_x(\bar{x}-\bar{x}_o)}\, dK_x$$

$$\frac{E_y}{k_o} = \int_{-\infty}^{\infty} A_o(K_x)\, e^{j\sqrt{1-K_x^2}(\bar{z}-\bar{z}_o)}\, e^{jK_x(\bar{x}-\bar{x}_o)}\, dK_x \tag{49}$$

$$\frac{E_z}{k_o} = \frac{E_{pz}}{k_o} - \int_{-\infty}^{\infty} B_o(K_x)\, \frac{K_x}{\sqrt{1-K_x^2}}\, e^{j\sqrt{1-K_x^2}(\bar{z}-\bar{z}_o)}\, e^{jK_x(\bar{x}-\bar{x}_o)}\, dK_x$$

where $A_o(K_x)$ and $B_o(K_x)$ are functions of K_x which are determined through the application of the boundary conditions on the total fields at $z = 0$.

In the crystal medium ($z < 0$) the mechanical displacement fields and the electric fields may be expressed as follows:

$$U_i = \int_{-\infty}^{\infty} U_i(K_x)\, e^{-jK_z(\bar{z}-\bar{z}_o)}\, e^{jK_x(\bar{x}-\bar{x}_o)}\, dK_x \qquad i = 1,2,3$$

$$\frac{E_i}{k_o} = \int_{-\infty}^{\infty} \psi_i(K_x)\, e^{-jK_z(\bar{z}-\bar{z}_o)}\, e^{jK_x(\bar{x}-\bar{x}_o)}\, dK_x \qquad i = 1,2,3 \tag{50}$$

When the above integral representations are substituted into the differential equations for the crystal, viz.

$$C_{ijk\ell}\, U_{k,\ell i} - e_{kij} E_{k,i} = \rho \ddot{U}_j \qquad j = 1,2,3$$

$$\nabla \times \nabla \times \vec{E} = \omega^2 \mu_o \vec{D} \tag{51}$$

there results a linear system of homogeneous equations for the amplitudes $U_i(K_x)$ and $\psi_i(K_x)$. The determinant of the coefficients of U_i and ψ_i must vanish for a non-trivial solution to exist, namely,

$$\det\begin{pmatrix}
\begin{matrix}-C_{11}K_x^2+2C_{15}K_xK_z\\ -C_{55}K_z^2+C^2\rho\end{matrix} & \begin{matrix}-C_{16}K_x^2-C_{45}K_z^2\\ +(C_{14}+C_{56})K_xK_z\end{matrix} & \begin{matrix}-C_{15}K_x^2-C_{35}K_z^2\\ +(C_{13}+C_{55})K_xK_z\end{matrix} & \begin{matrix}-e_{11}jK_x\\ +e_{15}jK_z\end{matrix} & \begin{matrix}-e_{21}jK_x\\ +e_{25}jK_z\end{matrix} & \begin{matrix}-e_{31}jK_x\\ +e_{35}jK_z\end{matrix}\\
\begin{matrix}-C_{16}K_x^2-C_{45}K_z^2\\ +(C_{14}+C_{56})K_xK_z\end{matrix} & \begin{matrix}-C_{66}K_x^2+2C_{46}K_xK_z\\ -C_{44}K_z^2+C^2\rho\end{matrix} & \begin{matrix}-C_{56}K_x^2-C_{34}K_z^2\\ +(C_{36}+C_{45})K_xK_z\end{matrix} & \begin{matrix}-e_{16}jK_x\\ +e_{14}jK_z\end{matrix} & \begin{matrix}-e_{26}jK_x\\ +e_{24}jK_z\end{matrix} & \begin{matrix}-e_{36}jK_x\\ +e_{34}jK_z\end{matrix}\\
\begin{matrix}-C_{15}K_x^2-C_{35}K_z^2\\ +(C_{13}+C_{55})K_xK_z\end{matrix} & \begin{matrix}-C_{56}K_x^2-C_{34}K_z^2\\ +(C_{36}+C_{45})K_xK_z\end{matrix} & \begin{matrix}-C_{55}K_x^2+2C_{35}K_xK_z\\ -C_{33}K_z^2+C^2\rho\end{matrix} & \begin{matrix}-e_{15}jK_x\\ +e_{13}jK_z\end{matrix} & \begin{matrix}-e_{25}jK_x\\ +e_{23}jK_z\end{matrix} & \begin{matrix}-e_{35}jK_x\\ +e_{33}jK_z\end{matrix}\\
C^2\mu_o[e_{11}jK_x-e_{15}jK_z] & \begin{matrix}C^2\mu_o[e_{16}jK_x\\ -e_{14}jK_z]\end{matrix} & \begin{matrix}C^2\mu_o[e_{15}jK_x\\ -e_{13}jK_z]\end{matrix} & \begin{matrix}C^2\mu_o\varepsilon_{11}\\ -K_z^2\end{matrix} & C^2\mu_o\varepsilon_{12} & \begin{matrix}C^2\mu_o\varepsilon_{13}\\ -K_xK_z\end{matrix}\\
C^2\mu_o[e_{21}jK_x-e_{25}jK_z] & \begin{matrix}C^2\mu_o[e_{26}jK_x\\ -e_{24}jK_z]\end{matrix} & \begin{matrix}C^2\mu_o[e_{25}jK_x\\ -e_{23}jK_z]\end{matrix} & C^2\mu_o\varepsilon_{21} & \begin{matrix}C^2\mu_o\varepsilon_{22}\\ -K_x^2-K_z^2\end{matrix} & C^2\mu_o\varepsilon_{23}\\
C^2\mu_o[e_{31}jK_x-e_{35}jK_z] & \begin{matrix}C^2\mu_o[e_{36}jK_x\\ -e_{34}jK_z]\end{matrix} & \begin{matrix}C^2\mu_o[e_{35}jK_x\\ -e_{33}jK_z]\end{matrix} & \begin{matrix}C^2\mu_o\varepsilon_{31}\\ -K_xK_z\end{matrix} & C^2\mu_o\varepsilon_{32} & \begin{matrix}C^2\mu_o\varepsilon_{33}\\ -K_x^2\end{matrix}
\end{pmatrix} = 0 \qquad (52)$$

In the above equation C is the velocity of light. Expanding this determinant leads to a 10'th order algebraic equation for K_z as a function of K_y.

For a given K_x in the range $-\infty < K_x < \infty$ there will be 10 values of K_z satisfying the determinental equation but only 5 will be admissable (representing field solutions that are bounded as $z \to -\infty$ and have the form of out going waves in the region $z < 0$). For each usable K_z it is necessary to solve the homogeneous system for the corresponding field amplitudes U_i, ψ_i.

Thus U_i and E_i can be expressed as follows:

$$U_i = \int_{-\infty}^{\infty} \sum_{n=1}^{5} A_n(K_x)\, U_i^{(n)}(K_x)\, e^{-jK_z^{(n)}(\bar{z}-\bar{z}_o)}\, e^{jK_x(x-x_o)}\, dK_x$$

$$\frac{E_i}{k_o} = \int_{-\infty}^{\infty} \sum_{n=1}^{5} A_n(K_x)\, \psi_i^{(n)}(K_x)\, e^{-jK_z^{(n)}(\bar{z}-\bar{z}_o)}\, e^{jK_x(\bar{x}-\bar{x}_o)}\, dK_x \quad . \tag{53}$$

where $A_n(K_x)$ are unknown amplitude coefficients to be determined by an application of the boundary conditions. The magnetic field in the crystal medium and in the vacuum can be written in a similar fashion and is derivable from the equation

$$\nabla \times \vec{E} = j\omega\mu_o\vec{H} \quad .$$

The boundary conditions imposed on the field solutions at $\bar{z} = 0$ are as follows:

Continuity of T_{3j} $\quad j = 1, 2, 3$

Continuity of E_1 and E_2

Continuity of H_1 and H_2 .

The imposition of these conditions leads to the following set of equations in the amplitude coefficients A_o, B_o, A_n. The limiting case $\bar{z}_o \to 0$ has been taken in the following since the electrodes will be located on the crystal surface at $\bar{z} = 0$.

Continuity of T_{3j} $\quad j = 1, 2, 3$

$$\sum_{n=1}^{5} \{[jK_xC_{15} - jK_z^{(n)}C_{55}]\,U_1^{(n)} + [jK_xC_{56} - jK_z^{(n)}C_{45}]\,U_2^{(n)}$$

$$+ [jK_xC_{55} - jK_z^{(n)}C_{35}]\,U_3^{(n)} - e_{15}\psi_1^{(n)} - e_{25}\psi_2^{(n)} - e_{35}\psi_3^{(n)}\}\,A_n = 0$$

$$\sum_{n=1}^{5} \{[jK_xC_{14} - jK_z^{(n)}C_{45}]\,U_1^{(n)} + [jK_xC_{46} - jK_z^{(n)}C_{44}]\,U_2^{(n)}$$

$$+ [jK_xC_{45} - jK_z^{(n)}C_{34}]\,U_3^{(n)} - e_{14}\psi_1^{(n)} - e_{24}\psi_2^{(n)} - e_{34}\psi_3^{(n)}\}\,A_n = 0 \quad (54)$$

$$\sum_{n=1}^{5} \{[jK_xC_{13} - jK_z^{(n)}C_{35}]\,U_1^{(n)} + [jK_xC_{36} - jK_z^{(n)}C_{34}]\,U_2^{(n)}$$

$$+ [jK_xC_{35} - jK_z^{(n)}C_{33}]\,U_3^{(n)} - e_{13}\psi_1^{(n)} - e_{23}\psi_2^{(n)} - e_{33}\psi_3^{(n)}\}\,A_n = 0$$

Continuity of E_1, E_2

$$\sum_{n=1}^{5} \psi_1^{(n)} A_n - B_o = -\frac{1}{4\pi}\sqrt{\frac{\mu_o}{\epsilon_o}}\sqrt{1-K_x^2} \quad (55)$$

$$\sum_{n=1}^{5} \psi_2^{(n)} A_n - A_o = 0$$

Continuity of H_1, H_2

$$\sum_{n=1}^{5} jK_z^{(n)}\,\psi_2^{(n)} A_n + j\;\; 1-K_x^2\;\; A_o = 0$$

$$\sum_{n=1}^{5} [jK_z^{(n)}\,\psi_1^{(n)} + jK_x\,\psi_3^{(n)}]\,A_n + \left[j\sqrt{1-K_x^2} + j\,\frac{K_x^2}{\sqrt{1-K_x^2}}\right] B_o = \frac{j}{4\pi}\sqrt{\frac{\mu_o}{\epsilon_o}}$$

(56)

In practice the solution of equations (54), (55), and (56) would be performed numerically on a computer as a function of K_x.

The integral expressions for the total mechanical displacements and electromagnetic field components follow from the solution of the system of equations described above. An asymptotic evaluation of the resulting integrals may be employed to obtain formal expressions for the physical quantities of significant interest such as the surface wave fields, power in the surface wave, total power input to the crystal by the transducer, and the bulk wave scattered amplitude pattern. The expressions for the aforementioned quantities are very formidable and would require an extensive amount of numerical computation to obtain even limited information. Consequently, it was decided to abandon this approach and the numerical implementation of the theoretical analysis was not carried out.

IV. COMPUTER PROGRAM OUTLINES

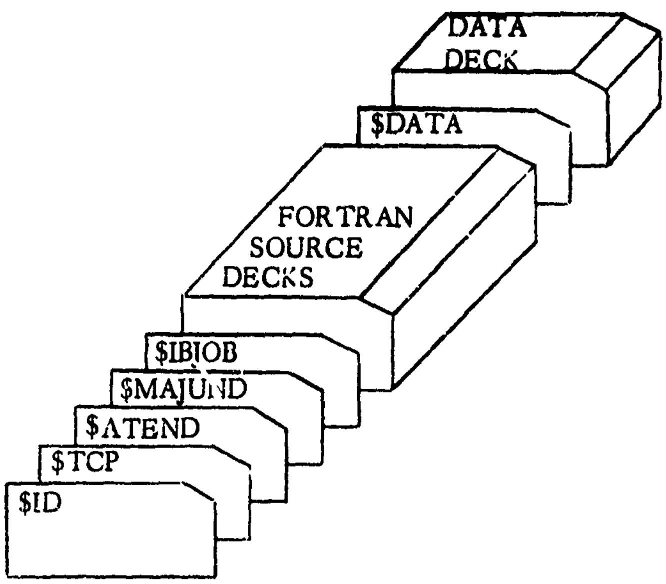
This section describes the use and programming format of the computer programs which were written to implement the numerical analysis of the various surface wave propagation problems described in Section II.

1. Surface Waves on Piezoelectric Crystals in the Presence of Infinitesimally Thin Electric and "Magnetic" Conductors

This computer program is divided into two parts: Part A is concerned with an isotropic elastic conductor (such as gold) of finite thickness above a piezoelectric substrate (such as lithium niobate); Part B is concerned with an infinitesimally thin electric or magnetic conductor above a piezoelectric substrate. All information necessary for the operation of the program is described below. For example, an initial guess of the surface wave velocity is required. From this information, the program refines the initial guess, resulting in a velocity accurate to input specifications.

The program is set up to run on an IBM 7094, and the form of input is FORTRAN Namelist input. Although it is discussed in this document, it is suggested that those not familiar with Namelist input read the appropriate sections in a Fortran manual.

Deck Setup



The $ID and $TCP control cards must be supplied by the user; the remaining control cards are already in the program deck. The data deck (i.e., the input data for the program) utilizes Namelist input. Two input sections are required: the first describes the parameters of the substrate crystal; the second provides the remainder of the information necessary for the execution of the program.

The first data set is called CONST. This set includes the piezoelectric, elastic and dielectric constants. Column 2 of the data card contains a dollar sign ($) and columns 3-7 contain the letters CONST. The constants begin in column 9 of the first card and continue up to column 72; they then continue to columns 2-72 of each succeeding card for as many cards as needed. The general form of the data to be input is:

Variable name = 1st value, 2nd value, ... , last value .

For example, the piezoelectric constants (e_{ip}), called P in the program, could be input as:

P = 0.,0.,0.,0.,3.7,-2.5,...,0. ,

There are 18 piezoelectric constants and they should be input in the following order:

e_{11}

e_{12}

e_{13}

e_{14}

e_{15}

e_{16}

e_{21}

e_{22}

e_{23}

e_{24}

e_{25}

e_{26}

e_{31}

e_{32}

e_{33}

e_{34}

e_{35}

e_{36}

i.e. $P = e_{11}, e_{12}, \ldots, e_{36}$, .

In the program the transformed piezoelectric constants e'_{ij} are printed out as

$E_1 = e'_{11}$

$E_2 = e'_{13}$

$E_3 = e'_{14}$

$E_4 = e'_{15}$

$E_5 = e'_{16}$

$E_6 = e'_{31}$

$E_7 = e'_{33}$

$E_8 = e'_{34}$

$E_9 = e'_{35}$

$E_{10} = e'_{36}$

$E_{11} = e'_{12}$

$E_{12} = e'_{32}$

$E_{13} = e'_{21}$

$E_{14} = e'_{23}$

$E_{15} = e'_{24}$

$E_{16} = e'_{25}$

$E_{17} = e'_{26}$

The transformed constant e'_{22} is never used and therefore is not printed out.

The elastic constants (C_{pq}), called G in the program, are next. Immediately following the comma (,) behind the last piezoelectric constant (excluding blanks), print:

G =

followed by the 21 values of the elastic constants in the following order, separating each variable by a comma:

C_{11}

C_{22}

C_{33}

C_{12}

C_{13}

C_{14}

C_{15}

C_{16}

C_{23}

C_{24}

C_{25}

C_{26}

C_{34}

C_{35}

C_{36}

C_{44}

C_{45}

C_{46}

C_{55}

C_{56}

C_{66}

In the program the transformed elastic constants C'_{ij} are printed out as:

$C_1 = c'_{11}$

$C_2 = c'_{13}$

$C_3 = c'_{14}$

$C_4 = c'_{15}$

$C_5 = c'_{33}$

$C_6 = c'_{34}$

$C_7 = c'_{35}$

$C_8 = c'_{36}$

$C_9 = c'_{44}$

$C_{10} = c'_{45}$

$C_{11} = c'_{46}$

$C_{12} = c'_{55}$

$C_{13} = c'_{56}$

$C_{14} = c'_{66}$

$C_{15} = c'_{16}$

$C_{16} = c'_{12}$

$C_{17} = c'_{25}$

$C_{18} = c'_{26}$

$C_{19} = c'_{24}$

$C_{20} = c'_{23}$

The transformed constant c'_{22} is not used and therefore not printed out.

The dielectric constants (ϵ_{ij}), called EPS in the program, are the last constants to be entered. They should be entered following the comma after the last value of the elastic coefficients, as

EPS =

followed by 9 values of EPS in the following order, separating each variable by a comma:

ϵ_{11}

ϵ_{12}

ϵ_{13}

ϵ_{21}

ϵ_{22}

ϵ_{23}

ϵ_{31}

ϵ_{32}

ϵ_{33}

In the program the transformed dielectric constants ϵ'_{ij} are printed out as:

$T_1 = \epsilon'_{11}$

$T_2 = \epsilon'_{13}$

$T_3 = \epsilon'_{33}$

$T_4 = \epsilon'_{21}$

$T_5 = \epsilon'_{23}$

The transformed constant ϵ'_{22} is never used and therefore is not printed out.

After the last value of EPS, namely ϵ_{33}, print a dollar sign (\$) instead of a comma. That is,

$$EPS = \epsilon_{11}, \epsilon_{12}, \epsilon_{13}, \epsilon_{21}, \epsilon_{22}, \epsilon_{23}, \epsilon_{31}, \epsilon_{32}, \epsilon_{33}\$ \quad .$$

This signals the end of the first data set.

The second data set is called "INPUT." $INPUT must be printed in columns 2-7 of the next card (following the EPS data). Then each input parameter should be entered, followed by a comma (except the last value, which should be followed by a dollar sign, $). The following is a definition of each input parameter (unless otherwise stated, the input parameters will refer to both Part A and Part B):

Input Name	Equation Names	Definition
MUA	μ (Part A)	Lame's constants for elastic conductor
LAMDAA	λ (Part A)	
RHOA	ρ	Mass density of elastic conductor
LAMDAB	λ (Part B)	Euler Angles
MUB	μ (Part B)	
NUB	ν (Part B)	
RHOB	ρ (Part B)	Mass density of crystal
VS	v_s	Initial guess to a velocity. This initial value will be used to find a final velocity, $\bar{v}_s$ such $\lvert f(\bar{v}_s) \rvert < \epsilon$, where ϵ is input.
KS	k_s	Can take on two values: $k_s = 0$ for Part B; $k_s = 1$ for Part A
EPSLON	ϵ	A positive number used as a convergence criterion. When $\lvert f(v_s) \rvert < \epsilon$, then v_s is assumed to be the root required.
WH	ωh	Normalized height of conducting wall or magnetic wall (Part B) Normalized thickness of elastic conductor (Part A). To input $\omega h = \infty$, set WH > 10^{10}.
WXA	ωx_a (Part A)	Normalized distance into elastic conductor
WXB	ωx_b (Part A)	Normalized distance into crystal
KL	K_L (Part B)	K_L is normally 0. However, if the electric wall case is being run (see K_M) and if wh = 0, then K_L should be set to 1.
KM	K_M (Part B)	This can take on two values: $K_M = 0$ electric wall; $K_M = 1$ magnetic wall

Input Name	Equation Names	Definition
MAX	--	Since an iteration scheme is used for convergence for a final root v_s, there must be an indication of how many iterations are to be executed before divergence is assumed. Hence, MAX should be the maximum number of iterations the user wishes the program to make (usually 10). If MAX is set to zero (MAX = 0) the determinant $\|f(\bar{v}_s)\|$ will be evaluated for the particular v_s value input – the iteration scheme will not be used. This option may be useful if there is difficulty in determining the range in which v_s lies.
ICHECK	--	A logical parameter which controls the use of a checkout option. If ICHECK = .FALSE., all FINAL ANSWERS* are computed in addition to the evaluation of the determinant $\|f(\bar{v}_s)\|$. If ICHECK = .TRUE., FINAL ANSWERS are not computed – evaluation of the determinant only. This option was included for use when MAX = 0.
DVS	Δv_s	Increment to be used for v_s when ICHECK = .TRUE. (DVS ≥ 0.)
VSMAX	$v_{s_{max}}$	Maximum value of v_s to be used when DVS ≠ 0.
EPSO	ϵ_o	Permittivity of free space
WX	ωx (Part B)	Normalized distance into crystal
DNU	$\Delta\nu$	If the user wishes to vary ν (NUB) from some initial value, ν, to some final value, ν_{max}, in steps of $\Delta\nu$, then set DNU equal to the steps desired; also, see NUMAX.
NUMAX	ν_{max}	The maximum value of ν (see DNU). ν_{max} is only used when DNU ≠ 0.
DWX	$\Delta\omega x$	An increment for ωx, similar to DNU. If DWX = 0, then ωx is not incremented.

*The FINAL ANSWERS consist of the partial field relative amplitudes (Eta), stress components, strain components, time average power flow, electric and mechanical displacements, electric potential, and electric field.

Input Name	Equation Names	Definition
WXMAX	ωx_{max}	The maximum value of ωx (see DWX). ωx_{max} is only used when DWX $\neq$ 0.
TITLE	--	An alphanumeric array of 24 characters or less used to describe the type of crystal, such as lithium niobate. This is input in the following manner: TITLE = nH name of crystal, where n is the number of characters following the H (including blanks). For example TITLE = 6HQUARTZ
REPEAT	--	REPEAT is a logical variable and in its usage, can take only one value: .TRUE. If there are no more cases to run after the current case, REPEAT does not need to be input. If there will be another case to follow, but the crystal coefficients remain the same, then, again, REPEAT does not need to be input. However, if another case is to be run and the coefficients are different, then REPEAT needs to be input as .TRUE. This means that the $CONST data will have to be input again (in the other cases above, $CONST would not have to be input again).
HXAGNL	--	Parameter which controls the calculation of betas (β's) for a hexagonal crystal (such as zinc oxide) .TRUE. hexagonal crystal (use special technique) .FALSE. non-hexagonal crystal (use normal procedure)
VSINC	--	VSINC = .TRUE. – New estimates of initial velocity (v_s) are computed using a linear fit to the two previous values. (Used when NUB varies over a range NUB, NUB + DNU, ... , NUMAX) VSINC = .FALSE. – The same initial estimate of velocity is used for all values over the specified range of NUB.

The following input parameters are all logical variables which are assumed to be false (.FALSE.) in the program. They are used as switches indicating whether or not intermediate calculations are to be printed. If any one, or any combination of these parameters are input as true (.TRUE.), then certain intermediate data will print, according to the following:

ROOTS	Print the roots of the polynomial each time they are calculated.
COEFF	Print the constants E, C, and T (the transformed piezoelectric, elastic, and dielectric constants) calculated from the constants P, G, and EPS.
DETERM	Print the L matrix and the value of the determinant.
POLY	Print the coefficients of the 8'th order polynomial.
BETA	Print the values of β_{ij}.
ALPHA	Print α_A's, Part A.
ALL	Print all of the above.

The manner in which the above listed parameters in the $INPUT data set are input is best illustrated by an example (assume Part B is being run):

```
$INPUT          MUB = 90., LAMDAB = 90., NUB = 100., RHOB = 4700.,
                VS = 3400., KS = 0, EPSLON = 1.E-11, WH = 0, KL = 1,
                KM = 0, WX = 0., DWX = 10., WXMAX = 100.,
                title = 15HLITHIUM NIOBATE
```

ωh is zero in the above example. To input $\omega h = \infty$, set $\omega h > 10^{10}$. Note that some of the values discussed in the list are not present in the above example. This is because either they are not required or the program assumed nominal values. A nominal value is a value that a parameter will take on if no other value is input. In the above example, MAX, EPSO, and DNU take on their nominal values of 10, 8.85×10^{-12}, and 0, respectively. It is not necessary to input NUMAX since DNU = 0; all parameters referring to Part A are not necessary since Part B is being run; and all the logical parameters take on their nominal value of false. The following is a complete list of nominal values:

Parameter	Nominal Value
MUA	2.85×10^{10}
LAMDAA	1.5×10^{11}
RHOA	1.888×10^{4}
VS	3000
EPSLON	1×10^{-11}
KL	0
KM	0
MAX	10
EPSO	8.85×10^{-12}
DNU	0
DWX	0
DVS	0
TITLE	15HLITHIUM NIOBATE
ROOTS	
COEFF	
DETERM	
POLY	.FALSE.
BETA	
ALL	
REPEAT	
ALPHA	

Sample Data Decks:

The following sample data deck, illustrated on the attached code sheet, gives an example of three data runs: the first is a 90-90-100 degree cut of lithium niobate. The second is the same, except for a new value of ωh. The third is a 0, 0, -90 cut of quartz (note that REPEAT is set to true in the second case, just prior to the case when new coefficients are to be input).

HUGHES AIRCRAFT COMPANY
7094 FORTRAN CODING SHEET

NAME ______ PHONE ______ DATE ______ LOG NO. ______ PROB. NO. ______ PROG. NO. ______ PAGE ______ OF ______

```
 $CONST F = 4*0., 3.7, -2.5, -2.5, 2.5, 0., 3.7, 0., 0., 0.2, 0.2,
      1.3, 0., 0., 0.,
      G = 2.03E11, 2.03E11, 2.45E11, 0.53E11, 0.75E11, 0.09E11,
      0., 0., 0.75E11, -0.09E11, 5*0., 0.60E11, 0., 0., 0.60E11,
      0.09E11, 0.75E11,
      EPS = 3.9E-10, 3*0., 3.9E-10, 3*0., 2.57E-10$
 $INPUT MUB = 90., LAMDAB = 90., NUB = 100., RHOB = 4700., VS = 3400.,
      KS = 0, WH = 0., KL = 1, WK = 0.$
 $INPUT WH = 2000., REPEAT = .TRUE.$
 $CONST F = 0.171, -0.171, 0., -0.0406, 6*0., 0.0406, -0.171, 6*0.,
      G = 2*3.674E10, 1.072E11, 6.99E9, 11.91E9, -17.91E9, 2*0.,
      11.91E9, 17.91E9, 5*0., 57.94E9, 2*0., 57.94E9, -17.91E9,
      39.00E9,
      EPS = 39.21E-12, 3*0., 39.21E-12, 3*0., 41.03E-12$
 $INPUT MUB = 0., LAMDAB = 0., NUB = -90., KTIME = 0$
```

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The following is a description of the program flow diagram provided at the end of this section.

First, the nominal data values[(1)] are set up in the program. These values are assumed by certain parameters in the program unless new values are specified. Following this the program reads in the elastic (G), piezoelectric (P), and dielectric (EPS) constants (CONST DATA) of the substrate medium. Finally, the remaining input data is read in (INPUT).

Next, subroutine SETCTE is called to perform the Euler transformation to obtain the elastic (CC), piezoelectric (CE), and dielectric (CT) constants relative to the input coordinate system as specified by the constants λ, μ, and ν. At this point subroutine ROOT is called to perform the calculations leading to the evaluation of the determinant of the boundary condition matrix (L matrix of the analysis). The determinant is referred to as F(VS) since it is evaluated as a function of velocity (VS).

There is an option in the program to use a root finding scheme to minimize |F(VS)| or simply to increment VS in steps of DVS and calculated F(VS) at each value. To perform these various calculations at a particular velocity (VS) ROOT calls subroutine F which is described in detail below (in <u>Determination of F(VS)</u>).

After exiting from ROOT and returning to the main program logical checks are made to establish the type of case considered in ROOT. Depending upon the results of these checks the values of the amplitudes of the partial surface wave fields are computed ($A^{(\ell)}$ of analysis section). In the program these are called ETA(1), ETA(2), etc. The program now proceeds to compute the magnitude (MAG U(I)) and phase (PHASE U(I)) of the mechanical displacements ($\bar{U}$(I), I=1, 2, 3) and electric potential ($\bar{U}$(4)). Next subroutine P1FUN is called to compute the time average flow (P1M, P2M) followed by the computation of the stress components (TW31, TW32, TW33, TW11, TW12, TW22) in subroutine TFUN. Subroutine SFUN then implements the calculation of the strain components (S11, S33, S12, S13, S23). Finally the electric field (E1, E3) and electric displacement (D1, D2, D3) are computed. All the above quantities are evaluated as a function of normalized distance (WX) into the crystal. They can be computed at incremented values of WX for any specified initial and final values.

The velocity (VS) can be incremented if desired (up to some specified maximum value, VSMAX) and the steps in ROOT and that which follows are repeated for each new velocity. Thus it is possible to plot the determinant as a function of velocity. After VSMAX is reached there is an option to increment the third Euler angle (NU) and repeat the steps from SETCTE on. When this has been completed the program returns to read in new CONST DATA if the crystal is being changed or to read in new INPUT DATA if the crystal is to remain unchanged but the orientation is to be changed. After all data from both sources has been exhausted the program stops.

Determination of F(VS)

Subroutine F calls subroutine STRIP to compute the coefficients of the eighth order polynomial equation in α.* Next subroutine CROOT calculates the 8 roots (ALFA(I), I = 1, 8) of the polynomial equation by Muller's method. If the medium is non-piezoelectric the solution for the roots involves two extraneous roots which are rendered useless by setting them equal to -10-10j. The roots with positive real part are chosen (ALFAB(I), I=1, K).

If the medium is piezoelectric and the number of roots with positive real part (K) is equal to 4 the program proceeds to calculate the relative amplitudes ($\beta_i^{(\ell)}$ of the analysis section) of the displacement and potential corresponding to each α. These amplitudes are referred to as BETAB(I,J) in the program. The matrix (ACAP, $\hat{A}$ for simplicity) of coefficients of the amplitudes ($\beta_i^{(\ell)}$) is set up for each α. If the crystal is not hexagonal non-degenerate cases are solved by setting $\beta_4^{(\ell)} = 1$ and solving the first three equations of the system for $\beta_i^{(\ell)}$ $i = 1, 2, 3$. If the crystal is hexagonal one of the α's naturally leads to an ill-conditioned system if the first three equations are solved for $\beta_1^{(\ell)}$, $\beta_2^{(\ell)}$, and $\beta_3^{(\ell)}$ in terms of $\beta_4^{(\ell)}$. Thus $\beta_1^{(\ell)}$ is set equal to 10^{-10} and the system composed of the second, third, and fourth equation are solved for $\beta_2^{(\ell)}$, $\beta_3^{(\ell)}$, $\beta_4^{(\ell)}$. If the case is degenerate the $\beta_i^{(\ell)}$'s are calculated in the fashion indicated in the analysis.

If $K < 4$ the procedure for calculating the $\beta_i^{(\ell)}$'s is dependent upon the value of K. If $K \leq 1$ the case terminates since no solution is possible with only one available value of α. If $K > 1$, $\hat{A}_{12}$ and $\hat{A}_{23}$ are investigated to see if they are identically zero.

*See Appendix IV.

If either $\hat{A}_{12}$ or $\hat{A}_{23}$ is not identically zero the case cannot be degenerate. The program proceeds in one of two possible ways. If the crystal is non-piezoelectric and K = 3 the $\beta_i^{(\ell)}$'s are calculated as indicated in the analysis (i.e. $\beta_4^{(\ell)} = 0$, $\beta_3^{(\ell)} = 1$ and the first two equations of the system are solved for $\beta_1^{(\ell)}$ and $\beta_2^{(\ell)}$). If either $K \neq 3$ or the crystal is piezoelectric the case terminates. This is due to the fact that if the crystal is piezoelectric and non-degenerate, four α's are necessary for a solution in the general case.

If both $\hat{A}_{12}$ and $\hat{A}_{23}$ are equal to zero, the non-piezoelectric case is degenerate and is treated as follows. $|\hat{A}_{22}|$ is calculated for each value of α. For K = 2 it is necessary that $|\hat{A}_{22}|$ be non-zero for both values of α (due to the large magnitude of the individual terms in $\hat{A}_{22}$ it is sufficient to compare $|\hat{A}_{22}|$ to 10^7). If $|\hat{A}_{22}| > 10^7$ for both values of α then we may set

$$\beta_2^{(\ell)} = 0, \ \beta_3^{(\ell)} = 10^{-10}, \quad \text{and} \quad \beta_1^{(\ell)} = -\frac{\hat{A}_{13}^{(\ell)}}{\hat{A}_{11}^{(\ell)}} \cdot 10^{-10} \quad .$$

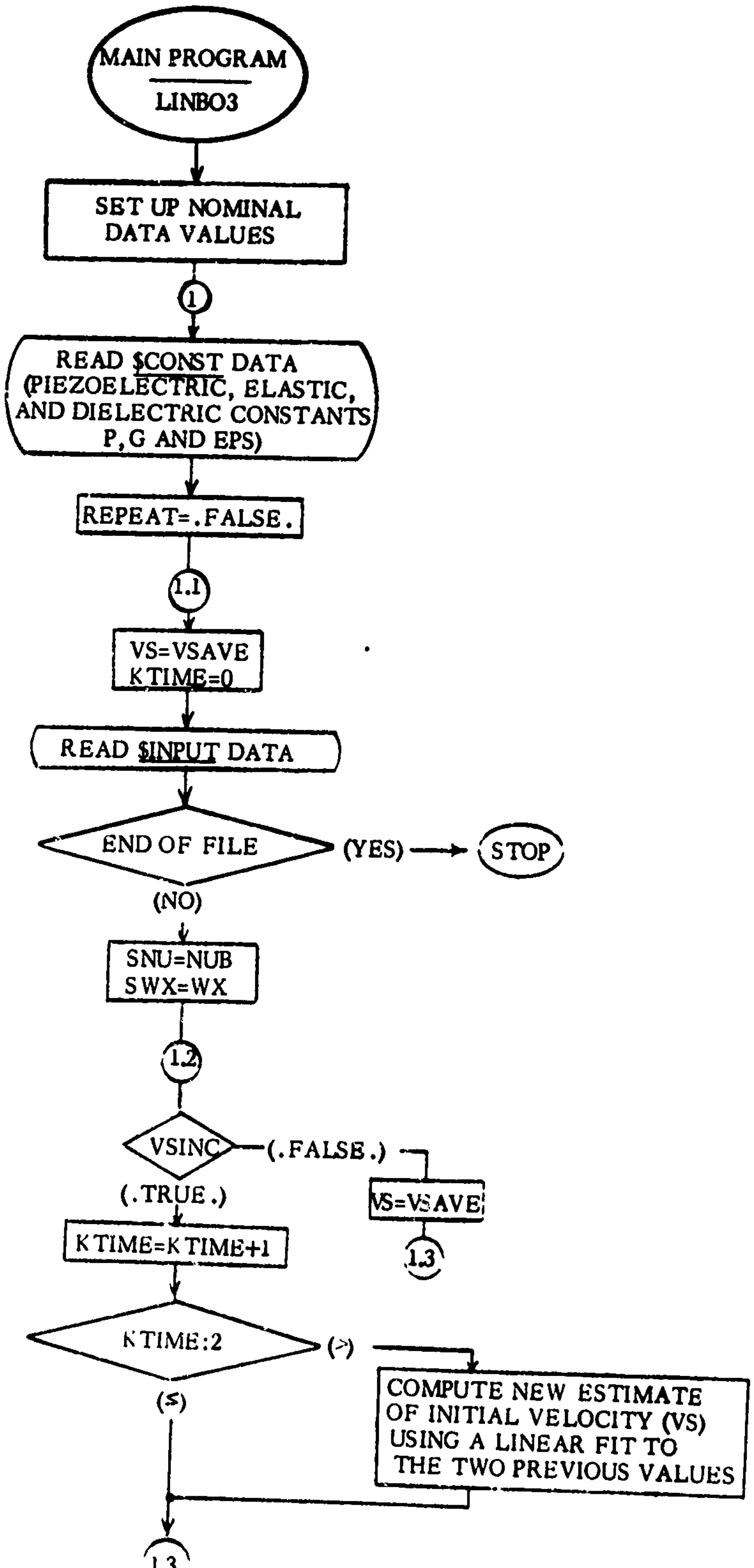
Otherwise the case is terminated. If K = 3 the minimum value of $|\hat{A}_{22}|$ is calculated and the corresponding α is discarded. The β's are then calculated for the other two α's from the above formulas.

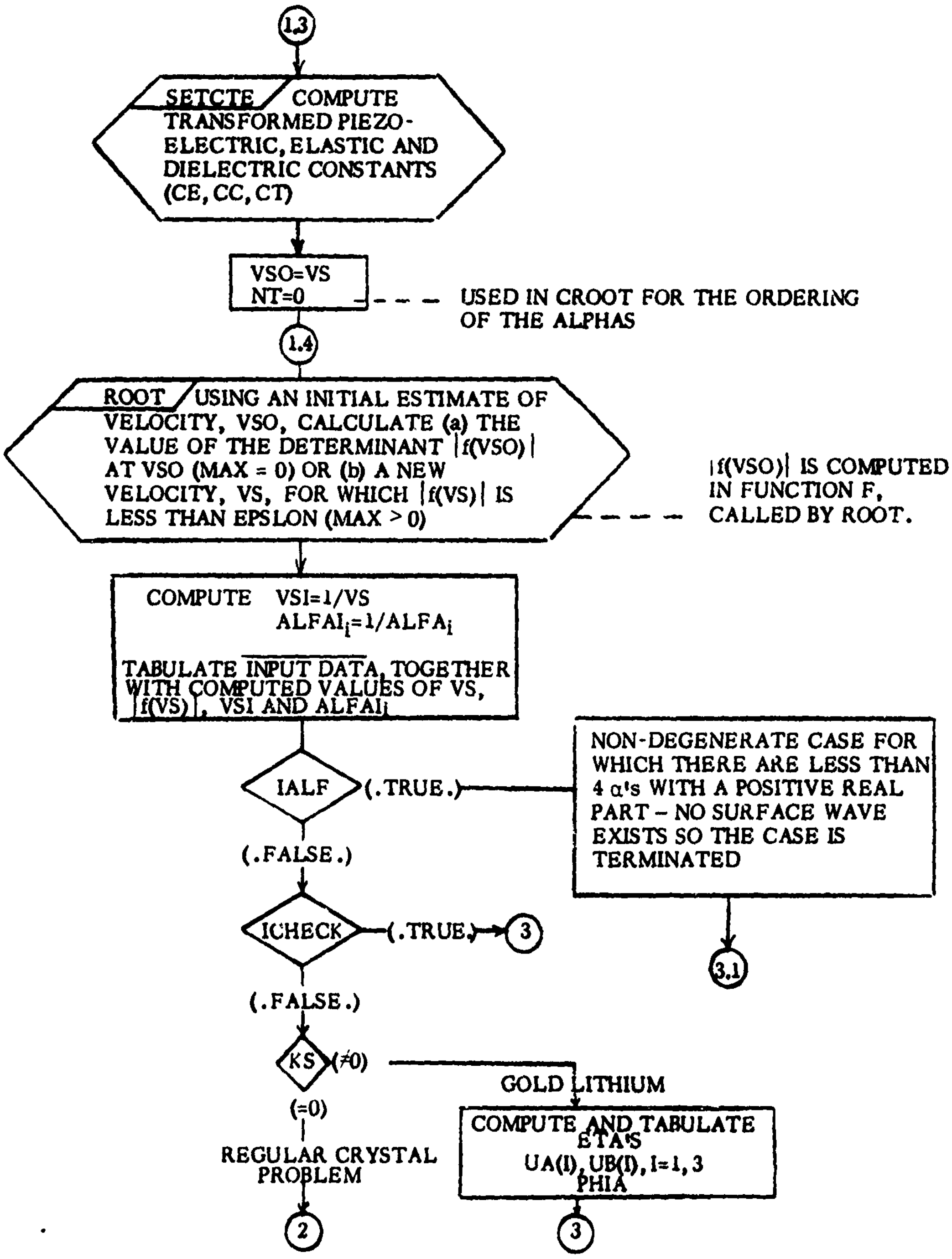
If $\hat{A}_{12}$ and $\hat{A}_{23}$ are identically equal to zero and the crystal is piezoelectric the program proceeds as follows. $\hat{A}_{24}$ is tested and if equal to zero the first degenerate case of the analysis section must be considered. The case is terminated if K = 2 but if K = 3 a check is made of $|\hat{A}_{22}|$. If $|\hat{A}_{22}| > 10^7$ for all three α's, the β's are calculated as indicated in the analysis. If $|\hat{A}_{22}| < 10^7$ for any of the α's the case is terminated.

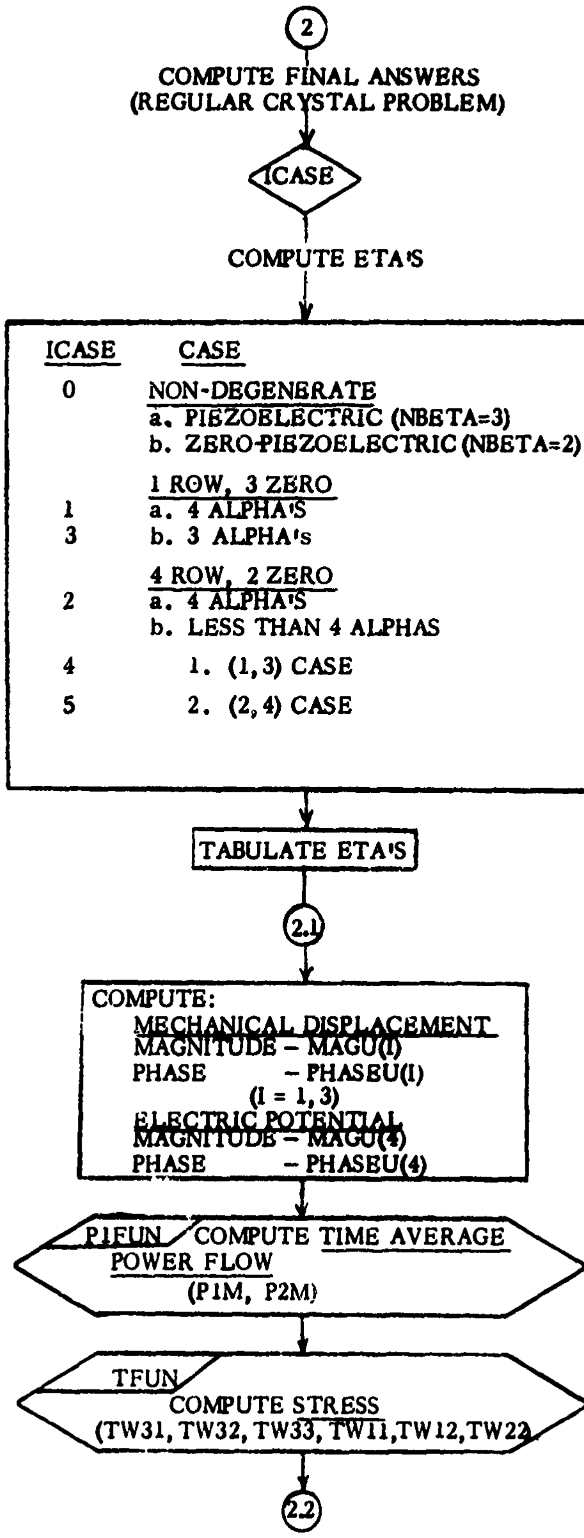
If $\hat{A}_{22} \neq 0$ a check of $\hat{A}_{14}$ and $\hat{A}_{34}$ is made. If they are not both identically equal to zero the case is terminated. If both are equal to zero the second degenerate case of the analysis is considered. In this case $|\hat{A}_{22}\hat{A}_{44} - \hat{A}_{24}^2|$ (TERMJ in the program) is calculated for each α. If TERMJ $\geq 10^{-5}$ for two of the α's the (β_1, β_3) split of the analysis arises and the β's are appropriately calculated. If TERMJ $< 10^{-5}$ for two of the α's the (β_2, β_4) split arises and the β's are appropriately calculated. Under any other conditions the case is terminated.

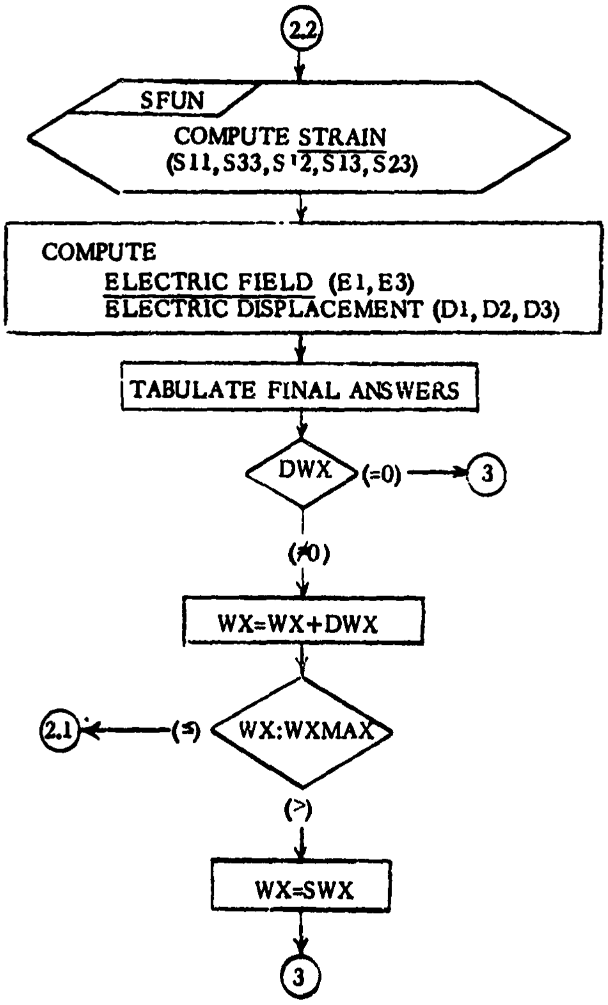
Now that the α's and β's are known the boundary condition matrix $(\hat{L})$ is set up and its determinant evaluated. If the problem of a conducting elastic medium in contact with a piezoelectric or elastic medium is being considered

the α's and β's appropriate to the conductor are first evaluated then the appropriate boundary condition matrix is set up and its determinant evaluated. This completes the computation of F(VS) whereupon the subroutine is exited back to the main program.

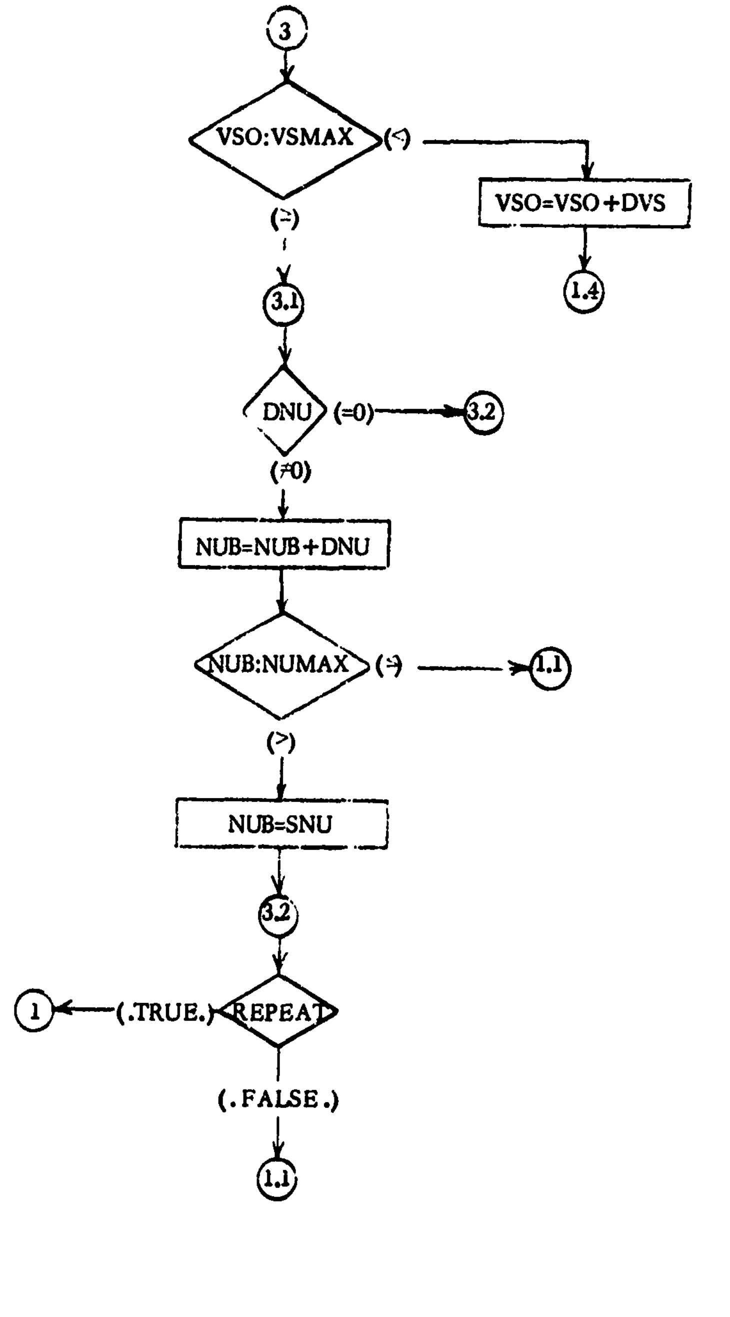

MAIN PROGRAM
LINBO3
SET UP NOMINAL DATA VALUES
1
READ $CONST DATA (PIEZOELECTRIC, ELASTIC, AND DIELECTRIC CONSTANTS P, G AND EPS)
REPEAT=.FALSE.
1.1
VS=VSAVE
KTIME=0
READ $INPUT DATA
END OF FILE
(YES)
STOP
(NO)
SNU=NUB
SWX=WX
1.2
VSINC
(.FALSE.)
VS=VSAVE
1.3
(.TRUE.)
KTIME=KTIME+1
KTIME:2
(>)
COMPUTE NEW ESTIMATE OF INITIAL VELOCITY (VS) USING A LINEAR FIT TO THE TWO PREVIOUS VALUES
(≤)
1.3

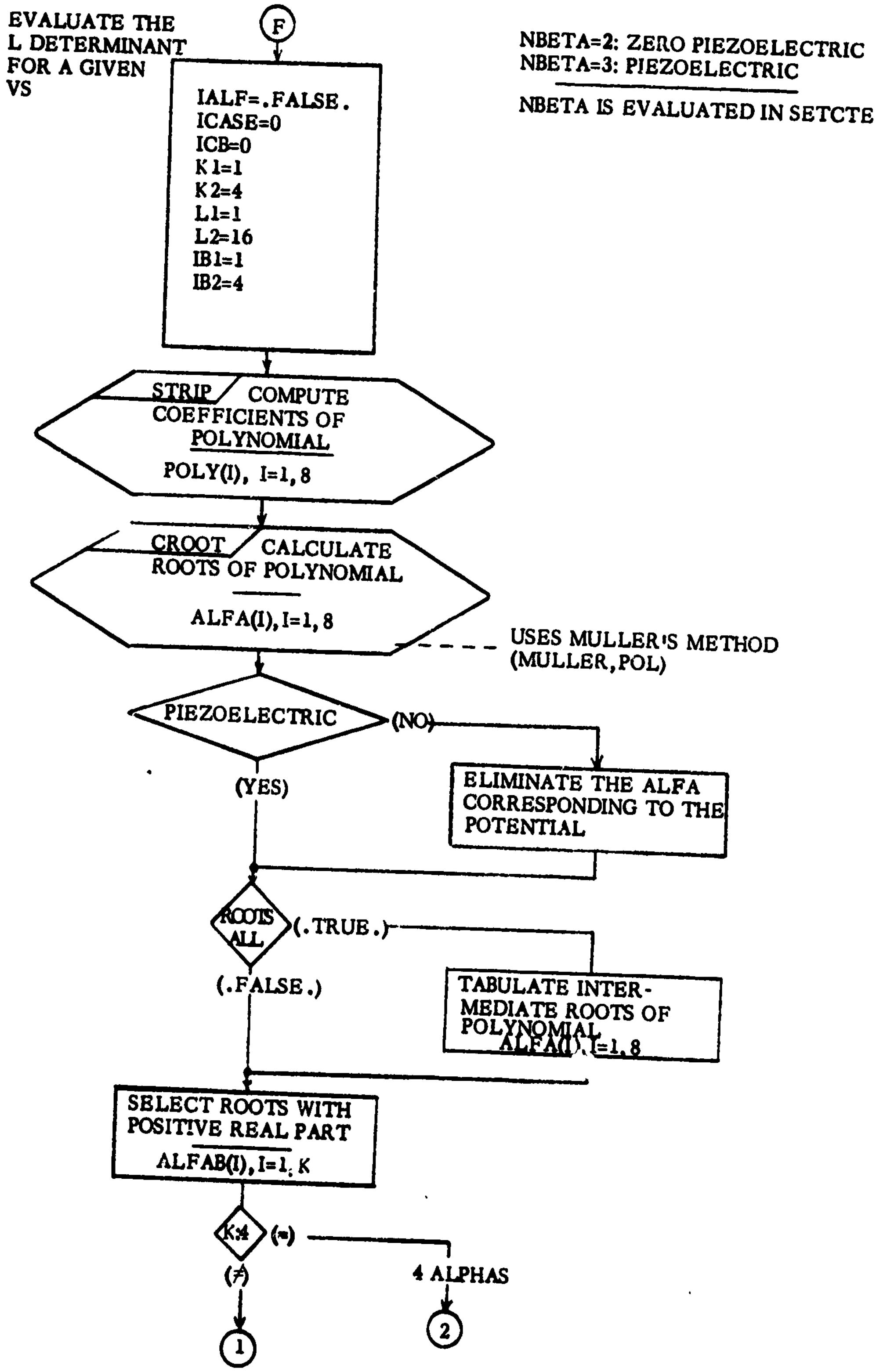

1.3
SETCTE
COMPUTE TRANSFORMED PIEZO-ELECTRIC, ELASTIC AND DIELECTRIC CONSTANTS (CE, CC, CT)
VSO=VS
NT=0
USED IN CROOT FOR THE ORDERING OF THE ALPHAS
1.4
ROOT
USING AN INITIAL ESTIMATE OF VELOCITY, VSO, CALCULATE (a) THE VALUE OF THE DETERMINANT |f(VSO)| AT VSO (MAX = 0) OR (b) A NEW VELOCITY, VS, FOR WHICH |f(VS)| IS LESS THAN EPSLON (MAX > 0)
|f(VSO)| IS COMPUTED IN FUNCTION F, CALLED BY ROOT.
COMPUTE VSI=1/VS
ALFAI_i=1/ALFA_i
TABULATE INPUT DATA, TOGETHER WITH COMPUTED VALUES OF VS, |f(VS)|, VSI AND ALFAI_i
IALF
(.TRUE.)
NON-DEGENERATE CASE FOR WHICH THERE ARE LESS THAN 4 α's WITH A POSITIVE REAL PART – NO SURFACE WAVE EXISTS SO THE CASE IS TERMINATED
3.1
(.FALSE.)
ICHECK
(.TRUE.)
3
(.FALSE.)
KS
(≠0)
GOLD LITHIUM
COMPUTE AND TABULATE
ETA'S
UA(I), UB(I), I=1, 3
PHIA
3
(=0)
REGULAR CRYSTAL PROBLEM
2

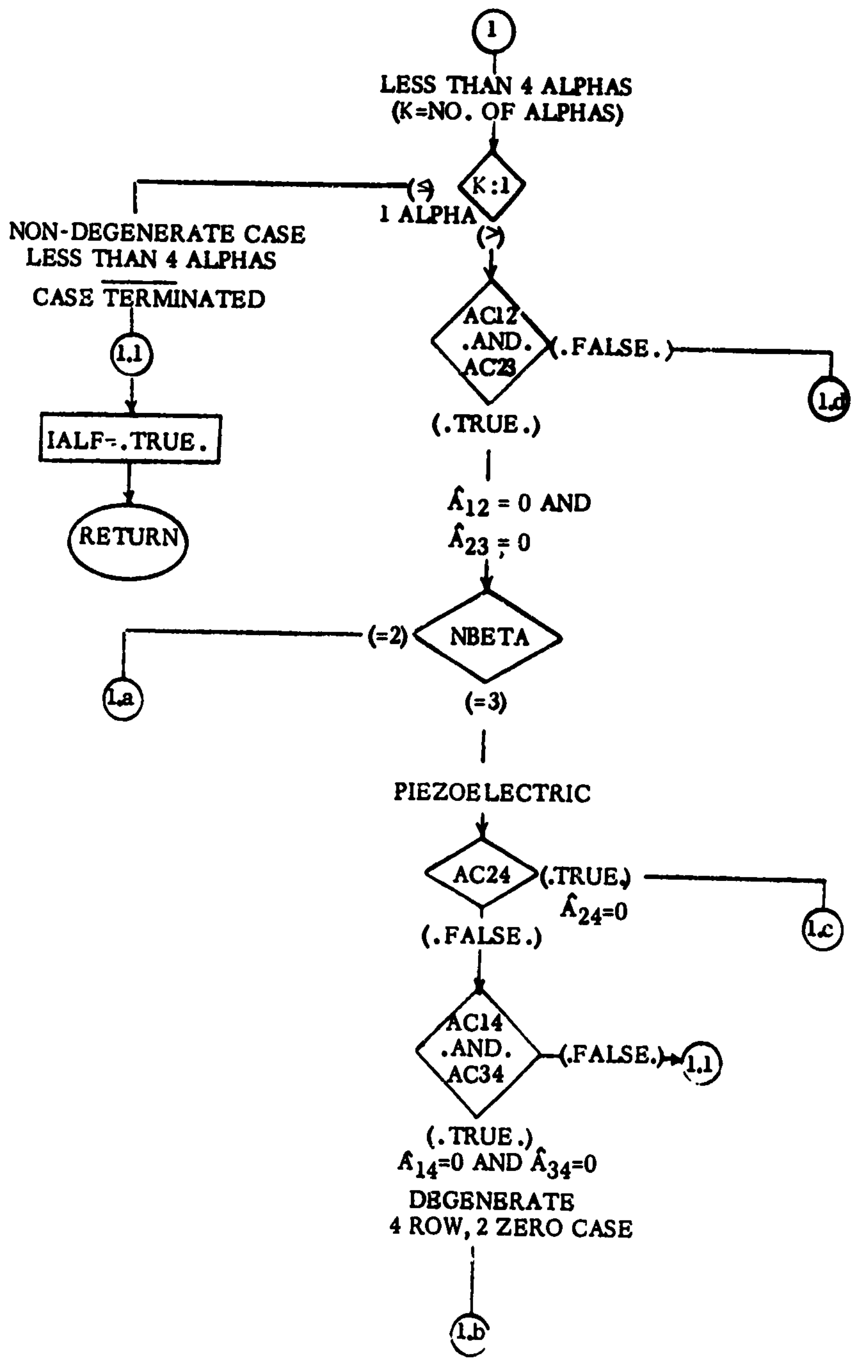

2
COMPUTE FINAL ANSWERS
(REGULAR CRYSTAL PROBLEM)
ICASE
COMPUTE ETA'S
ICASE CASE
0 NON-DEGENERATE
a. PIEZOELECTRIC (NBETA=3)
b. ZERO-PIEZOELECTRIC (NBETA=2)
1 ROW, 3 ZERO
1 a. 4 ALPHA'S
3 b. 3 ALPHA's
4 ROW, 2 ZERO
2 a. 4 ALPHA'S
b. LESS THAN 4 ALPHAS
4 1. (1, 3) CASE
5 2. (2, 4) CASE
TABULATE ETA'S
2.1
COMPUTE:
MECHANICAL DISPLACEMENT
MAGNITUDE – MAGU(I)
PHASE – PHASEU(I)
(I = 1, 3)
ELECTRIC POTENTIAL
MAGNITUDE – MAGU(4)
PHASE – PHASEU(4)
P1FUN
COMPUTE TIME AVERAGE POWER FLOW
(P1M, P2M)
TFUN
COMPUTE STRESS
(TW31, TW32, TW33, TW11, TW12, TW22)
2.2

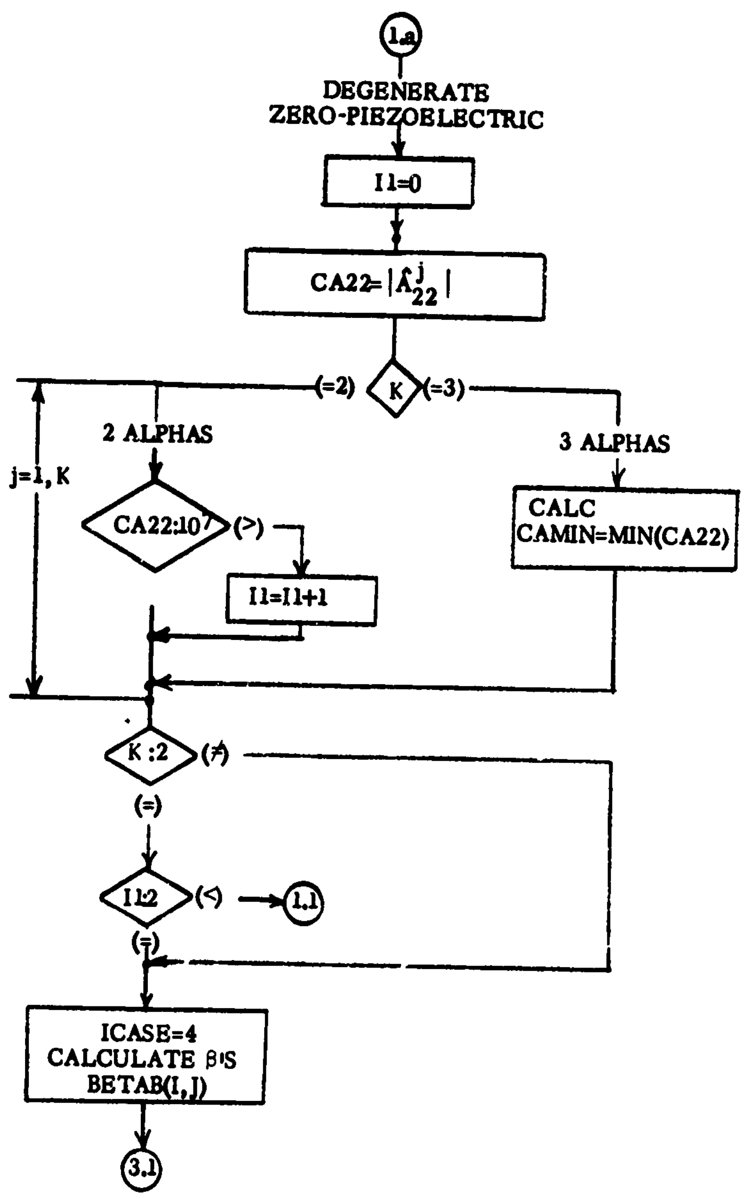


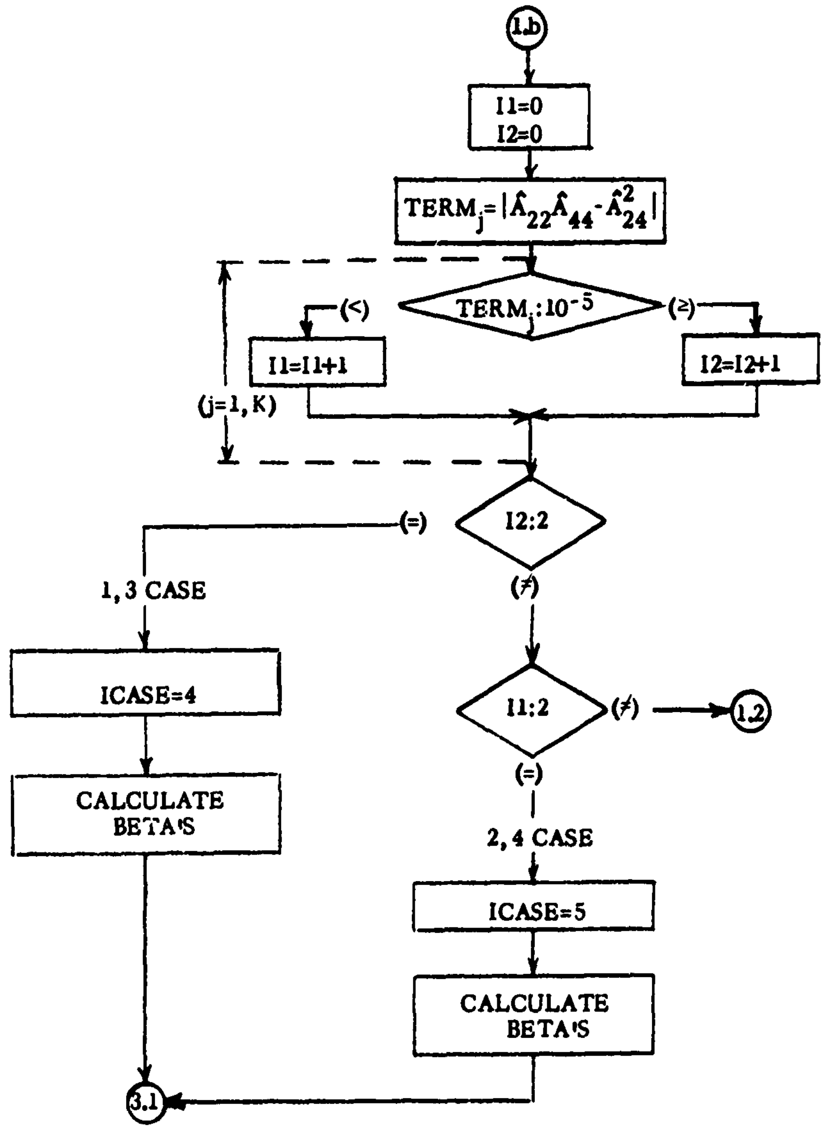
2.2
SFUN
COMPUTE STRAIN
(S11, S33, S12, S13, S23)
COMPUTE
ELECTRIC FIELD (E1, E3)
ELECTRIC DISPLACEMENT (D1, D2, D3)
TABULATE FINAL ANSWERS
DWX
(=0)
3
(≠0)
WX=WX+DWX
WX:WXMAX
(≤)
2.1
(>)
WX=SWX
3

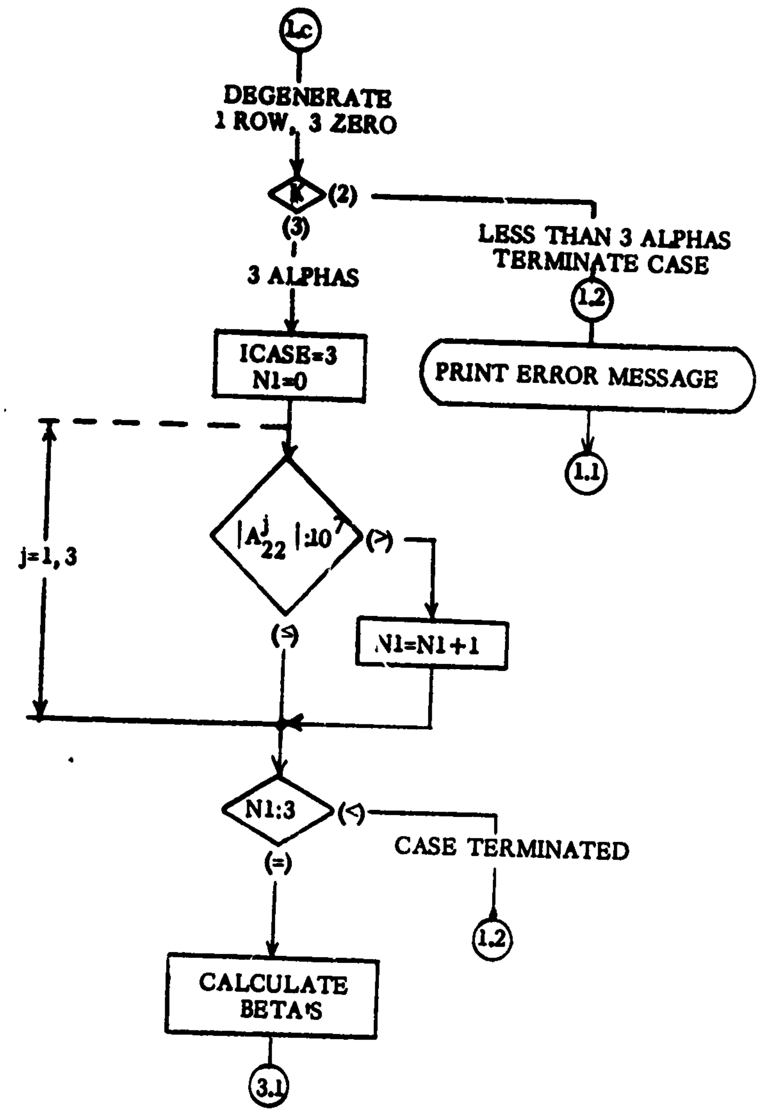

3
VSO:VSMAX
(<)
VSO=VSO+DVS
1.4
(≥)
3.1
DNU
(=0)
3.2
(≠0)
NUB=NUB+DNU
NUB:NUMAX
(≤)
1.1
(>)
NUB=SNU
3.2
1
(.TRUE.)
REPEAT
(.FALSE.)
1.1

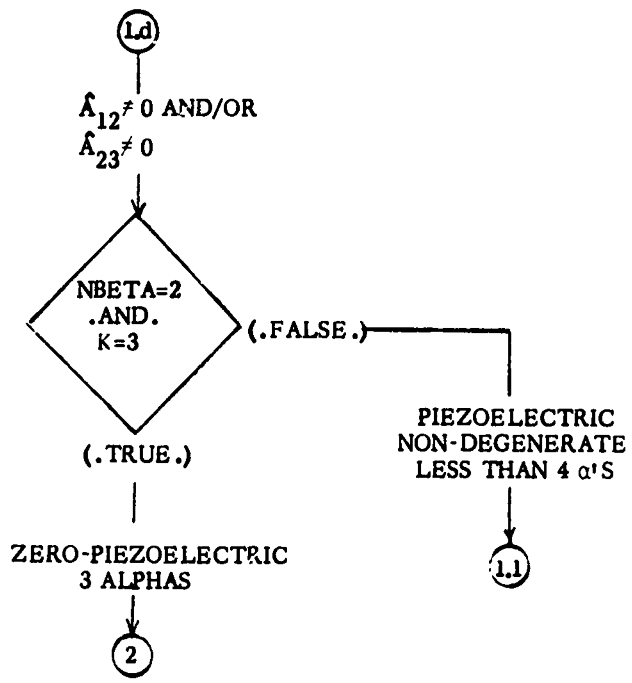

EVALUATE THE L DETERMINANT FOR A GIVEN VS
F
NBETA=2: ZERO PIEZOELECTRIC
NBETA=3: PIEZOELECTRIC
NBETA IS EVALUATED IN SETCTE
IALF=.FALSE.
ICASE=0
ICB=0
K1=1
K2=4
L1=1
L2=16
IB1=1
IB2=4
STRIP
COMPUTE COEFFICIENTS OF POLYNOMIAL
POLY(I), I=1, 8
CROOT
CALCULATE ROOTS OF POLYNOMIAL
ALFA(I), I=1, 8
USES MULLER'S METHOD (MULLER, POL)
PIEZOELECTRIC
(NO)
(YES)
ELIMINATE THE ALFA CORRESPONDING TO THE POTENTIAL
ROOTS ALL
(.TRUE.)
(.FALSE.)
TABULATE INTERMEDIATE ROOTS OF POLYNOMIAL
ALFA(I), I=1, 8
SELECT ROOTS WITH POSITIVE REAL PART
ALFAB(I), I=1, K
K:4
(=)
(≠)
4 ALPHAS
1
2

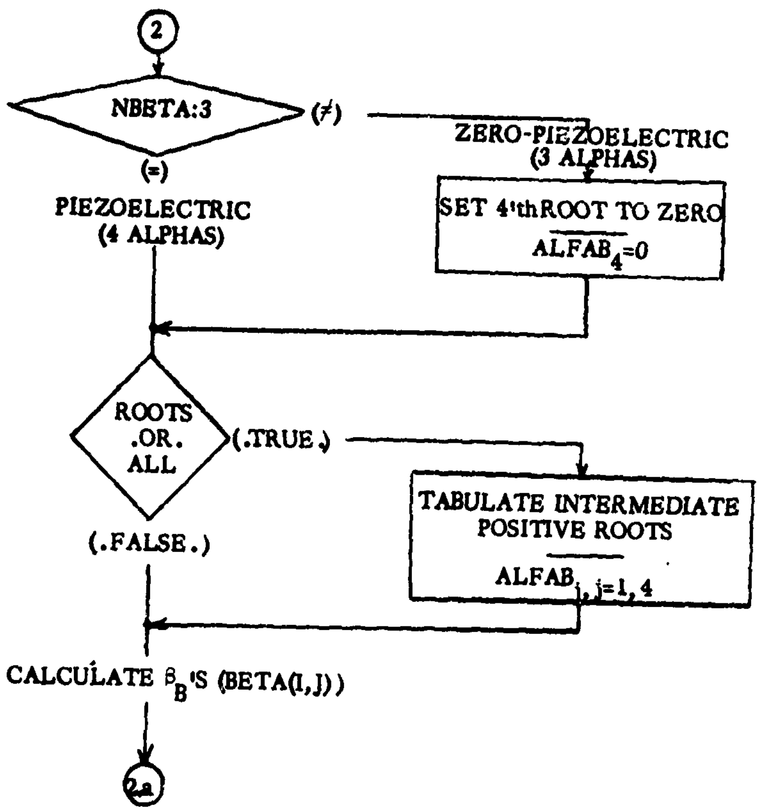

1
LESS THAN 4 ALPHAS
(K=NO. OF ALPHAS)
K:1
(≤)
1 ALPHA
(>)
NON-DEGENERATE CASE
LESS THAN 4 ALPHAS
CASE TERMINATED
1.1
IALF=.TRUE.
RETURN
AC12
.AND.
AC23
(.FALSE.)
1.d
(.TRUE.)
Â12 = 0 AND
Â23 = 0
NBETA
(=2)
1.a
(=3)
PIEZOELECTRIC
AC24
(.TRUE.)
Â24=0
1.c
(.FALSE.)
AC14
.AND.
AC34
(.FALSE.)
1.1
(.TRUE.)
Â14=0 AND Â34=0
DEGENERATE
4 ROW, 2 ZERO CASE
1.b

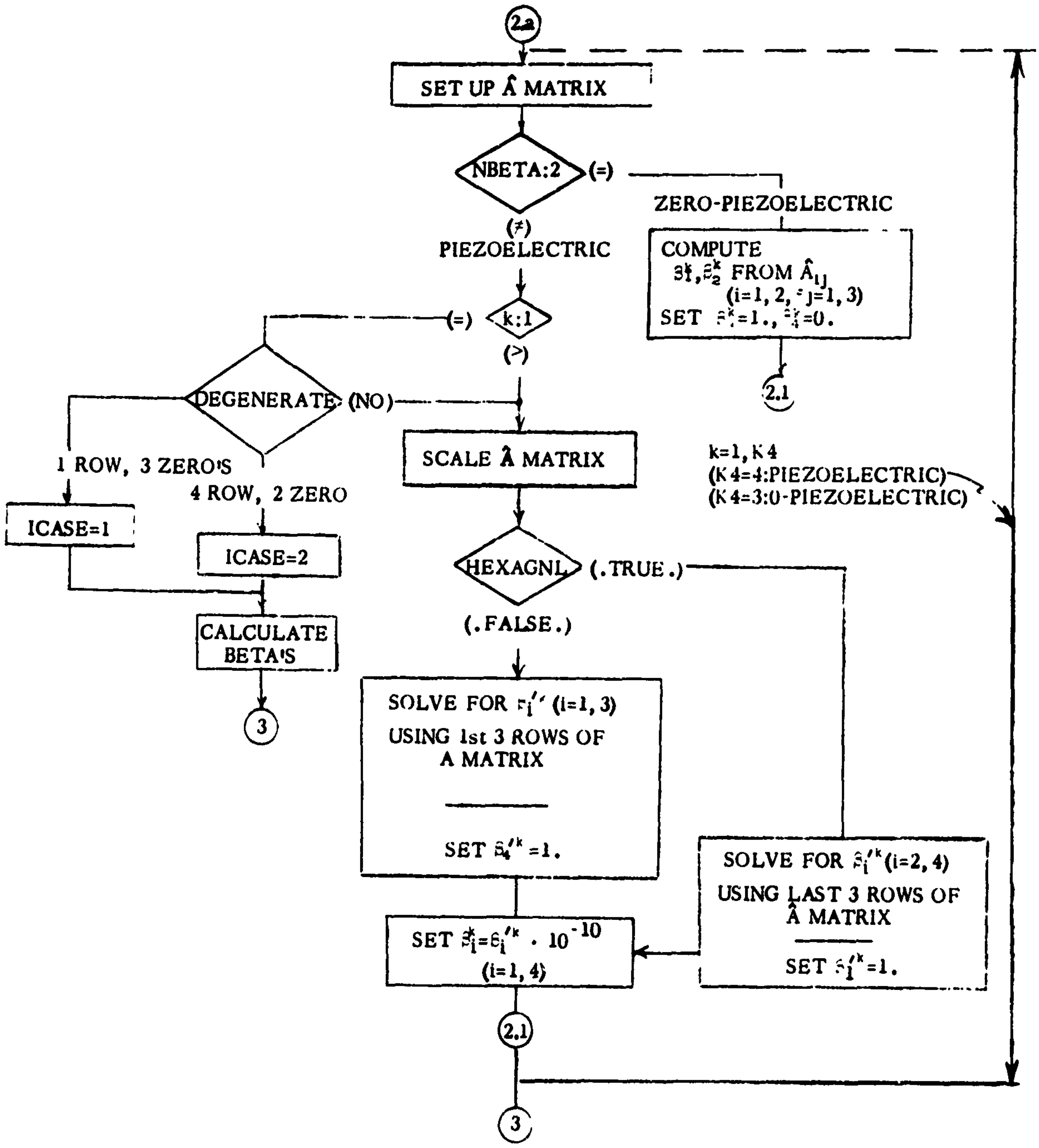

1.a
DEGENERATE
ZERO-PIEZOELECTRIC
I1=0
CA22=|Â^j_22|
(=2)
K
(=3)
2 ALPHAS
3 ALPHAS
j=1, K
CA22:10^7
(>)
I1=I1+1
CALC
CAMIN=MIN(CA22)
K:2
(≠)
(=)
I1:2
(<)
1.1
(=)
ICASE=4
CALCULATE β'S
BETAB(I,J)
3.1

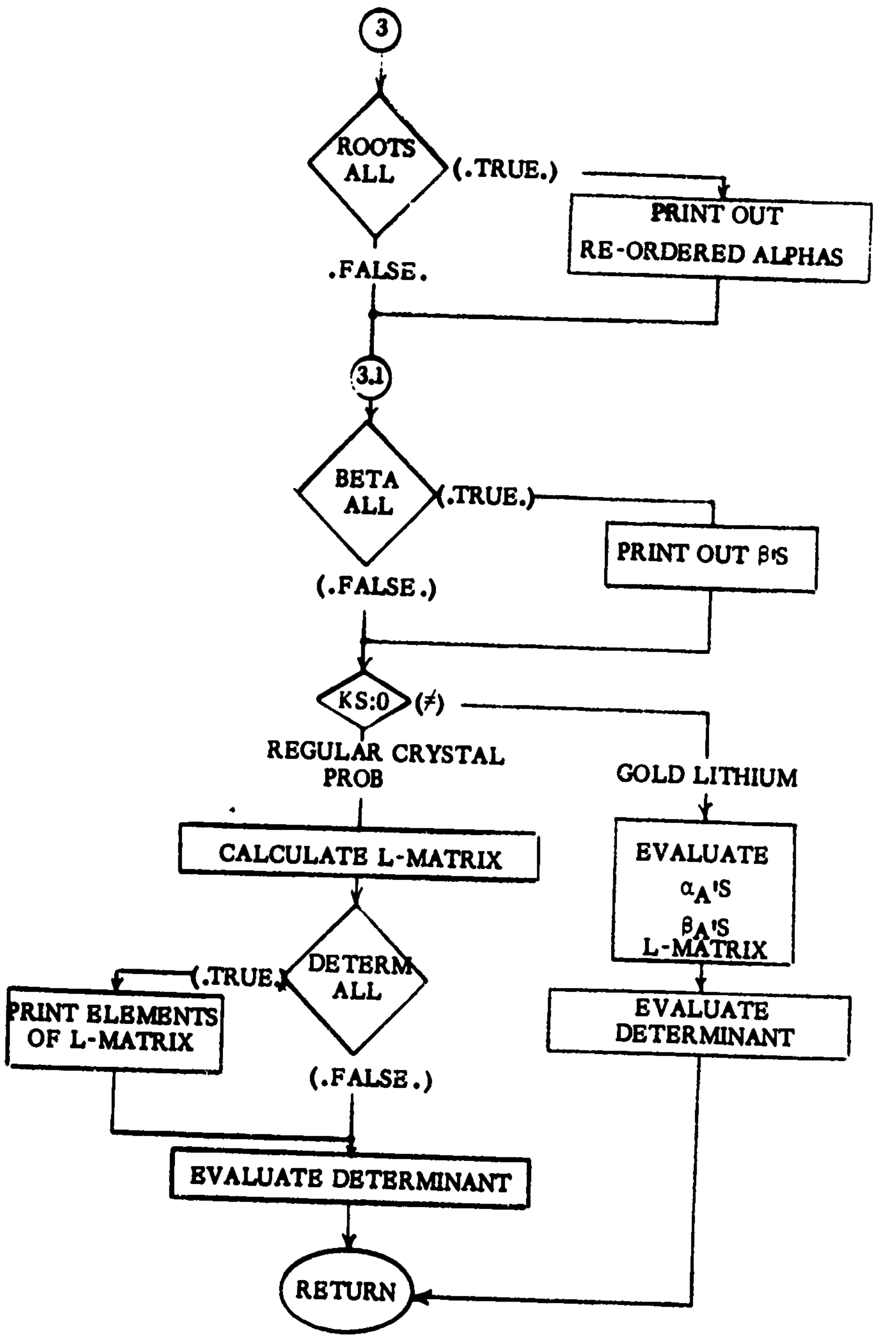

1.b
I1=0
I2=0
TERM_j = |Â22 Â44 - Â24^2|
TERM_j : 10^-5
(<)
(≥)
I1=I1+1
I2=I2+1
(j=1, K)
I2:2
(=)
(≠)
1, 3 CASE
ICASE=4
CALCULATE BETA'S
I1:2
(≠)
1.2
(=)
2, 4 CASE
ICASE=5
CALCULATE BETA'S
3.1


1.c
DEGENERATE
1 ROW, 3 ZERO
K
(2)
(3)
LESS THAN 3 ALPHAS
TERMINATE CASE
1.2
3 ALPHAS
ICASE=3
N1=0
PRINT ERROR MESSAGE
1.1
j=1, 3
$|A^j_{22}|:10^{-7}$
(>)
(≤)
N1=N1+1
N1:3
(<)
(=)
CASE TERMINATED
1.2
CALCULATE
BETA'S
3.1


1.d
Â12≠ 0 AND/OR
Â23≠ 0
NBETA=2
.AND.
K=3
(.FALSE.)
(.TRUE.)
PIEZOELECTRIC
NON-DEGENERATE
LESS THAN 4 α'S
1.1
ZERO-PIEZOELECTRIC
3 ALPHAS
2


2
NBETA:3
(≠)
(=)
ZERO-PIEZOELECTRIC
(3 ALPHAS)
PIEZOELECTRIC
(4 ALPHAS)
SET 4'th ROOT TO ZERO
ALFAB4=0
ROOTS
.OR.
ALL
(.TRUE.)
(.FALSE.)
TABULATE INTERMEDIATE
POSITIVE ROOTS
ALFABj, j=1, 4
CALCULATE βB'S (BETA(I,J))
2a


2a
SET UP Â MATRIX
NBETA:2
(=)
ZERO-PIEZOELECTRIC
(≠)
PIEZOELECTRIC
COMPUTE
FROM Â
(i=1, 2, j=1, 3)
SET
2.1
k:1
(=)
(>)
DEGENERATE
(NO)
1 ROW, 3 ZERO'S
4 ROW, 2 ZERO
ICASE=1
ICASE=2
CALCULATE BETA'S
3
SCALE Â MATRIX
k=1, K4
(K4=4:PIEZOELECTRIC)
(K4=3:0-PIEZOELECTRIC)
HEXAGNL
(.TRUE.)
(.FALSE.)
SOLVE FOR (i=1, 3)
USING 1st 3 ROWS OF A MATRIX
SET =1.
SOLVE FOR (i=2, 4)
USING LAST 3 ROWS OF Â MATRIX
SET =1.
SET · 10^-10
(i=1, 4)
2.1
3



3
ROOTS ALL
(.TRUE.)
PRINT OUT RE-ORDERED ALPHAS
.FALSE.
3.1
BETA ALL
(.TRUE.)
PRINT OUT β'S
(.FALSE.)
KS:0
(≠)
REGULAR CRYSTAL PROB
GOLD LITHIUM
CALCULATE L-MATRIX
EVALUATE
α_A'S
β_A'S
L-MATRIX
DETERM ALL
(.TRUE.)
PRINT ELEMENTS OF L-MATRIX
EVALUATE DETERMINANT
(.FALSE.)
EVALUATE DETERMINANT
RETURN

2. Surface Waves at the Boundary Between a Fluid Medium and Piezoelectric Crystal – Program Description

The purpose of this program is to determine the complex velocity of propagation of surface waves at the interface between a semi-infinite fluid and a piezoelectric substrate. Input parameters which define the fluid, the piezoelectric medium, and control the use of the program are described on the following pages.

The program is set up to run on the IBM 7094, using FORTRAN IV and Namelist input and the deck set up is identical with that given for the preceding program.

As in the proceding program, two input sections are required: the first describes the material constants of the piezoelectric crystal and the second describes the orientation of the crystal as well as other information pertinent to the execution of the program. The first data set is called CONST and is identical with that presented in Section IV.1. The following is a definition of each input parameter in the "INPUT" data set. Medium A refers to the dielectric (elastic) layer and medium B, to the piezoelectric substrate.

Input Name	Equation Name	Type	Definition
LAMDAB	λ_B	real	Euler angles for Medium B.
MUB	μ_B	real	
NUB	ν_B	real	
DNU	$\Delta\nu$	real	If the user wishes to vary ν (NUB) from some initial value, ν, to some final value, ν_{max}, in steps of $\Delta\nu$, then set DNU equal to the steps desired; also, see NUMAX. (See VSINC)
NUMAX	ν_{max}	real	The maximum value of ν (see DNU). ν_{max} is only used when DNU $\neq 0$.
VS	v_s	real	Initial estimate of velocity. This initial value will be used to find a final velocity, $\bar{v}_s$, such that $\|f(\bar{v}_s)\| < \epsilon$, where ϵ is input.
DVS	Δv_s	real	If the user does not care to use the root-finding scheme in determining a final value for v_s, but wishes, instead, to evaluate the determinant $\|f(v_s)\|$ for particular values of v_s in the range from v_s to $v_{s_{max}}$ in steps of Δv_s, then set DVS equal to the step size desired. (For use when MAX = 0.)
VSMAX	$v_{s_{max}}$	real	Maximum value of v_s to be used when DVS $\neq 0$.
LAMDAA	λ_A	real	Lame constants for Medium A.
MUA	μ_A	real	
RHOA	ρ_A	real	Mass density of Medium A.
RHOB	ρ_B	real	Mass density of Medium B.
EPSLON	ϵ	real	A positive number used as a convergence criterion by the root-finding scheme (MAX > 0). If $\|f(v_s)\| < \epsilon$, then v_s is assumed to be the root required.
EPSO	ϵ_o	real	Permittivity of free space.

Input Name	Equation Name	Type	Definition
EPSA	ϵ_A	real	Dielectric constant for Medium A.
WH	ωh	real	Frequency thickness product.
WXA	ωx_A	real	Normalized distance into Medium A.
DWXA	$\Delta\omega x_A$	real	In order to vary ωx_A (WXA) from an initial value, ωx_A, to a final value $\omega x_{A_{max}}$, DWXA must be set equal to the desired step size. See WXAMAX.
WXAMAX	$\omega x_{A_{max}}$	real	The maximum value of ωx_A to be used when DWXA $\neq 0$.
WXB	ωx_B	real	Normalized distance into Medium B.
DWXB	$\Delta\omega x_B$	real	In order to vary ωx_B from an initial value, ωx_B, to a final value, $\omega x_{B_{max}}$, DWXB must be set equal to the step size desired. See WXBMAX.
WXBMAX	$\omega x_{B_{max}}$	real	The maximum value of ωx_B to be used when DWXB $\neq 0$.
ICHECK	----	logical	ICHECK = .TRUE. – All FINAL ANSWERS* are computed in addition to the evaluation of the determinant $\lvert f(\overline{v}_s)\rvert$. ICHECK = .FALSE. – FINAL ANSWERS are not computed; evaluate determinant only.
MAX	----	integer	Since an iteration scheme is used for convergence for a final root v_s, there must be an indication of how many iterations are to be executed before divergence is assumed. Hence, MAX should be the maximum number of iterations the user wishes the program to make (usually 15). If MAX is set to zero (MAX = 0) the determinant $\lvert f(\overline{v}_s)\rvert$ will be evaluated for the particular v_s value input – the iteration scheme will not be used. This option may be useful if there is difficulty in determining the range in which v_s lies.

*The FINAL RESULTS, which are computed for all values of WXA (dielectric layer) and WXB (piezoelectric layer), include the following:

- Stress Components
- Strain Components
- Time Average Power Flow
- Electric Displacement
- Mechanical Displacement
- Electric Potential Magnitude
- Electric Field

Input Name	Equation Name	Type	Definition
TITLE	----	BCD	An alphanumeric array of 24 characters or less used to describe the type of crystal, such as lithium niobate. This is input in the following manner: TITLE = nH name of crystal, where n is the number of characters following the H (including blanks). For example TITLE = 6HQUARTZ
HXAGNL	----	logical	Parameter which controls the calculation of betas (β's) for a hexagonal crystal (such as zinc oxide) .TRUE. hexagonal crystal (use special technique) .FALSE. non-hexagonal crystal (use normal procedure)
VSINC	----	logical	VSINC = .TRUE. – New estimates of initial velocity (v_S) are computed using a linear fit to the two previous values. (Used when NUB varies over a range NUB, NUB + DNU, ... , NUMAX) VSINC = .FALSE. – The same initial estimate of velocity is used for all values over the specified range of NUB.
IOP	----	integer	Degenerate case options (used when exactly four α's with positive real part occur). IOP = 1 – seek modes of propagation of the Quasi-Rayleigh or Sesawa type. IOP = 2 – seek modes of propagation of Love type.
REPEAT	----	logical	REPEAT is a logical variable and in its usage, can take only one value: .TRUE. If there are no more cases to run after the current case, REPEAT does not need to be input. If there will be another case to follow, but the crystal coefficients remain the same, then, again, REPEAT does not need to be input. However, if another case is to be run and the coefficients are different, then REPEAT needs to be input

Input Name	Equation Name	Type	Definition
			as .TRUE. This means that the $CONST data will have to be input again (in the other cases above, $CONST would not have to be input again).

The following input parameters are all logical variables which are assumed to be false (.FALSE.) in the program. They are used as switches indicating whether or not intermediate calculations are to be printed. If any one, or any combination of these parameters are input as true (.TRUE.), then certain intermediate data will print, according to the following:

TABCTE	Print the constants E, C, and T (the transformed piezoelectric, elastic, and dielectric constants) calculated from the constants P, G, and EPS.
ROOTS	Print the roots of the polynomial each time they are calculated.
BETA	Print the values of β_{ij}.
DETERM	Print the value of the determinant.
COEFF	Print the coefficients of the 8'th order polynomial.
TABL	Print the L matrix (or $\hat{P}$, $\hat{Q}$, $\hat{R}$, etc., when used).
ALPHA	Print the roots of the polynomial (α_B^j's) and the re-ordered roots for degenerate cases.
ALL	Print all of the above.

Data items may be excluded from the input stream at the discretion of the user. Items omitted from the first data set will take on nominal values (i.e.: values assigned within the program). Items omitted from succeeding data sets will take on previously assigned values. The following is a complete list of nominal values:*

*All logical parameters have a nominal value of .FALSE.

Parameter	Nominal Value
LAMDAB	0.
MUB	0.
NUB	0.
DNU	0.
NUMAX	0.
VS	3000.
DVS	0.
VSMAX	0.
LAMDAA	1.5×10^{11}
MUA	2.85×10^{10}
RHOA	1.888×10^{4}
RHOB	4700.
EPSLON	$1. \times 10^{-11}$
EPSO	8.85×10^{-12}
EPSA	44.25×10^{-12}
WH	0.
WXA	0.
DWXA	0.
WXAMAX	0.
WXB	0.
DWXB	0.
WXBMAX	0.
MAX	15.
TITLE	LITHIUM NIOBATE
IOP	1

The computer program flow described below shares a good many features of the program described in Section IV.1. As in the preceding programs the nominal data values are set up first. Next the piezoelectric (P), elastic (G), and dielectric (EPS) constants are read in (CONST DATA). Following this the rest of the input data is entered (INPUT DATA).

At this point subroutine SETCTE is called to compute the transformed piezoelectric (CE), elastic (CC), and dielectric (CT) constants. Next subroutine ROOT is called. ROOT performs the calculations and calls the subroutines necessary to perform the following tasks:

(a) Compute F(VS), the boundary condition determinant,

(b) implement a complex root finding scheme to minimize |F(VS)| as a function of complex velocity (VS),

(c) perform the perturbation analysis.

ROOT calls subroutine F to perform the manipulations necessary to compute the boundary condition determinant. F will be discussed in some detail below.

After exiting from ROOT and returning to the main program the trial velocity (VS) can be incremented if it is desired only to compute F(VS) at specified velocities rather than implement the root finding scheme or the perturbation scheme. When this has been completed the third Euler angle (NU) can be incremented and all of the steps, from the point where SETCTE is called to calculate the transformed piezoelectric, elastic, and dielectric constants, are repeated for each value of NU.

Following this the program returns to read in new data in either of the following fashions:

(a) If the crystal is to remain the same but the orientation of the crystal face is changed (new Euler angles) the new data comes from INPUT DATA.

(b) If the crystal itself is changed as well as the Euler angles the data comes from CONST DATA and INPUT DATA in that order.

When all the data has been exhausted the program stops.

Subroutine F

Upon entering subroutine F a check is made to determine whether the perturbation scheme is to be used. If it is not to be used, VS is set equal to VSO (the input value). If the perturbation scheme is to be used a loop is begun which will allow the numerical derivatives of the various determinants to be taken by setting VS = VSO + DEL, VS = VSO - DEL and VS = VSO, in that order. For each of these velocities the necessary determinants needed to evaluate Δv_s (the perturbed velocity) are computed. The convergence of the numerical derivatives is checked by noting the differences in the computed determinants and Δv_s as DEL is allowed to take on subsequently smaller values.

The loop is begun by setting counters INIT and IDX both equal to 1. DEL = EPS(INIT) · VSO is calculated and VS is set equal to VSO + DEL. EPS(INIT) is a small number depending upon the value of INIT. Three values EPS(1), EPS(2), and EPS(3) will be used eventually as a convergence test on the numerical derivatives. The program now proceeds to set up the M, N, and P matrices as a function of VS (the P matrix is the cofactor of M_{46} and its determinant has been referred to as K in the analysis section, K = det(P)).

When the determinant of the P matrix (DP(IDX)) has been evaluated for the first time (IDX = 1) a logical check notes that IDX $\neq$ 3 and proceeds to evaluate the determinant of the N matrix (DN(IDX)). At this point another logical check notes that IDX = 1 and proceeds to set VS = VSO - DEL and IDX = 2. The program then returns to set up the new M, N, and P matrixes and evaluate DP(IDX) = DP(2). The logical check following the evaluation of DP(IDX) again notes that IDX $\neq$ 3 and therefore evaluates DN(IDX) = DN(2). The logical check following evaluation of DN(IDX) now notes that IDX $\neq$ 1 as it was before. The program therefore proceeds to calculate the numerical derivatives ((det N)$'$ and (det P)$'$) by the approximate formulas

$$\underbrace{\mathrm{DNP(INIT)}}_{(\det N)'} = \frac{\mathrm{DN(1)-DN(2)}}{2 \cdot \mathrm{DEL}} \quad \text{and} \quad \underbrace{\mathrm{DPP(INIT)}}_{(\det P)'} = \frac{\mathrm{DP(1)-DP(2)}}{2 \cdot \mathrm{DEL}} .$$

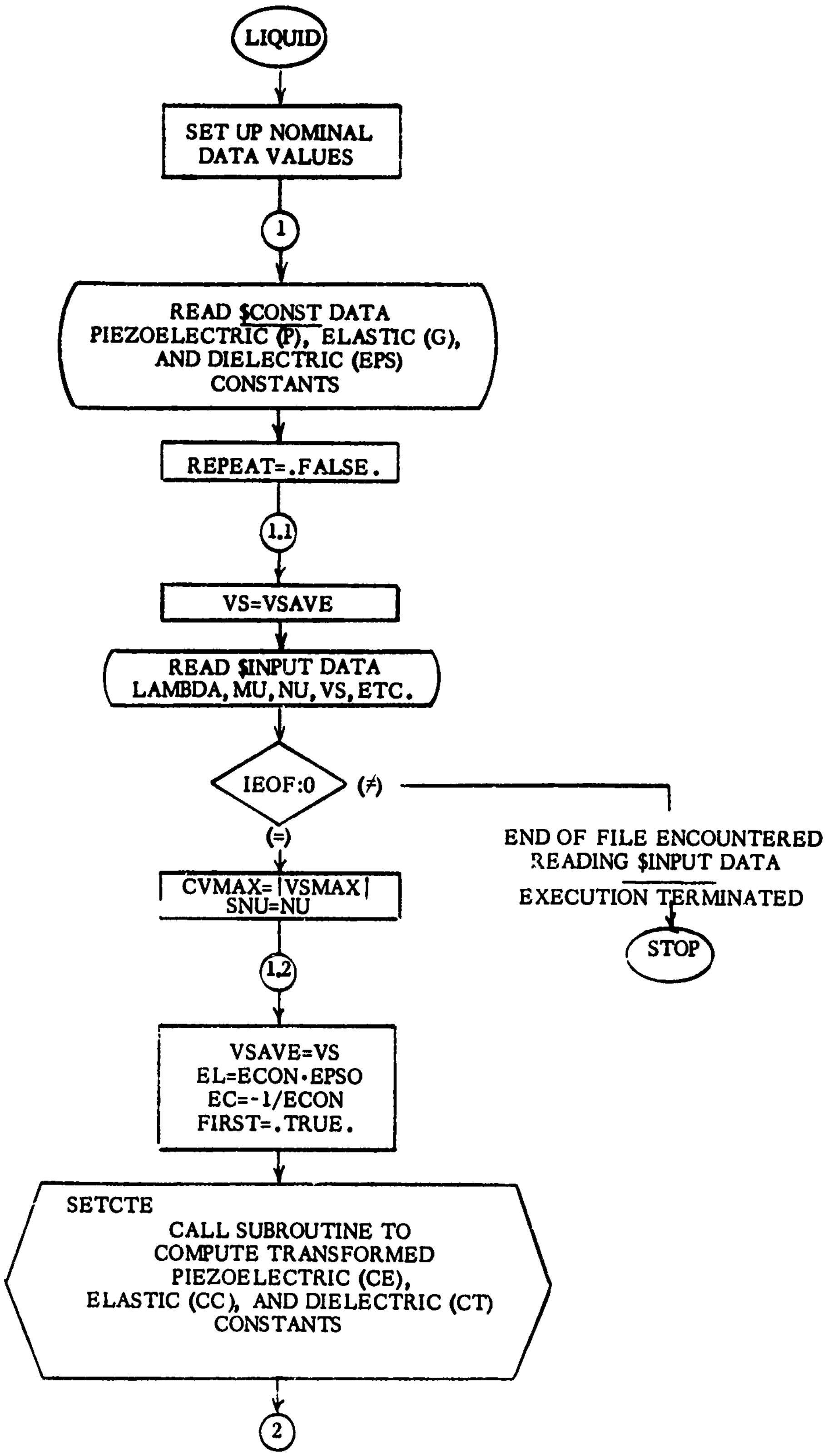
Next a new logical check notes that INIT $\neq$ 3 (it is still 1) and returns to the point where INIT and IDX were originally initialized. It now increments INIT by one (i.e. INIT = 2 now) and resets IDX = 1. DEL = EPS(2) · VSO and VS = VSO + DEL are evaluated and all the steps from this point are repeated until finally the numerical derivatives are again taken with the new value of DEL.

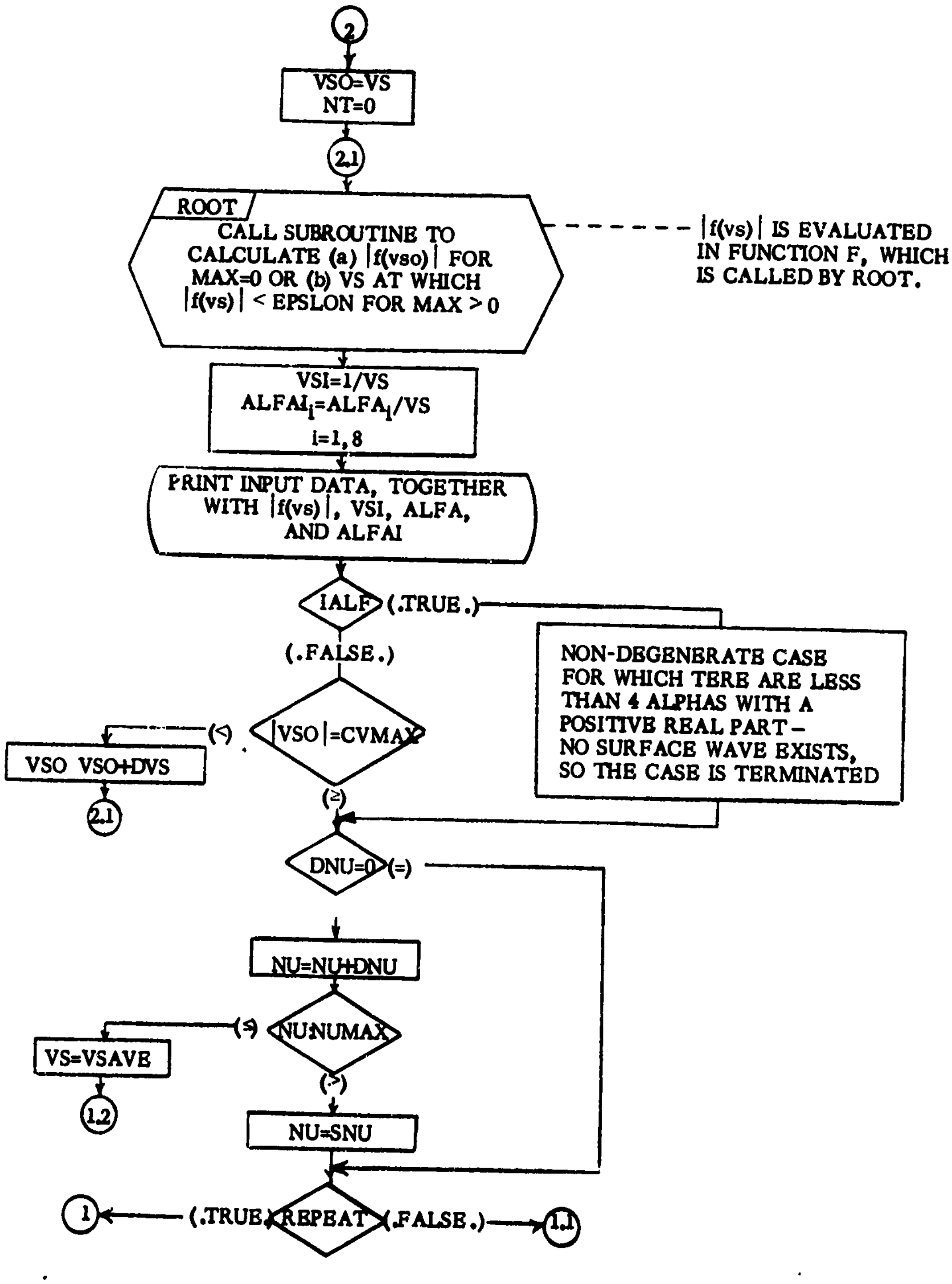
Again the logical check at this point notes INIT $\neq$ 3 (INIT = 2) and again the point of initialization of INIT and IDX is entered. INIT is again incremented by one (INIT = 3 now) and IDX is reset to 1. DEL = EPS(3) $\cdot$ VSO and VS = VSO + DEL are evaluated again and the subsequent steps again taken until the numerical derivatives are taken again at this third value of DEL. The logical check following the evaluation of the numerical derivatives now notes that INIT = 3 and thus sets IDX = 3 and returns to the point where VS was first set equal to VSO. The perturbation loop has been bypassed and VS = VSO is now used to evaluate M, N, P matrix elements. When the determinant of the P matrix (DP(IDX)) has been evaluated the subsequent logical check notes that IDX = 3 and all the quantities needed have been evaluated. It therefore proceeds to print out the results. After this the main program is re-entered to look for new cases.

The steps for any particular velocity (VS) taken to evaluate the elements of the M, N, or P matrices will now be discussed. First quantities VSQ = $(\mathrm{VS})^2$ and $\left\{\begin{array}{l}\mathrm{RVS = RHOB \cdot VSQ}\\ \mathrm{RVSQ = RHOL \cdot VSQ}\end{array}\right\}$ are set up. RHOL is the mass density of the liquid while RHOB is the mass density of the crystal. Next subroutine STRIP is called to compute the coefficients of the eighth order polynomial in α just as was done in the first program. Subroutine CROOT is now called to calculate the roots (α) of the polynomial. If a non-piezoelectric case is being considered the two extraneous roots are eliminated as in the first program. The roots with positive real part are now selected (ALFAB(I) I=1,K). If K $\leq$ 1 the case terminates. If K = 2 or 3 checks are made of various elements of the matrix of coefficients ($\hat{A}$) of the relative field amplitudes ($\beta_i^{(\ell)}$). $\hat{A}_{12}$ and $\hat{A}_{23}$ are tested to see if they are identically zero. If they are not both identically zero a degenerate case cannot exist. If the crystal is piezoelectric, then, there are insufficient α's and the case terminates. If the crystal is non-piezoelectric and K = 2 the case terminates also (insufficient α's). If the crystal is non-piezoelectric and K = 3 the appropriate β's are calculated as indicated in the analysis of the first problem. If, however, $\hat{A}_{12}$ and $\hat{A}_{23}$ are both identically zero and a non-piezoelectric case is being considered, it is a degenerate case and is so treated. If the crystal is piezoelectric a check of $\hat{A}_{24}$ is made. If $\hat{A}_{24}$ is zero and K = 2 the program terminates but if K = 3 the first degenerate case of the analysis section of the first problem has arisen (β_1, β_3, $\beta_4 \neq 0$, $\beta_2 = 0$) and is treated appropriately.

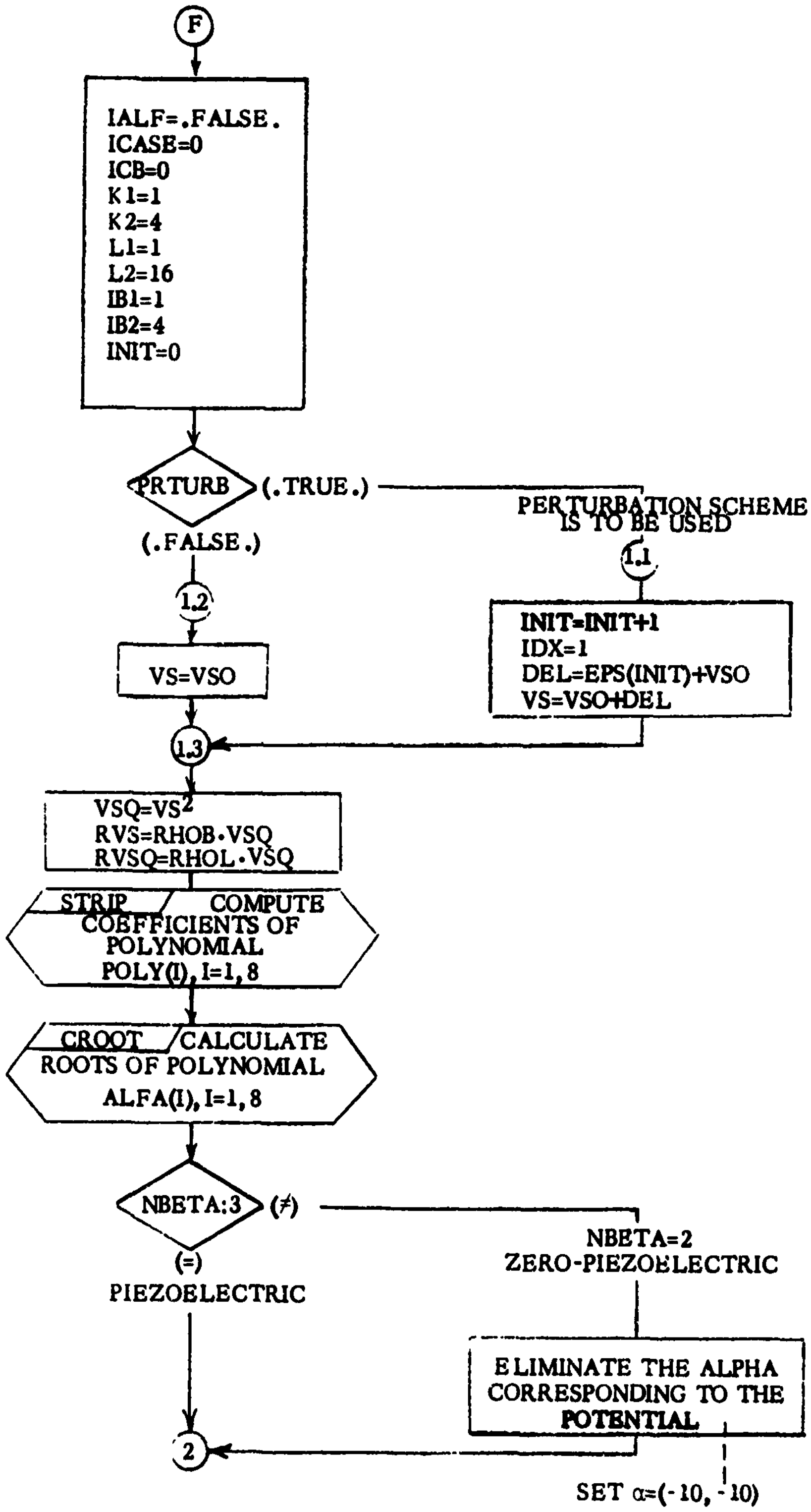
If $\hat{A}_{24}$ is not identically zero a check of $\hat{A}_{14}$ and $\hat{A}_{34}$ is made. If they are not both equal to zero the case terminates since no degenerate case has arisen. If both are identically zero the second degenerate case of the analysis section of the first problem has arisen and is treated accordingly.

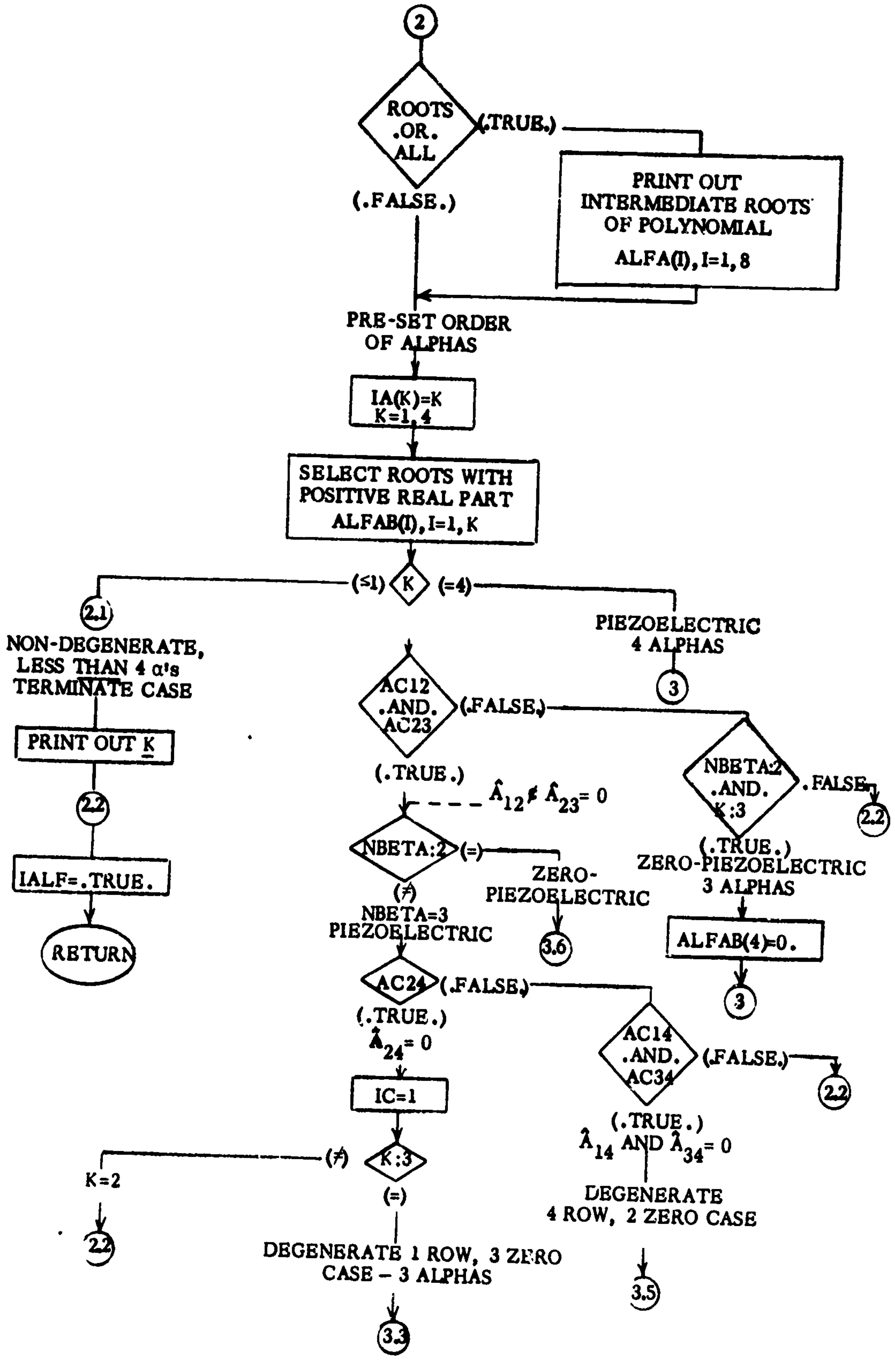
If K = 4 and a piezoelectric case is being considered or K = 3 and a non-piezoelectric case is being considered the β's are derived in the fashion indicated in the first program. After the β's have been computed the quantity $\text{ARAD} = 1 - \frac{\text{RVSQ}}{\text{LAMDAL}}$ is computed where LAMDAL (λ_ℓ) is the modulus of compression of the fluid. If no perturbation scheme is to be used the program computes $\text{ALFAL} = \sqrt{\text{ARAD}}$ and tests to see if the imaginary part of ALFAL (Im (ALFAL)) is equal to zero. If Im (ALFAL) = 0 the negative square root is taken; otherwise the root is taken so that Im (ALFAL) < 0 (this was necessary in order that a velocity with positive imaginary part result as a solution). The elements of the M matrix are set up next and the determinant evaluated (det (M) = F(VS)). If the perturbation scheme is to be used the various matrices indicated are set up as indicated earlier and the velocity perturbation Δv_s is calculated.

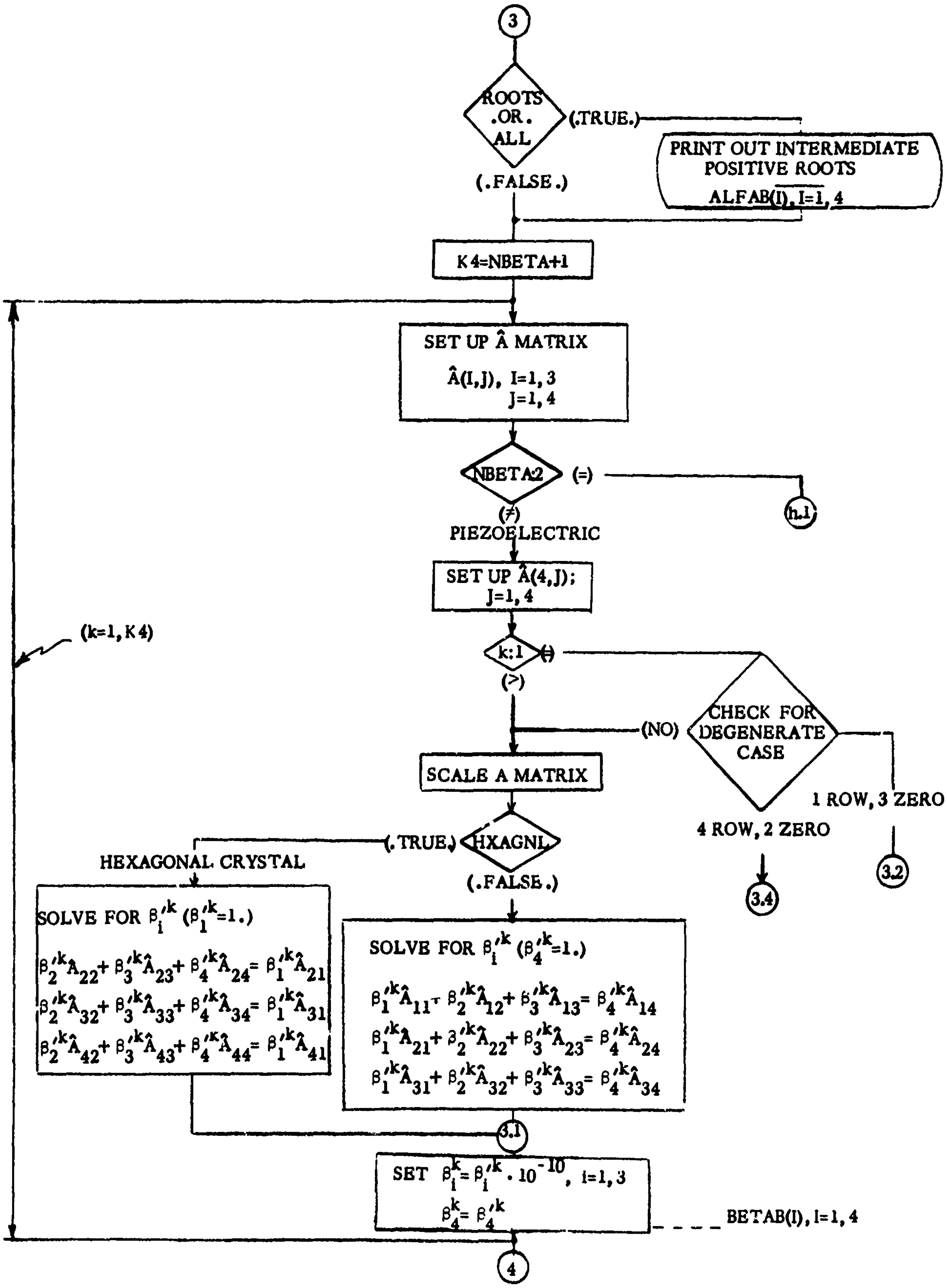

LIQUID
SET UP NOMINAL DATA VALUES
1
READ $CONST DATA
PIEZOELECTRIC (P), ELASTIC (G), AND DIELECTRIC (EPS) CONSTANTS
REPEAT=.FALSE.
1.1
VS=VSAVE
READ $INPUT DATA
LAMBDA, MU, NU, VS, ETC.
IEOF:0
(≠)
(=)
END OF FILE ENCOUNTERED READING $INPUT DATA
EXECUTION TERMINATED
STOP
CVMAX=|VSMAX|
SNU=NU
1.2
VSAVE=VS
EL=ECON·EPSO
EC=-1/ECON
FIRST=.TRUE.
SETCTE
CALL SUBROUTINE TO COMPUTE TRANSFORMED PIEZOELECTRIC (CE), ELASTIC (CC), AND DIELECTRIC (CT) CONSTANTS
2



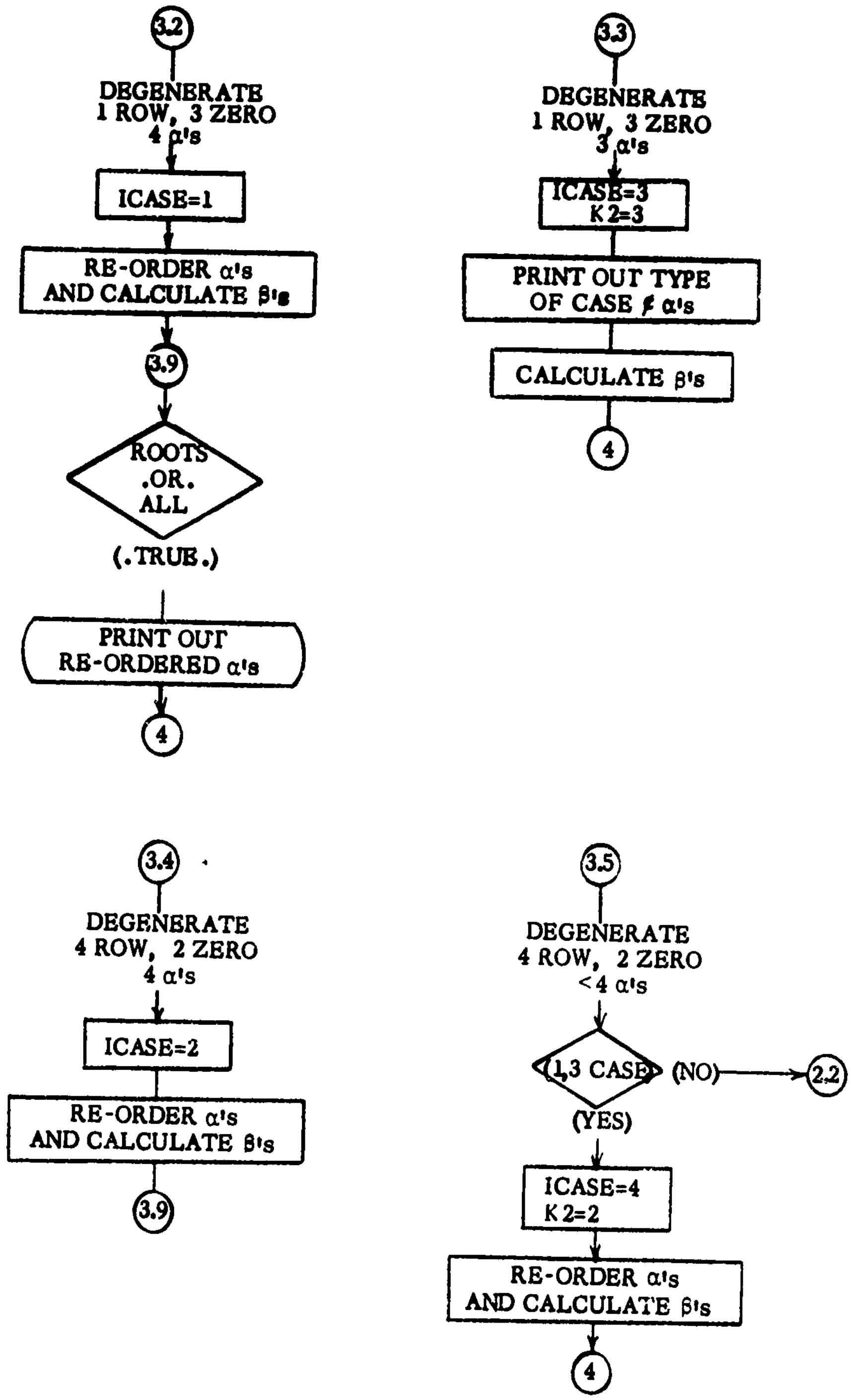
2
VSO=VS
NT=0
2.1
ROOT
CALL SUBROUTINE TO CALCULATE (a) |f(vso)| FOR MAX=0 OR (b) VS AT WHICH |f(vs)| < EPSLON FOR MAX > 0
|f(vs)| IS EVALUATED IN FUNCTION F, WHICH IS CALLED BY ROOT.
VSI=1/VS
ALFAI$_i$=ALFA$_i$/VS
i=1, 8
PRINT INPUT DATA, TOGETHER WITH |f(vs)|, VSI, ALFA, AND ALFAI
IALF
(.TRUE.)
(.FALSE.)
NON-DEGENERATE CASE FOR WHICH TERE ARE LESS THAN 4 ALPHAS WITH A POSITIVE REAL PART – NO SURFACE WAVE EXISTS, SO THE CASE IS TERMINATED
|VSO|=CVMAX
(<)
VSO VSO+DVS
2.1
(≥)
DNU=0
(=)
NU=NU+DNU
NU:NUMAX
(≤)
VS=VSAVE
1.2
(>)
NU=SNU
REPEAT
(.TRUE.)
1
(.FALSE.)
1.1

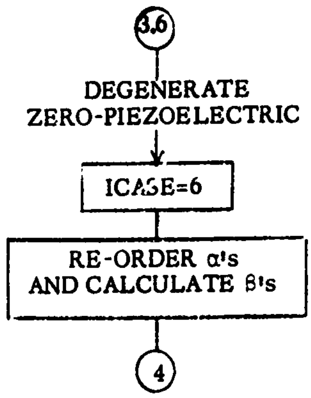

F
IALF=.FALSE.
ICASE=0
ICB=0
K1=1
K2=4
L1=1
L2=16
IB1=1
IB2=4
INIT=0
PRTURB
(.TRUE.)
PERTURBATION SCHEME IS TO BE USED
1.1
INIT=INIT+1
IDX=1
DEL=EPS(INIT)+VSO
VS=VSO+DEL
(.FALSE.)
1.2
VS=VSO
1.3
VSQ=VS2
RVS=RHOB·VSQ
RVSQ=RHOL·VSQ
STRIP
COMPUTE COEFFICIENTS OF POLYNOMIAL POLY(I), I=1, 8
CROOT
CALCULATE ROOTS OF POLYNOMIAL ALFA(I), I=1, 8
NBETA:3
(≠)
NBETA=2
ZERO-PIEZOELECTRIC
ELIMINATE THE ALPHA CORRESPONDING TO THE POTENTIAL
SET α=(-10, -10)
(=)
PIEZOELECTRIC
2

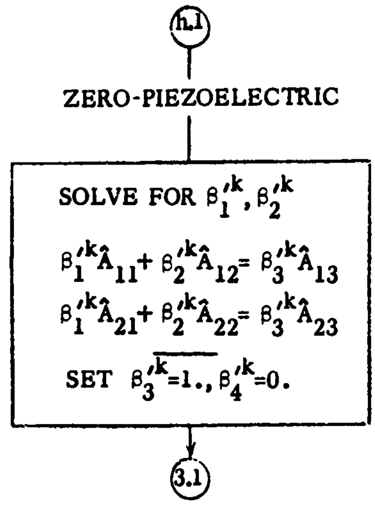

2
ROOTS .OR. ALL
(.TRUE.)
(.FALSE.)
PRINT OUT INTERMEDIATE ROOTS OF POLYNOMIAL
ALFA(I), I=1, 8
PRE-SET ORDER OF ALPHAS
IA(K)=K K=1, 4
SELECT ROOTS WITH POSITIVE REAL PART
ALFAB(I), I=1, K
(≤1) K (=4)
2.1
NON-DEGENERATE, LESS THAN 4 α's TERMINATE CASE
PRINT OUT K
2.2
IALF=.TRUE.
RETURN
PIEZOELECTRIC 4 ALPHAS
3
AC12 .AND. AC23
(.FALSE.)
(.TRUE.)
$\hat{A}_{12} \neq \hat{A}_{23} = 0$
NBETA:2
(=)
ZERO-PIEZOELECTRIC
3.6
(≠)
NBETA=3 PIEZOELECTRIC
NBETA:2 .AND. K:3
.FALSE.
2.2
(.TRUE.)
ZERO-PIEZOELECTRIC 3 ALPHAS
ALFAB(4)=0.
3
AC24
(.FALSE.)
(.TRUE.)
$\hat{A}_{24} = 0$
IC=1
K:3
(≠)
K=2
2.2
(=)
DEGENERATE 1 ROW, 3 ZERO CASE – 3 ALPHAS
3.3
AC14 .AND. AC34
(.FALSE.)
2.2
(.TRUE.)
$\hat{A}_{14}$ AND $\hat{A}_{34} = 0$
DEGENERATE 4 ROW, 2 ZERO CASE
3.5

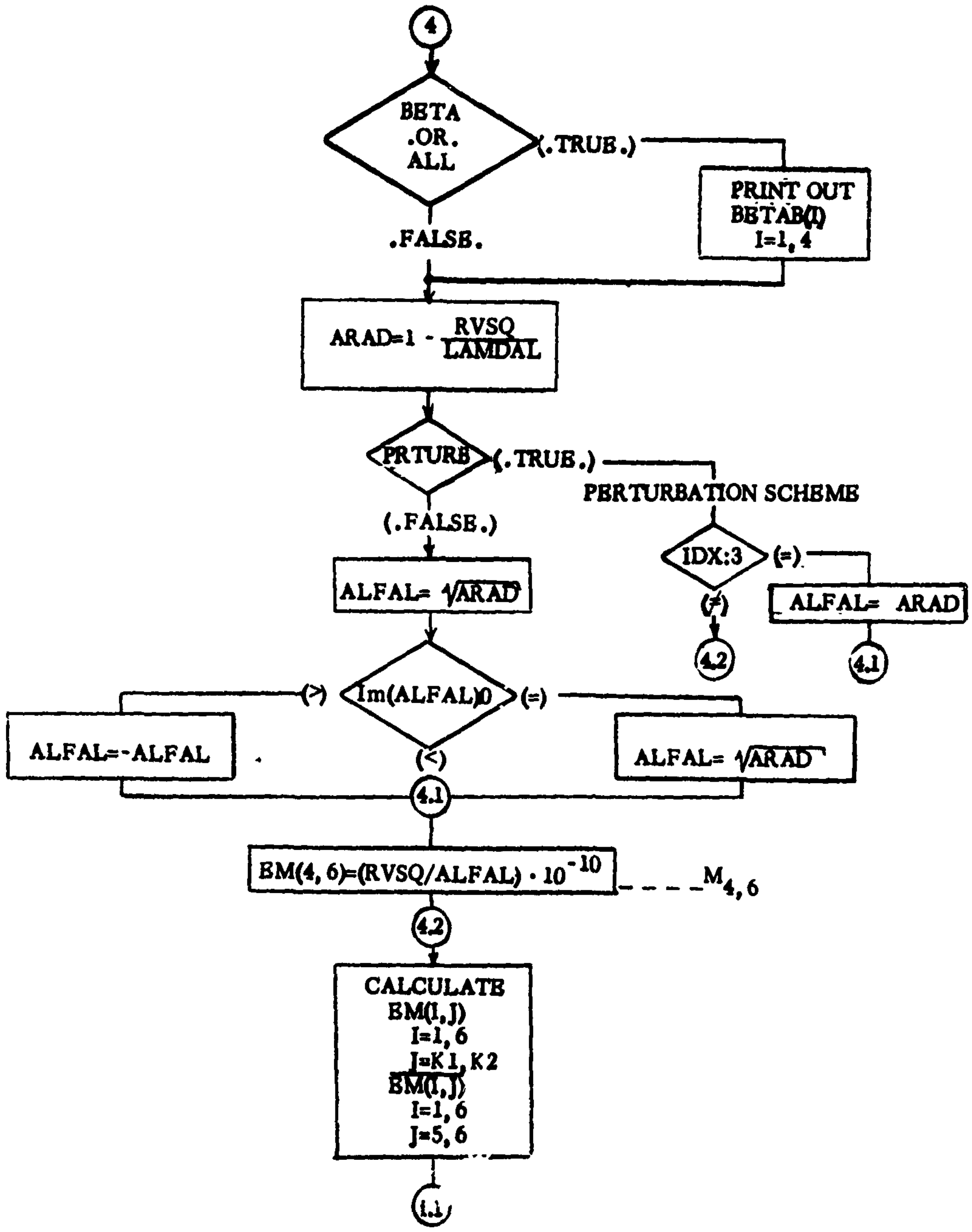


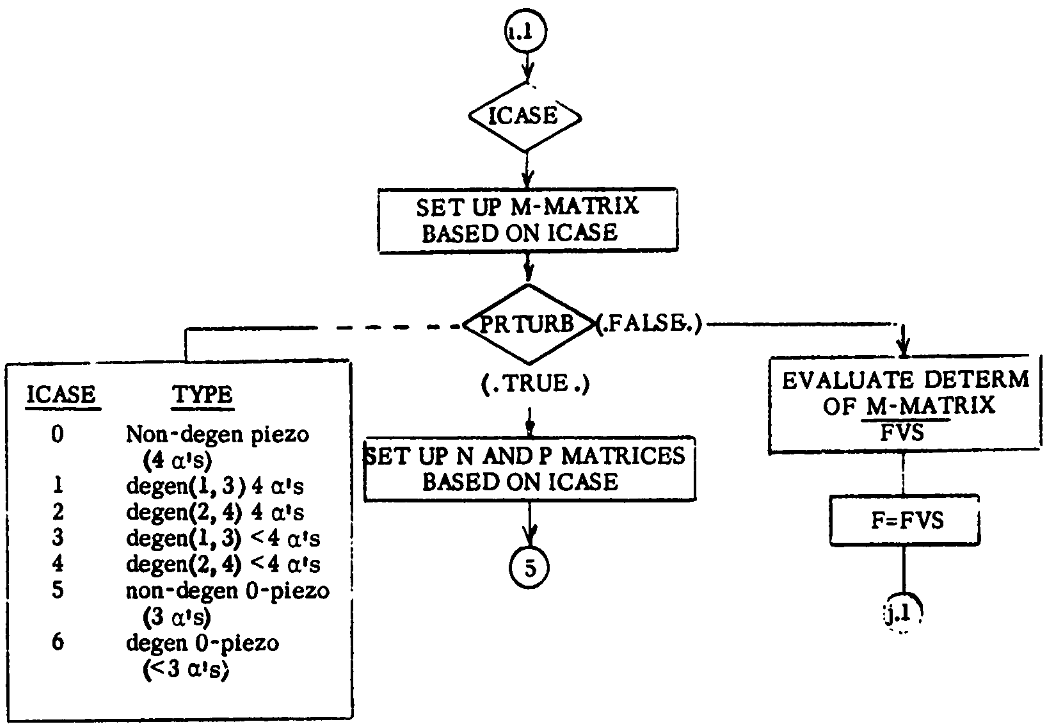
3
ROOTS .OR. ALL
(.TRUE.)
PRINT OUT INTERMEDIATE POSITIVE ROOTS
ALFAB(I), I=1, 4
(.FALSE.)
K4=NBETA+1
SET UP Â MATRIX
Â(I,J), I=1, 3
J=1, 4
NBETA:2
(=)
h.1
(≠)
PIEZOELECTRIC
SET UP Â(4,J);
J=1, 4
(k=1, K4)
k:1
(=)
(>)
CHECK FOR DEGENERATE CASE
(NO)
SCALE A MATRIX
1 ROW, 3 ZERO
4 ROW, 2 ZERO
3.2
3.4
HXAGNL
(.TRUE.)
HEXAGONAL CRYSTAL
(.FALSE.)
SOLVE FOR $\beta_i'^k$ ($\beta_1'^k$=1.)
$\beta_2'^k\hat{A}_{22}+\beta_3'^k\hat{A}_{23}+\beta_4'^k\hat{A}_{24}=\beta_1'^k\hat{A}_{21}$
$\beta_2'^k\hat{A}_{32}+\beta_3'^k\hat{A}_{33}+\beta_4'^k\hat{A}_{34}=\beta_1'^k\hat{A}_{31}$
$\beta_2'^k\hat{A}_{42}+\beta_3'^k\hat{A}_{43}+\beta_4'^k\hat{A}_{44}=\beta_1'^k\hat{A}_{41}$
SOLVE FOR $\beta_i'^k$ ($\beta_4'^k$=1.)
$\beta_1'^k\hat{A}_{11}+\beta_2'^k\hat{A}_{12}+\beta_3'^k\hat{A}_{13}=\beta_4'^k\hat{A}_{14}$
$\beta_1'^k\hat{A}_{21}+\beta_2'^k\hat{A}_{22}+\beta_3'^k\hat{A}_{23}=\beta_4'^k\hat{A}_{24}$
$\beta_1'^k\hat{A}_{31}+\beta_2'^k\hat{A}_{32}+\beta_3'^k\hat{A}_{33}=\beta_4'^k\hat{A}_{34}$
3.1
SET $\beta_i^k=\beta_i'^k\cdot 10^{-10}$, i=1, 3
$\beta_4^k=\beta_4'^k$
BETAB(I), I=1, 4
4

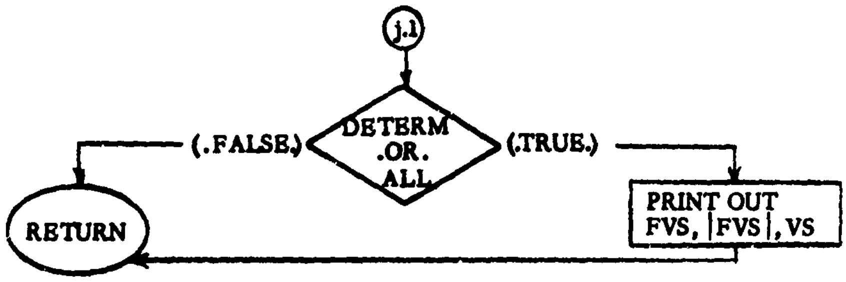

3.2
DEGENERATE
1 ROW, 3 ZERO
4 α's
ICASE=1
RE-ORDER α's
AND CALCULATE β's
3.9
ROOTS
.OR.
ALL
(.TRUE.)
PRINT OUT
RE-ORDERED α's
4
3.3
DEGENERATE
1 ROW, 3 ZERO
3 α's
ICASE=3
K 2=3
PRINT OUT TYPE
OF CASE # α's
CALCULATE β's
4
3.4
DEGENERATE
4 ROW, 2 ZERO
4 α's
ICASE=2
RE-ORDER α's
AND CALCULATE β's
3.9
3.5
DEGENERATE
4 ROW, 2 ZERO
<4 α's
1,3 CASE
(NO)
2.2
(YES)
ICASE=4
K 2=2
RE-ORDER α's
AND CALCULATE β's
4

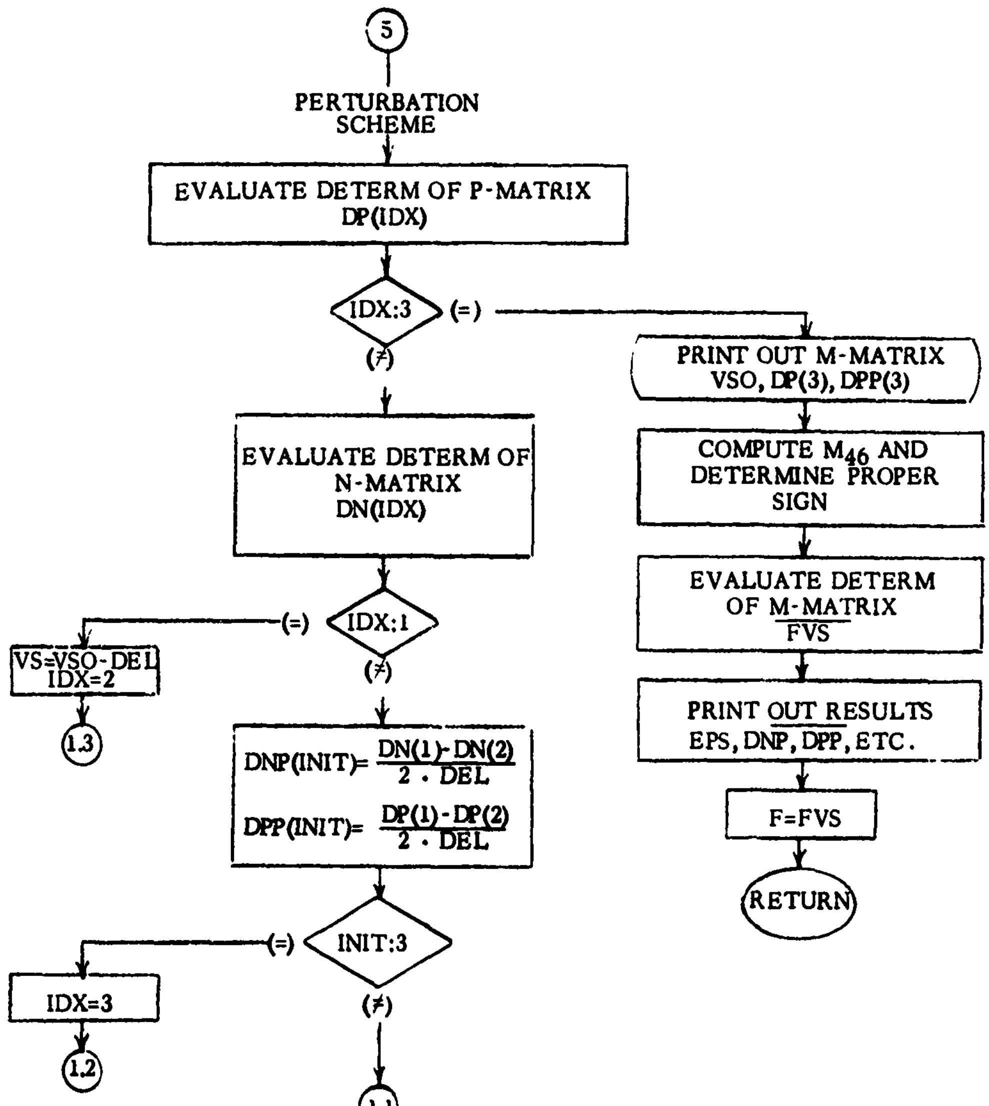

3.6
DEGENERATE
ZERO-PIEZOELECTRIC
ICASE=6
RE-ORDER α's
AND CALCULATE β's
4


h.1
ZERO-PIEZOELECTRIC
SOLVE FOR $\beta_1'^k, \beta_2'^k$
$\beta_1'^k\hat{A}_{11} + \beta_2'^k\hat{A}_{12} = \beta_3'^k\hat{A}_{13}$
$\beta_1'^k\hat{A}_{21} + \beta_2'^k\hat{A}_{22} = \beta_3'^k\hat{A}_{23}$
SET $\beta_3'^k = 1.$, $\beta_4'^k = 0.$
3.1


4
BETA .OR. ALL
(.TRUE.)
PRINT OUT BETAB(I) I=1, 4
.FALSE.
ARAD=1 - RVSQ/LAMDAL
PRTURB
(.TRUE.)
PERTURBATION SCHEME
(.FALSE.)
IDX:3
(=)
(≠)
ALFAL= ARAD
4.2
4.1
ALFAL= √ARAD
Im(ALFAL):0
(>)
(=)
(<)
ALFAL=-ALFAL
ALFAL= √ARAD
4.1
BM(4, 6)=(RVSQ/ALFAL) · 10^-10
M4, 6
4.2
CALCULATE BM(I,J) I=1, 6 J=K1, K2
BM(I,J) I=1, 6 J=5, 6
1.1


i.1
ICASE
SET UP M-MATRIX BASED ON ICASE
PRTURB
(.FALSE.)
(.TRUE.)
ICASE TYPE
0 Non-degen piezo (4 α's)
1 degen(1, 3) 4 α's
2 degen(2, 4) 4 α's
3 degen(1, 3) <4 α's
4 degen(2, 4) <4 α's
5 non-degen 0-piezo (3 α's)
6 degen 0-piezo (<3 α's)
SET UP N AND P MATRICES BASED ON ICASE
5
EVALUATE DETERM OF M-MATRIX
FVS
F=FVS
j.1


j.1
DETERM
.OR.
ALL.
(.FALSE.)
(.TRUE.)
RETURN
PRINT OUT
FVS, |FVS|, VS


5
PERTURBATION SCHEME
EVALUATE DETERM OF P-MATRIX DP(IDX)
IDX:3
(=)
(≠)
PRINT OUT M-MATRIX VSO, DP(3), DPP(3)
EVALUATE DETERM OF N-MATRIX DN(IDX)
COMPUTE M_{46} AND DETERMINE PROPER SIGN
EVALUATE DETERM OF M-MATRIX FVS
IDX:1
(=)
(≠)
VS=VSO-DEL IDX=2
1.3
PRINT OUT RESULTS EPS, DNP, DPP, ETC.
$DNP(INIT)= \frac{DN(1)-DN(2)}{2 \cdot DEL}$
$DPP(INIT)= \frac{DP(1)-DP(2)}{2 \cdot DEL}$
F=FVS
RETURN
INIT:3
(=)
(≠)
IDX=3
1.2
1.1

3. Isotropic, Elastic, Dielectric Layer on Piezoelectric Substrate – Program Description

The purpose of this program is to determine the complex velocity of propagation of surface waves at the interface between a semi-infinite fluid and a piezoelectric substrate. The necessary input and control parameters are described on the following pages.

As with the proceeding program, this program is set up to run on the IBM 7094, using FORTRAN IV and Namelist input and the deck set up is again identical with that described in Section IV.1. Again, two input sections are required: the first describes the material constants of the piezoelectric crystal and the second describes the orientation of the crystal as well as other information pertinent to the execution of the program. The first data set is called CONST and is identical with that described in Section IV.1. The second data set is called "INPUT," and the following is a definition of each input parameter. Medium A refers to the dielectric (elastic) layer and medium B, to the piezoelectric substrate.

Input Name	Equation Name	Type	Definition
LAMDAB	λ_B	real	Euler angles for Medium B.
MUB	μ_B	real	
NUB	ν_B	real	
DNU	$\Delta\nu$	real	If the user wishes to vary ν (NUB) from some initial value, ν, to some final value, ν_{max}, in steps of $\Delta\nu$, then set DNU equal to the steps desired; also, see NUMAX. (See VSINC)
NUMAX	ν_{max}	real	The maximum value of ν (see DNU). ν_{max} is only used when DNU $\neq 0$.
VS	v_s	real	Initial estimate of velocity. This initial value will be used to find a final velocity, $\overline{v}_s$, such that $\|f(\overline{v}_s)\| < \epsilon$, where ϵ is input.
DVS	Δv_s	real	If the user does not care to use the root-finding scheme in determining a final value for v_s, but wishes, instead, to evaluate the determinant $\|f(v_s)\|$ for particular values of v_s in the range from v_s to $v_{s_{max}}$ in steps of Δv_s, then set DVS equal to the step size desired. (For use when MAX = 0.)
VSMAX	$v_{s_{max}}$	real	Maximum value of v_s to be used when DVS $\neq 0$.
LAMDAA	λ_A	real	Lame constants for Medium A.
MUA	μ_A	real	
RHOA	ρ_A	real	Mass density of Medium A.
RHOB	ρ_B	real	Mass density of Medium B.
EPSLON	ϵ	real	A positive number used as a convergence criterion by the root-finding scheme (MAX > 0). If $\|f(v_s)\| < \epsilon$, then v_s is assumed to be the root required.
EPSO	ϵ_0	real	Permittivity of free space.

Input Name	Equation Name	Type	Definition
EPSA	ϵ_A	real	Dielectric constant for Medium A.
WH	ωh	real	Frequency thickness product.
WXA	ωx_A	real	Normalized distance into Medium A.
DWXA	$\Delta\omega x_A$	real	In order to vary ωx_A (WXA) from an initial value, ωx_A, to a final value $\omega x_{A_{max}}$, DWXA must be set equal to the desired step size. See WXAMAX.
WXAMAX	$\omega x_{A_{max}}$	real	The maximum value of ωx_A to be used when DWXA $\neq 0$.
WXB	ωx_B	real	Normalized distance into Medium B.
DWXB	$\Delta\omega x_B$	real	In order to vary ωx_B from an initial value, ωx_B, to a final value, $\omega x_{B_{max}}$, DWXB must be set equal to the step size desired. See WXBMAX.
WXBMAX	$\omega x_{B_{max}}$	real	The maximum value of ωx_B to be used when DWXB $\neq 0$.
ICHECK	----	logical	ICHECK = .TRUE. – All FINAL ANSWERS* are computed in addition to the evaluation of the determinant $\lvert f(\bar{v}_S)\rvert$. ICHECK = .FALSE. – FINAL ANSWERS are not computed; evaluate determinant only.
MAX	----	integer	Since an iteration scheme is used for convergence for a final root v_S, there must be an indication of how many iterations are to be executed before divergence is assumed. Hence, MAX should be the maximum number of iterations the user wishes the program to make (usually 15). If MAX is set to zero (MAX = 0) the determinant $\lvert f(\bar{v}_S)\rvert$ will be evaluated for the particular v_S value input – the iteration scheme will not be used. This option may be useful if there is difficulty in determining the range in which v_S lies.

*The FINAL RESULTS, which are computed for all values of WXA (dielectric layer) and WXB (piezoelectric layer), include the following:

- Stress Components
- Strain Components
- Time Average Power Flow
- Electric Displacement
- Mechanical Displacement
- Electric Potential Magnitude
- Electric Field

Input Name	Equation Name	Type	Definition
TITLE	----	BCD	An alphanumeric array of 24 characters or less used to describe the type of crystal, such as lithium niobate. This is input in the following manner: TITLE = nH name of crystal, where n is the number of characters following the H (including blanks). For example TITLE = 6HQUARTZ
HXAGNL	----	logical	Parameter which controls the calculation of betas (β's) for a hexagonal crystal (such as zinc oxide) .TRUE. hexagonal crystal (use special technique) .FALSE. non-hexagonal crystal (use normal procedure)
VSINC	----	logical	VSINC = .TRUE. – New estimates of initial velocity (v_S) are computed using a linear fit to the two previous values. (Used when NUB varies over a range NUB, NUB + DNU, ... , NUMAX) VSINC = .FALSE. – The same initial estimate of velocity is used for all values over the specified range of NUB.
IOP	----	integer	Degenerate case options (used when exactly four α's with positive real part occur). IOP = 1 – seek modes of propagation of the Quasi-Rayleigh or Sesawa type. IOP = 2 – seek modes of propagation of Love type.
REPEAT	----	logical	REPEAT is a logical variable and in its usage, can take only one value: .TRUE. If there are no more cases to run after the current case, REPEAT does not need to be input. If there will be another case to follow, but the crystal coefficients remain the same, then, again, REPEAT does not need to be input. However, if another case is to be run and the coefficients are different, then REPEAT needs to be input

Input Name	Equation Name	Type	Definition
			as .TRUE. This means that the $CONST data will have to be input again (in the other cases above, $CONST would not have to be input again).

The following input parameters are all logical variables which are assumed to be false (.FALSE.) in the program. They are used as switches indicating whether or not intermediate calculations are to be printed. If any one, or any combination of these parameters are input as true (.TRUE.), then certain intermediate data will print, according to the following:

TABCTE	Print the constants E, C, and T (the transformed piezoelectric, elastic, and dielectric constants) calculated from the constants P, G, and EPS.
ROOTS	Print the roots of the polynomial each time they are calculated.
BETA	Print the values of β_{ij}.
DETERM	Print the value of the determinant.
COEFF	Print the coefficients of the 8'th order polynomial.
TABL	Print the L matrix (or $\hat{P}$, $\hat{Q}$, $\hat{R}$, etc., when used).
ALPHA	Print the roots of the polynomial (α_B^j's) and the re-ordered roots for degenerate cases.
ALL	Print all of the above.

Data items may be excluded from the input stream at the discretion of the user. Items omitted from the first data set will take on nominal values (i.e.: values assigned within the program). Items omitted from succeeding data sets will take on previously assigned values. The following is a complete list of nominal values:*

*All logical parameters have a nominal value of .FALSE.

Parameter	Nominal Value
LAMDAB	0.
MUB	0.
NUB	0.
DNU	0.
NUMAX	0.
VS	3000.
DVS	0.
VSMAX	0.
LAMDAA	1.5×10^{11}
MUA	2.85×10^{10}
RHOA	1.888×10^{4}
RHOB	4700.
EPSLON	$1. \times 10^{-11}$
EPSO	8.85×10^{-12}
EPSA	44.25×10^{-12}
WH	0.
WXA	0.
DWXA	0.
WXAMAX	0.
WXB	0.
DWXB	0.
WXBMAX	0.
MAX	15.
TITLE	LITHIUM NIOBATE
IOP	1

The following is a description of the computer program flow diagram provided at the end of this section.

The computer program for this problem shares many common features with the programs described in the preceding sections. Again the nominal data values are set up first. The piezoelectric (P), elastic (G), and dielectric (EPS) constants are read in next (CONST DATA). Following this the rest of the input data is entered (INPUT DATA).

Subroutine SETCTE computes the transformed piezoelectric (CE), elastic (CC), and dielectric (CT) constants. Next, subroutine ROOT performs the calculations and calls the subroutines necessary to computer F(VS), the boundary condition determinant. ROOT will either minimize F(VS) (root finding scheme) or simply compute it at velocity VS depending on the setting of the counter MAX. Upon returning to the main program an option to increment VS followed by an option to increment the third Euler angle (NU) is available as in the first two programs. As NU is incremented it is also possible to update the initial velocity (VS) if the root finding scheme is being employed so that a closer initial estimate will be had as NU varies. The setting of a logical variable (VSINC) dictates whether this updating scheme is to be used or not.

When the correct velocity of propagation has been found (either from the root finding scheme or from plotting F(VS) as a function of velocity) the relative amplitudes of the partial surface wave fields are calculated (ETA(1), ETA(2), etc.). Next the various quantities of interest are calculated in medium A (the dielectric) as a function of normalized distance (WX_A) into the medium. Following this the same parameters are calculated in medium B (crystal) as a function of normalized distance (WX_B) into the crystal. For both media an incrementation scheme may be used to increase WX_A or WX_B in equal increments (DWXA or DWXB) from some initial value to some final value.

The quantities of interest mentioned above are as follows:

a) Mechanical Displacement (magnitude and phase) referred to as MAGU(I) and PHASEU(I), I=1 to 3 in the program.

b) The electric potential (magnitude and phase) referred to as MAGU(4) and PHASEU(4).

c) The time average power flow computed in the subroutine P1FUN.

d) The stresses computed in the subroutine TFUN.

e) The strains computed in subroutine SFUN.

f) The electric fields (E1, E3) and electric displacement (D1, D2, and D3).

Calculation of F(VS)

Subroutine ROOT calls subroutine F to evaluate the determinant of the boundary condition matrix. F first calls subroutine STRIP to calculate the coefficients of the eighth order equation in α. Next subroutine CROOT computes the roots of the polynomial (ALFA(I), I=1 to 8). The roots with positive real parts (ALFAB(I), I=1 to K) are selected as in the other programs and the extraneous roots in the non-piezoelectric case are eliminated.

If $K = 0$ the case terminates since no solution is possible. If $K \neq 0$ a search for degeneracies follows. $K = 1$ presents a possibility now which was not present in the previous problems.* First $\hat{A}_{12}$ and $\hat{A}_{23}$ are checked ($\hat{A}$ is the matrix of the coefficients of the unknown amplitudes $\beta_i^{(\ell)}$ as in the previous problems). If $\hat{A}_{12}$ and $\hat{A}_{23}$ are not both equal to zero the case cannot be degenerate. A check is made to see if the following two conditions both hold:

a) The case is non-piezoelectric

b) $K \neq 3$

If both of these conditions hold the case terminates. If it is not true that both hold then a test is made to see if the following two conditions hold:

a) The case is piezoelectric

b) $K \neq 4$.

If both of these conditions hold the case terminates. Otherwise the program continued for now we must have either a non-piezoelectric case with $K = 3$ or a piezoelectric case with $K = 4$, both of which are proper non-degenerate cases.

If $\hat{A}_{12}$ and $\hat{A}_{23}$ are both identically zero and the case is non-piezoelectric, the program proceeds to the section where degenerate, non-piezoelectric cases

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*Page 37 of analysis.

are handled. If, however, the case is piezoelectric $\hat{A}_{24}$ is checked. If $\hat{A}_{24}$ is zero the first type of degenerate case of the analysis section arises and the program proceeds to that section which treats such cases. If $\hat{A}_{24}$ is not identically zero then $\hat{A}_{14}$ and $\hat{A}_{34}$ are checked. If they are identically zero, the second degenerate case of the analysis section arises and the program proceeds to the section that treats these cases. If they are not both zero the program goes through the test mentioned above (i.e. is it simultaneously true that (a) the case is piezoelectric and (b) $K \neq 4$.

If (a) and (b) are not simultaneously true then we are dealing with a non-degenerate, piezoelectric case and the program proceeds accordingly.

Non-degenerate Cases

The $\hat{A}$ matrix is set up for each value of α (k = 1, K4) where K4 = 4 for piezoelectric cases and K4 = 3 for non-piezoelectric cases. If the case is non-piezoelectric the program sets $\beta_4 = 0$, $\beta_3 = 10^{-10}$ and solves the first two equations for β_1 and β_2. If the crystal is piezoelectric and not hexagonal β_4 is set equal to 1 and the first three equations of the set are solved for β_1, β_2, and β_3. If the crystal is piezoelectric and hexagonal β_1 is set equal to 10^{-10} and the second, third, and fourth equations of the set are solved for β_2, β_3, and β_4.

Degenerate Case 1

This case is characterized by a decoupling of the equations for $\beta_1^{(\ell)}$ so that three of the equations involve β_1, β_3, and β_4 only and one involves β_2 only as discussed in the analysis. If there are four α's with positive real part (K = 4) the program calculates $|\hat{A}_{22}|$ for each α and determines which α leads to a minimum of this function. This now becomes $\alpha^{(1)}$ while the other α's become $\alpha^{(2)}$, $\alpha^{(3)}$, and $\alpha^{(4)}$. The program proceeds to work with the relabeled α's and computes the β's as follows:

$$\beta_2^{(1)} = 10^{-10}\,,\ \beta_i^{(1)} = 0 \qquad i = 1,3,4; \qquad \beta_2^{(\ell)} = 0\,,\ \beta_4^{(\ell)} = 1$$

$$\beta_1^{(\ell)} = \frac{\hat{A}_{13}^{(\ell)}\hat{A}_{34}^{(\ell)} - \hat{A}_{14}^{(\ell)}\hat{A}_{33}^{(\ell)}}{\hat{A}_{11}^{(\ell)}\hat{A}_{33}^{(\ell)} - \hat{A}_{13}^{(\ell)2}}\,, \quad \text{and} \quad \beta_3^{(\ell)} = \frac{\hat{A}_{13}^{(\ell)}\hat{A}_{14}^{(\ell)} - \hat{A}_{11}^{(\ell)}\hat{A}_{34}^{(\ell)}}{\hat{A}_{11}^{(\ell)}\hat{A}_{33}^{(\ell)} - \hat{A}_{13}^{(\ell)2}}$$

$\ell = 2, 3, 4$. The program now proceeds to set up either the M(10 x 10) or

N(3 x 3) matrix discussed in the analysis and evaluates its determinant.

If there are less than four α's with positive real part ($K < 4$) the program calculates $|\hat{A}_{22}|$ for each α and counts the number (N1) of α's for which $|\hat{A}_{22}| < 10^7$ (this is close enough to zero considering the magnitude of the individual terms in $\hat{A}_{22}$). If K = 1 and N1 = 0 the case terminates (case 1c2). If K = 1 and N1 = 1 the program sets $\beta_2^{(1)} = 10^{-10}$ $\beta_i^{(1)} = 0$ $i = 1, 3, 4$ and then proceeds to set up the N matrix and evaluate its determinant (case 1c1).

If K = 2 and N1 = 0 the case terminates (case 1b3). If K = 2 and N1 = 1 the α that yielded $|\hat{A}_{22}| < 10^7$ becomes $\alpha^{(1)}$. The program now sets $\beta_2^{(1)} = 10^{-10}$ and $\beta_i^{(1)} = 0$ $i = 1, 3, 4$ and then proceeds to set up the N matrix and evaluate its determinant (case 1b1). If K = 2 and N1 = 2 this represents an impossible case and the case terminates (1b2).

If K = 3 and N1 = 0 then all three roots correspond to the $(\beta_1, \beta_3, \beta_4)$ split. The α's become $\alpha^{(2)}$, $\alpha^{(3)}$, and $\alpha^{(4)}$. The β's are calculated as they are in the four α case for $\alpha^{(2)}$, $\alpha^{(3)}$, and $\alpha^{(4)}$. Only the M matrix can be set up and evaluated for this case (case 1a1). If K = 3 and N1 = 1 the corresponding α becomes $\alpha^{(1)}$ and $\beta_2^{(1)} = 10^{-10}$ while $\beta_i^{(1)} = 0$ $i = 1, 3, 4$. Only the N matrix is set up and evaluated (case 1a2). If K = 3 and N1 = 2 or more the case terminates since this is physically impossible.

Degenerate Case 2

This case is characterized by a decoupling of the equations for $\beta_i^{(\ell)}$ so that two of the equations involve β_1 and β_3 only while the other two involve β_2 and β_4 only. If there are four α's with positive real part (K = 4) the program calculates $|\hat{A}_{22}\hat{A}_{44} - \hat{A}_{24}^2|$ for each α. The (2) values of $\alpha^{(\ell)}$ which lead to minimum values of $|\hat{A}_{22}\hat{A}_{24} - \hat{A}_{24}^2|$ are selected and become $\alpha^{(3)}$ and $\alpha^{(4)}$. The other (2) values of α become $\alpha^{(1)}$ and $\alpha^{(2)}$. The program then computes the β's as follows:

$$\beta_3^{(1)} = \beta_3^{(2)} = 10^{-10}; \quad \beta_1^{(1)} = \frac{-\hat{A}_{13}^{(1)}}{\hat{A}_{11}^{(1)}} \cdot 10^{-10}, \quad \beta_1^{(2)} = \frac{-\hat{A}_{13}^{(2)}}{\hat{A}_{11}^{(2)}} \cdot 10^{-10} \ ;$$

$$\beta_2^{(1)} = \beta_2^{(2)} = 0 \ ;$$

$$\beta_4^{(1)} = \beta_4^{(2)} = 0 \ ;$$

$$\beta_4^{(3)} = \beta_4^{(4)} = 1; \quad \beta_2^{(3)} = \frac{-\hat{A}_{24}^{(3)}}{\hat{A}_{22}^{(3)}}, \quad \beta_2^{(4)} = \frac{-A_{24}^{(4)}}{A_{22}^{(4)}} \ ;$$

$$\beta_1^{(3)} = \beta_1^{(4)} = 0; \quad \beta_3^{(3)} = \beta_3^{(4)} = 0 \quad .$$

The program now proceeds to set up either the P or the Q matrix and evaluates its determinant.

If there are less than four α's with positive real part ($K < 4$) the program proceeds as follows:

If $K = 1$ the case terminates (case 2c). If $K = 2$ or 3 the program computes the quantity $|\hat{A}_{22}\hat{A}_{44} - \hat{A}_{24}^2|$ for each α and counts the number (I1) of α's for which this quantity $< 10^{-5}$ and the number (I2) of α's for which this quantity $\geq 10^{-5}$. 10^{-5} is close enough to zero due to the magnitudes of the individual terms in the quantity. If $K = 3$ and $I1 = 2$ the α's become $\alpha^{(3)}$ and $\alpha^{(4)}$ and the β's are calculated as they were above for $\alpha^{(3)}$ and $\alpha^{(4)}$. Only the Q matrix is set up and evaluated (case 2a1). If $K = 3$ and $I2 = 2$ the α's become $\alpha^{(1)}$ and $\alpha^{(2)}$ and the β's are calculated as above for $\alpha^{(1)}$ and $\alpha^{(2)}$. Only the P matrix is set up and evaluated (case 2a2). If $K = 3$ while $I1 \neq 2$ and $I2 \neq 2$ the case terminates (case 2a3). If $K = 2$ and $I1 = 2$ the β's are handled as above (case 2b1). If $K = 2$ and $I2 = 2$ the β's are likewise handled as above (case 2b2). If $K = 2$ while $I1 \neq 2$ and $I2 \neq 2$ the case terminates (case 2b3).

Degenerate Non-piezoelectric Case

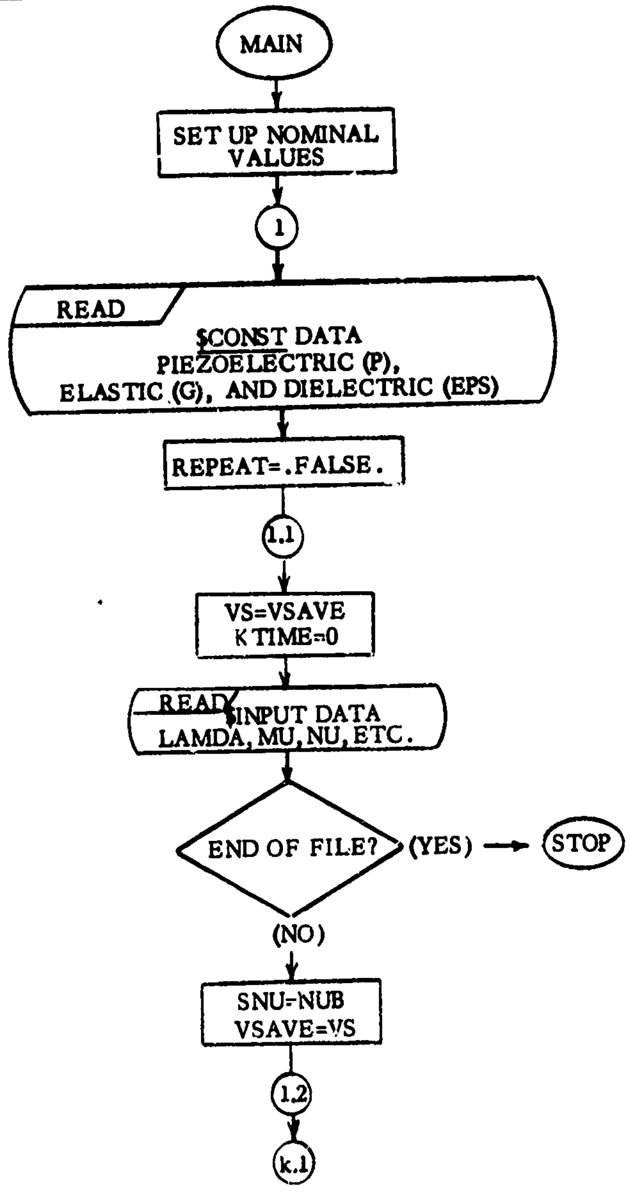
This case is characterized by a decoupling of the equations for $\beta_i^{(\ell)}$ such that two of the equations involve β_1 and β_3 only and one of the equations involves

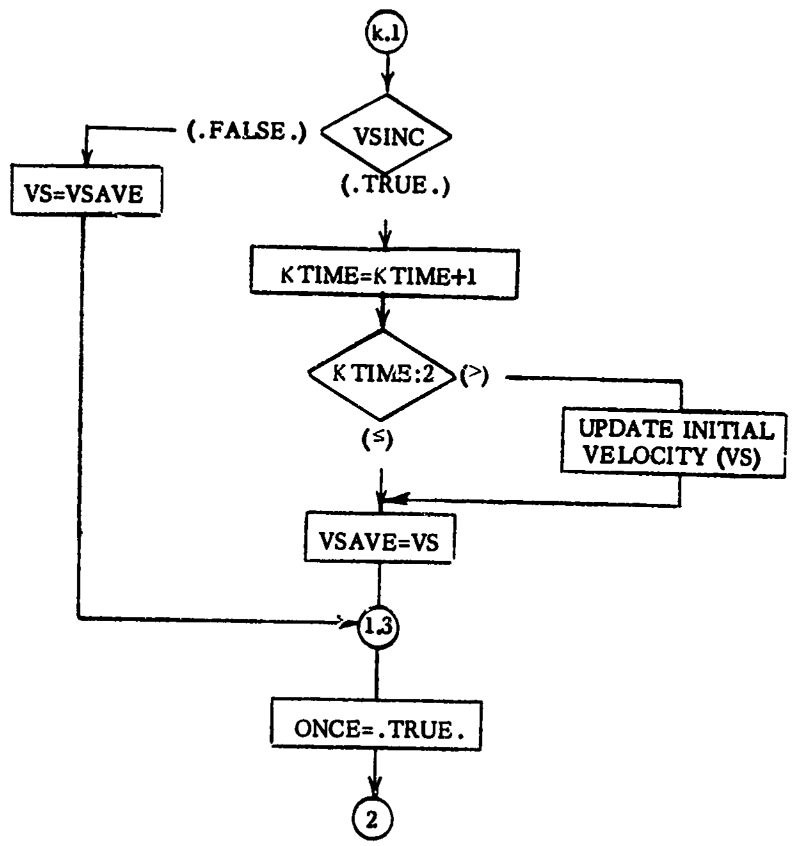
β_2 only. If there are three α's with positive real part $|\hat{A}_{22}|$ is evaluated for each α. The α leading to the minimum value of $|\hat{A}_{22}|$ becomes $\alpha^{(1)}$ while the others become $\alpha^{(2)}$ and $\alpha^{(3)}$. The program sets $\beta_2^{(1)} = 10^{-10}$, $\beta_i^{(1)} = 0$ $i = 1$ and 3; $\beta_2^{(\ell)} = 0$, $\beta_3^{(\ell)} = 10^{-10}$, $\beta_1^{(\ell)} = -\hat{A}_{13}^{(\ell)}/\hat{A}_{11}^{(\ell)} \cdot 10^{-10}$ $\ell = 2$ and 3. Either the R or N matrix can be set up and evaluated depending on the type of wave sought.

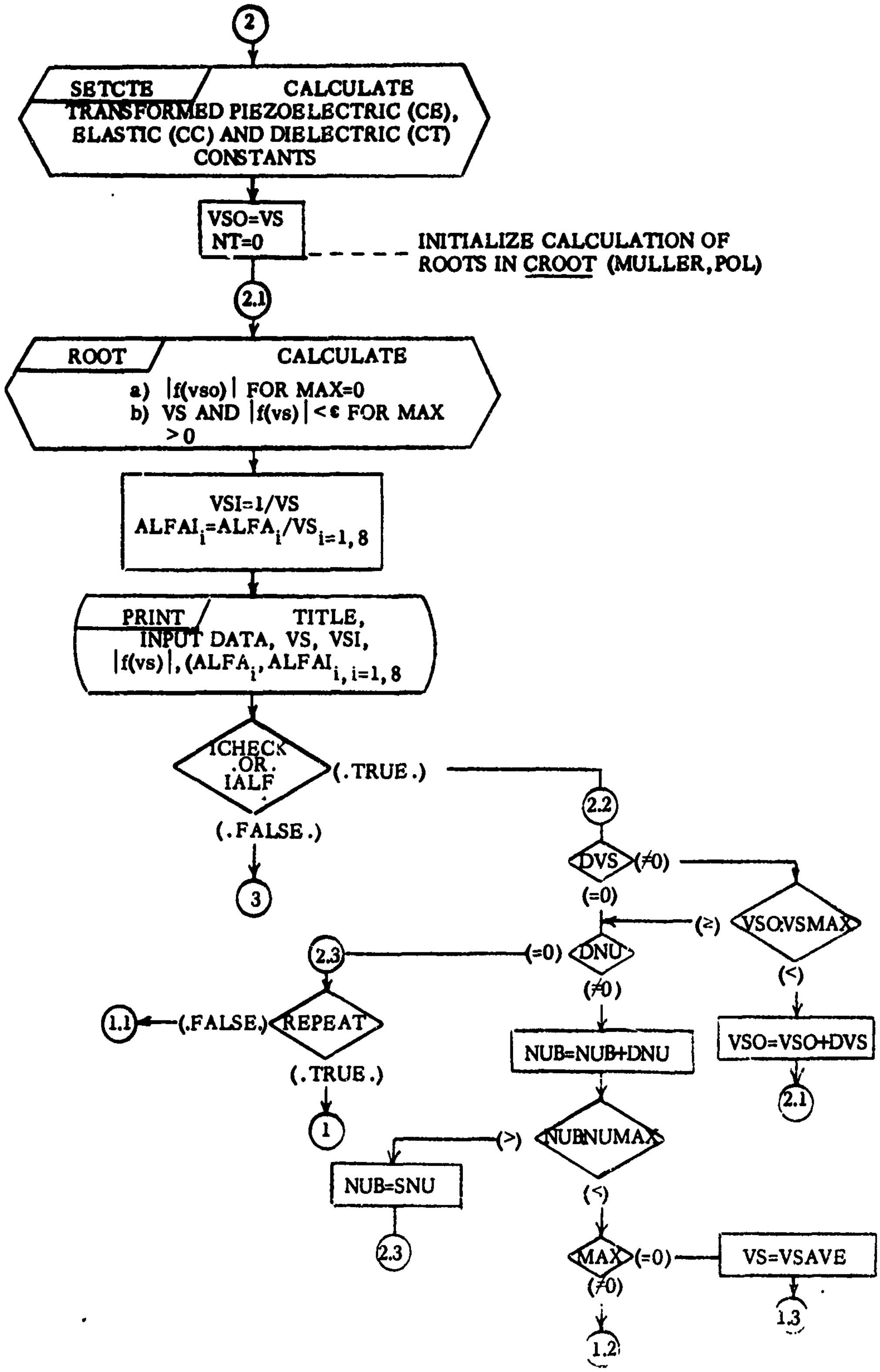
If there are two α's with positive real part $|\hat{A}_{22}|$ is evaluated for both α's and compared with 10^7. (This is close enough to zero considering the magnitudes of the individual terms of $\hat{A}_{22}$). If $|\hat{A}_{22}| > 10^7$ for both α's they become $\alpha^{(2)}$ and $\alpha^{(3)}$ and the β's are calculated as above. Only the R matrix is set up and its determinant evaluated (case xa1). If $|\hat{A}_{22}| \le 10^7$ for one of the α's this becomes $\alpha^{(1)}$ and the β's are the same as above. Only the N matrix is set up and its determinant evaluated (case xa2). If $|\hat{A}_{22}| \le 10^7$ for both α's the case is terminated (case xa3).

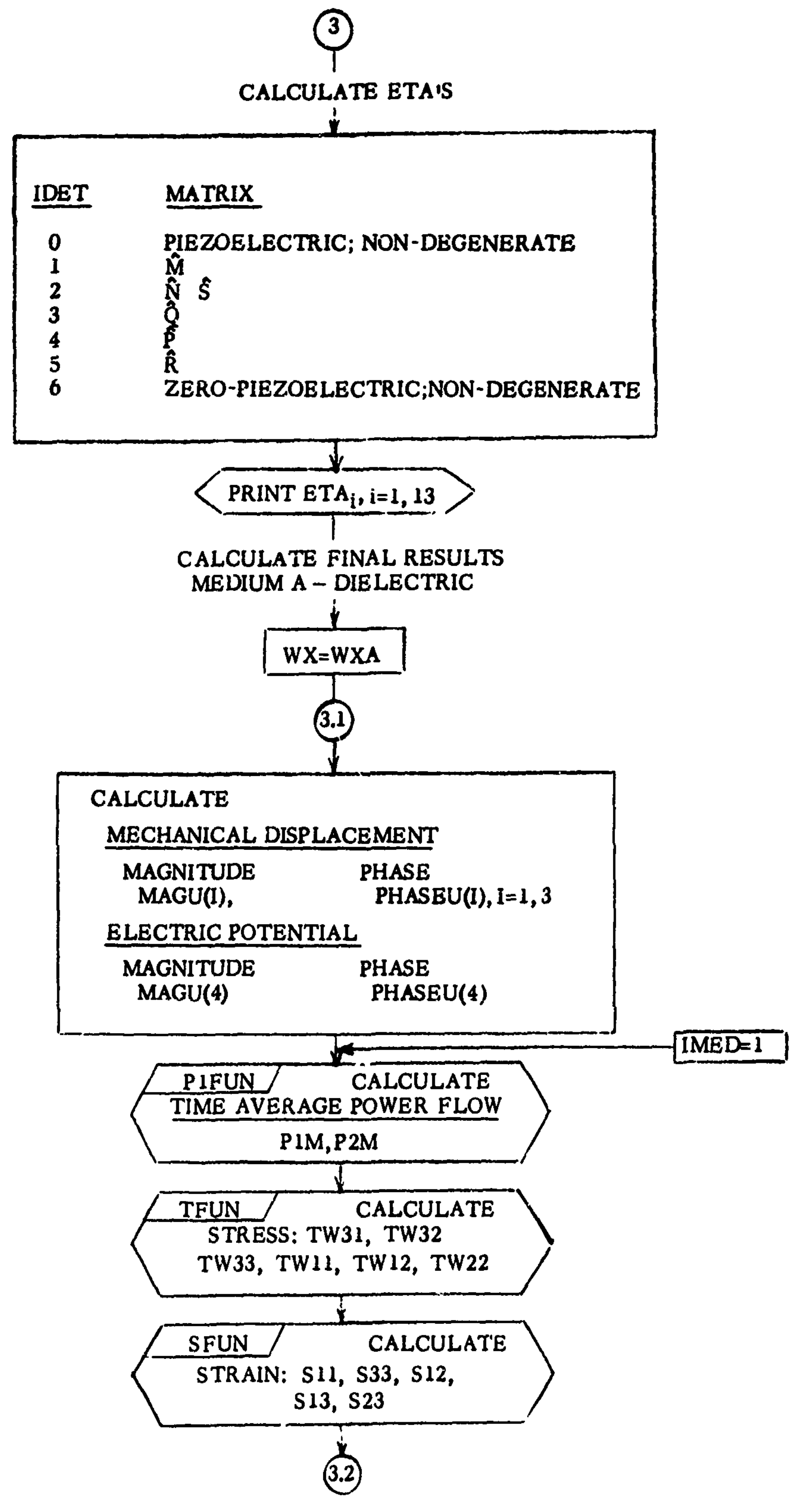
If there is one α with positive real part $|\hat{A}_{22}|$ is evaluated and compared to 10^7. If $|\hat{A}_{22}| \le 10^7$ the program sets $\beta_2^{(1)} = 10^{-10}$ $\beta_i^{(1)} = 0$ $i = 1$ and 3. Only the N matrix is set up and its determinant evaluated (case ya1). If $|A_{22}| > 10^7$ the case is terminated (case ya2).

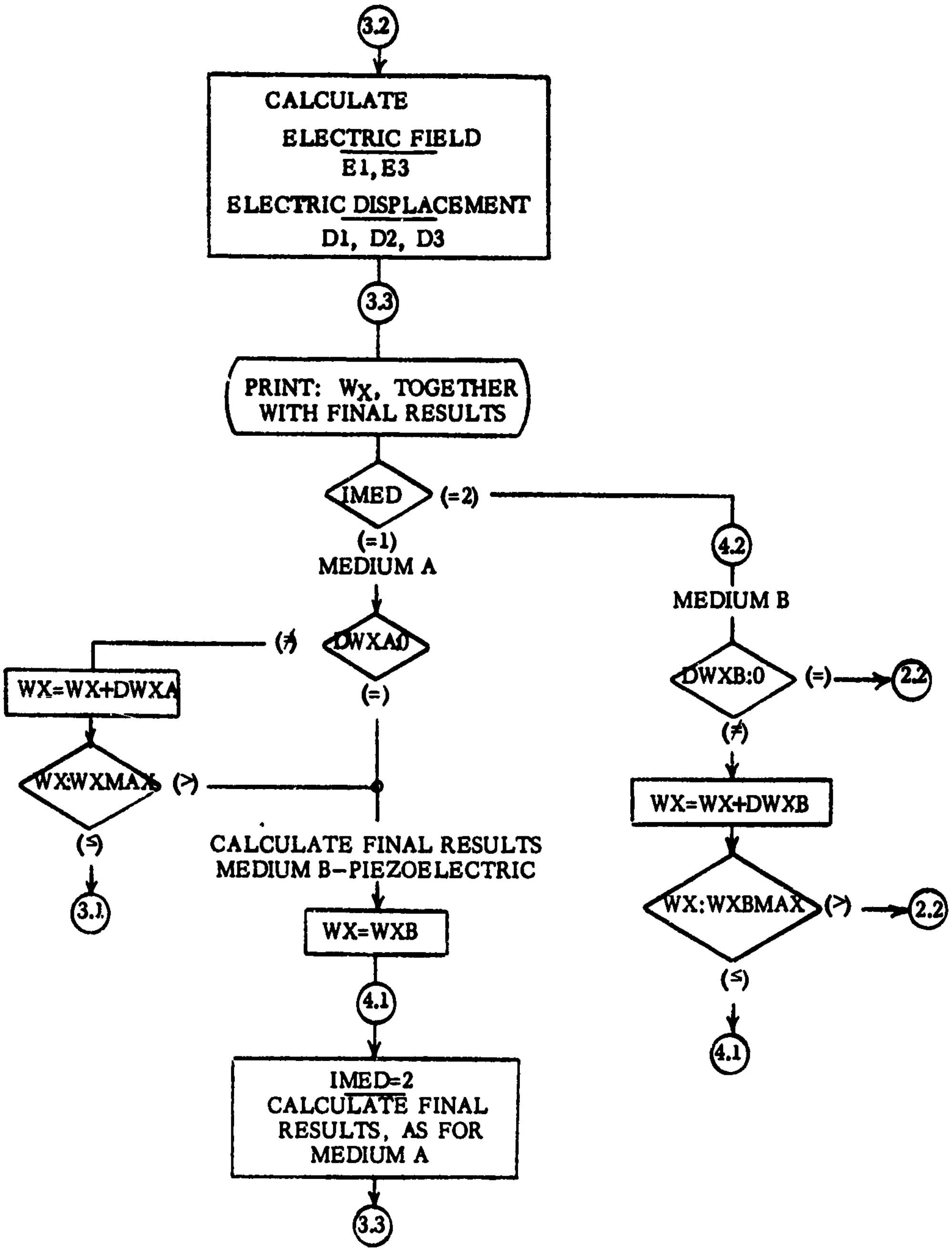
DIELECTRIC

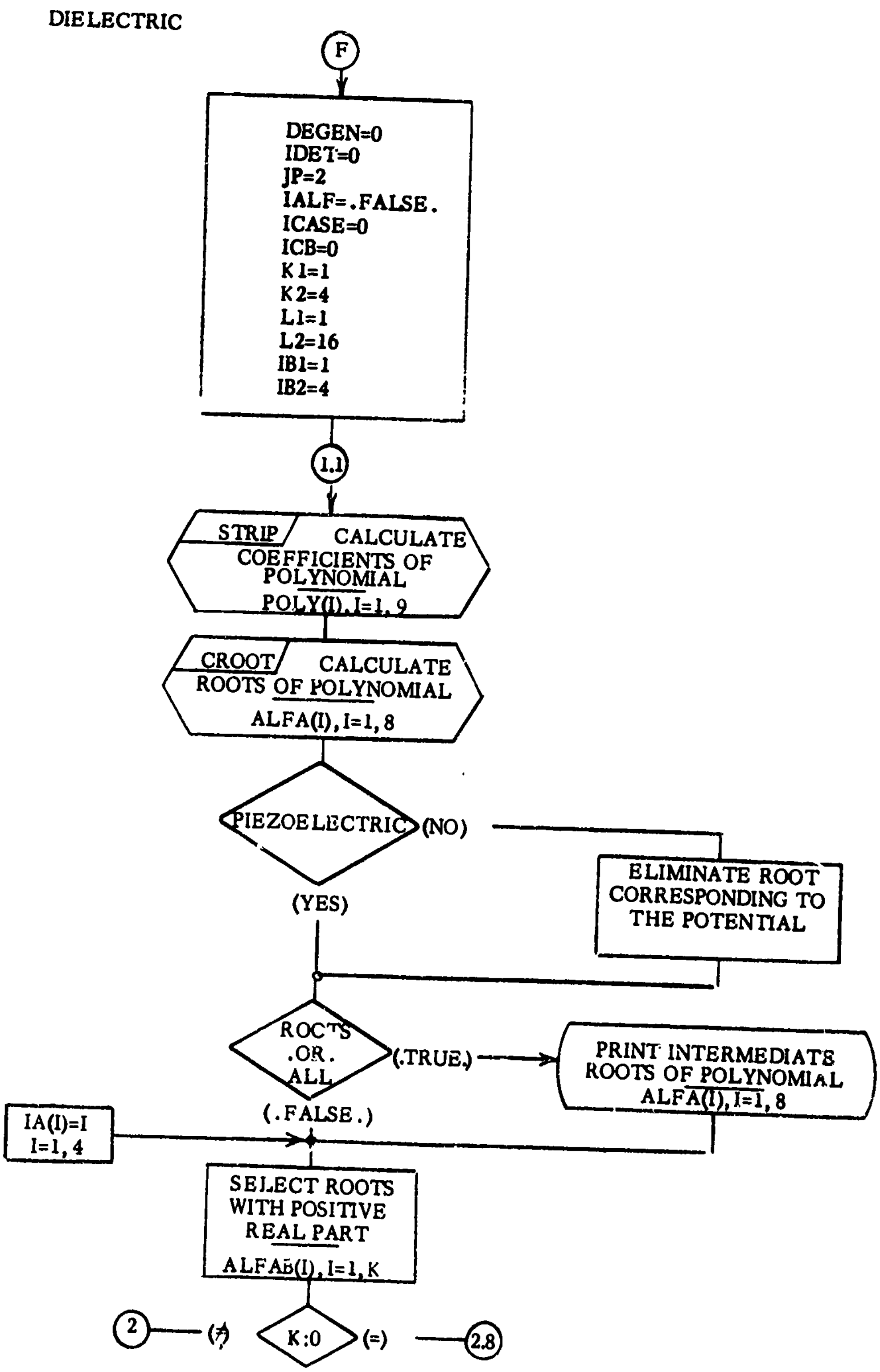

MAIN
SET UP NOMINAL VALUES
1
READ
$CONST DATA
PIEZOELECTRIC (P),
ELASTIC (G), AND DIELECTRIC (EPS)
REPEAT=.FALSE.
1.1
VS=VSAVE
KTIME=0
READ
$INPUT DATA
LAMDA, MU, NU, ETC.
END OF FILE?
(YES)
STOP
(NO)
SNU=NUB
VSAVE=VS
1.2
k.1


k.1
VSINC
(.FALSE.)
VS=VSAVE
(.TRUE.)
KTIME=KTIME+1
KTIME:2
(>)
(≤)
UPDATE INITIAL
VELOCITY (VS)
VSAVE=VS
1.3
ONCE=.TRUE.
2


2
SETCTE
CALCULATE TRANSFORMED PIEZOELECTRIC (CE), ELASTIC (CC) AND DIELECTRIC (CT) CONSTANTS
VSO=VS
NT=0
INITIALIZE CALCULATION OF ROOTS IN CROOT (MULLER,POL)
2.1
ROOT
CALCULATE
a) |f(vso)| FOR MAX=0
b) VS AND |f(vs)| < ε FOR MAX > 0
VSI=1/VS
ALFAI_i=ALFA_i/VS_i=1, 8
PRINT
TITLE, INPUT DATA, VS, VSI, |f(vs)|, (ALFA_i, ALFAI_i, i=1, 8
ICHECK .OR. IALF
(.TRUE.)
(.FALSE.)
3
2.2
DVS
(≠0)
(=0)
VSO:VSMAX
(≥)
(<)
VSO=VSO+DVS
2.1
DNU
(=0)
(≠0)
2.3
REPEAT
(.FALSE.)
1.1
(.TRUE.)
1
NUB=NUB+DNU
NUB:NUMAX
(>)
(<)
NUB=SNU
2.3
MAX
(=0)
(≠0)
VS=VSAVE
1.3
1.2


3
CALCULATE ETA'S
IDET MATRIX
0 PIEZOELECTRIC; NON-DEGENERATE
1 $\hat{M}$
2 $\hat{N}$ $\hat{S}$
3 $\hat{Q}$
4 $\hat{P}$
5 $\hat{R}$
6 ZERO-PIEZOELECTRIC;NON-DEGENERATE
PRINT ETA_i, i=1, 13
CALCULATE FINAL RESULTS
MEDIUM A – DIELECTRIC
WX=WXA
3.1
CALCULATE
MECHANICAL DISPLACEMENT
MAGNITUDE
MAGU(I),
PHASE
PHASEU(I), I=1, 3
ELECTRIC POTENTIAL
MAGNITUDE
MAGU(4)
PHASE
PHASEU(4)
IMED=1
P1FUN
CALCULATE
TIME AVERAGE POWER FLOW
P1M, P2M
TFUN
CALCULATE
STRESS: TW31, TW32
TW33, TW11, TW12, TW22
SFUN
CALCULATE
STRAIN: S11, S33, S12,
S13, S23
3.2


3.2
CALCULATE
ELECTRIC FIELD
E1, E3
ELECTRIC DISPLACEMENT
D1, D2, D3
3.3
PRINT: W_X, TOGETHER WITH FINAL RESULTS
IMED
(=2)
(=1)
MEDIUM A
4.2
MEDIUM B
DWXA:0
(≠)
(=)
WX=WX+DWXA
WX:WXMAX
(>)
(≤)
3.1
CALCULATE FINAL RESULTS
MEDIUM B–PIEZOELECTRIC
WX=WXB
4.1
IMED=2
CALCULATE FINAL RESULTS, AS FOR MEDIUM A
3.3
DWXB:0
(=)
2.2
(≠)
WX=WX+DWXB
WX:WXBMAX
(>)
2.2
(≤)
4.1


DIELECTRIC
F
DEGEN=0
IDET=0
JP=2
IALF=.FALSE.
ICASE=0
ICB=0
K1=1
K2=4
L1=1
L2=16
IB1=1
IB2=4
1.1
STRIP
CALCULATE COEFFICIENTS OF POLYNOMIAL
POLY(I), I=1, 9
CROOT
CALCULATE ROOTS OF POLYNOMIAL
ALFA(I), I=1, 8
PIEZOELECTRIC
(NO)
(YES)
ELIMINATE ROOT CORRESPONDING TO THE POTENTIAL
ROOTS .OR. ALL
(.TRUE.)
PRINT INTERMEDIATE ROOTS OF POLYNOMIAL
ALFA(I), I=1, 8
(.FALSE.)
IA(I)=I
I=1, 4
SELECT ROOTS WITH POSITIVE REAL PART
ALFAB(I), I=1, K
2
(≠)
K:0
(=)
2.8

$$\beta_3^{(1)} = \beta_3^{(2)} = 10^{-10};\ \beta_1^{(1)} = \frac{-\hat{A}_{13}^{(1)}}{\hat{A}_{11}^{(1)}} \cdot 10^{-10},\ \beta_1^{(2)} = \frac{-\hat{A}_{13}^{(2)}}{\hat{A}_{11}^{(2)}} \cdot 10^{-10}\ ;$$

$$\beta_2^{(1)} = \beta_2^{(2)} = 0\ ;$$

$$\beta_4^{(1)} = \beta_4^{(2)} = 0\ ;$$

$$\beta_4^{(3)} = \beta_4^{(4)} = 1;\ \beta_2^{(3)} = \frac{-\hat{A}_{24}^{(3)}}{\hat{A}_{22}^{(3)}},\ \beta_2^{(4)} = \frac{-A_{24}^{(4)}}{A_{22}^{(4)}}\ ;$$

$$\beta_1^{(3)} = \beta_1^{(4)} = 0;\ \beta_3^{(3)} = \beta_3^{(4)} = 0\ .$$

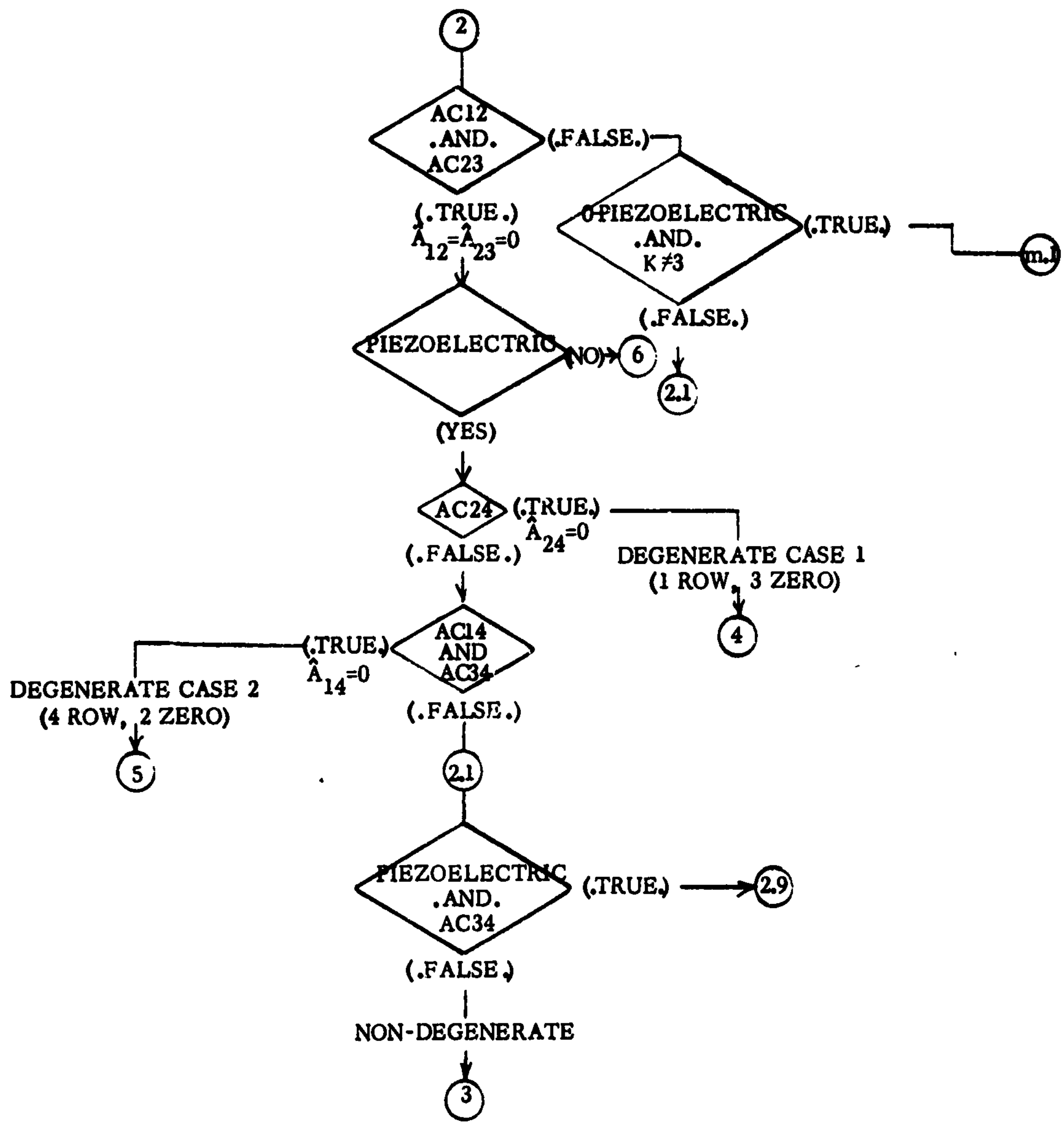
The program now proceeds to set up either the P or the Q matrix and evaluates its determinant.

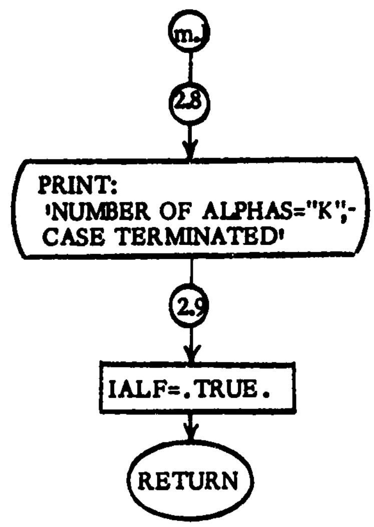
If there are less than four α's with positive real part ($K < 4$) the program proceeds as follows:

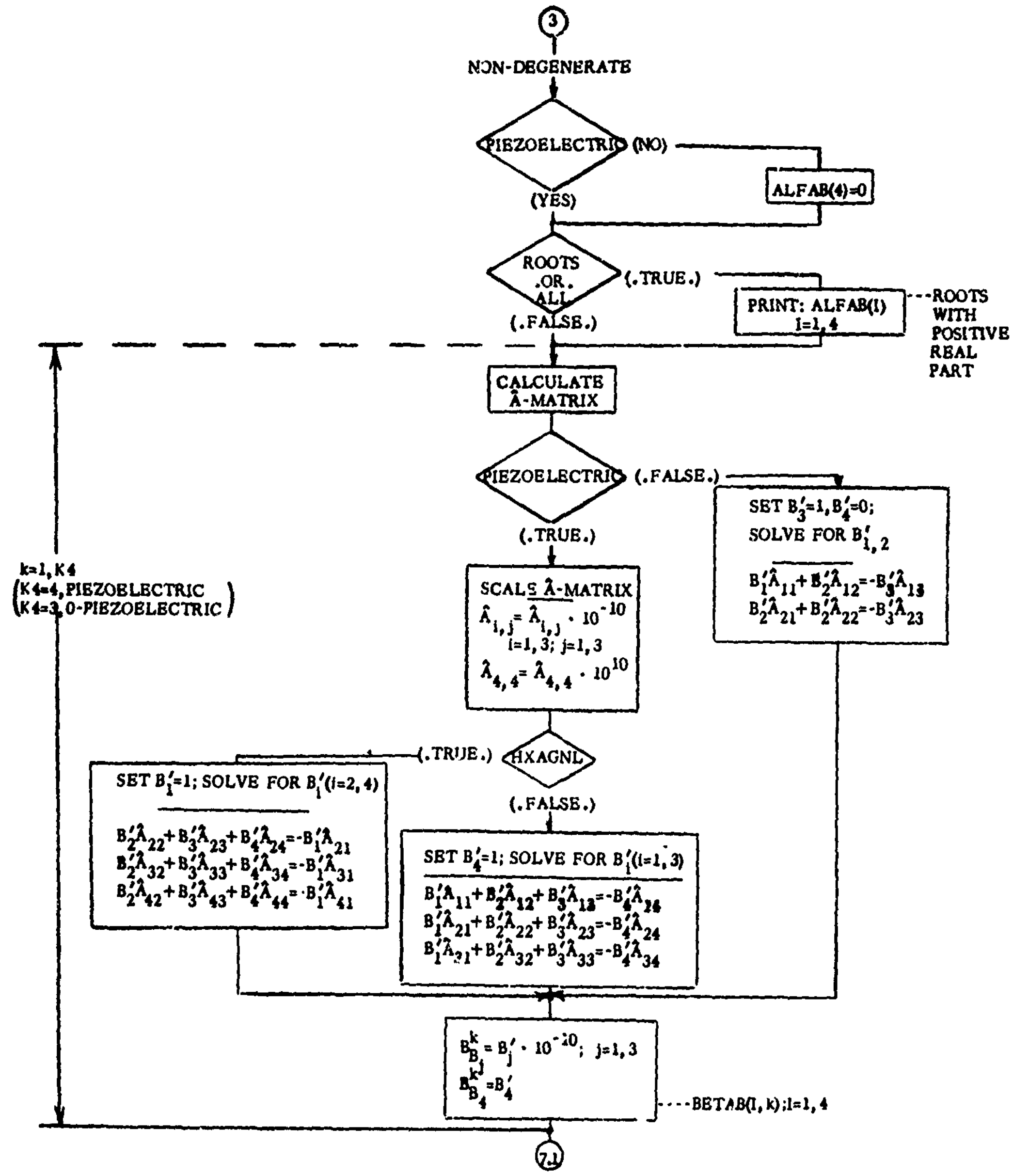
If $K = 1$ the case terminates (case 2c). If $K = 2$ or 3 the program computes the quantity $|\hat{A}_{22}\hat{A}_{44} - \hat{A}_{24}^2|$ for each α and counts the number (I1) of α's for which this quantity $< 10^{-5}$ and the number (I2) of α's for which this quantity $\geq 10^{-5}$. 10^{-5} is close enough to zero due to the magnitudes of the individual terms in the quantity. If $K = 3$ and $I1 = 2$ the α's become $\alpha^{(3)}$ and $\alpha^{(4)}$ and the β's are calculated as they were above for $\alpha^{(3)}$ and $\alpha^{(4)}$. Only the Q matrix is set up and evaluated (case 2a1). If $K = 3$ and $I2 = 2$ the α's become $\alpha^{(1)}$ and $\alpha^{(2)}$ and the β's are calculated as above for $\alpha^{(1)}$ and $\alpha^{(2)}$. Only the P matrix is set up and evaluated (case 2a2). If $K = 3$ while $I1 \neq 2$ and $I2 \neq 2$ the case terminates (case 2a3). If $K = 2$ and $I1 = 2$ the β's are handled as above (case 2b1). If $K = 2$ and $I2 = 2$ the β's are likewise handled as above (case 2b2). If $K = 2$ while $I1 \neq 2$ and $I2 \neq 2$ the case terminates (case 2b3).

Degenerate Non-piezoelectric Case

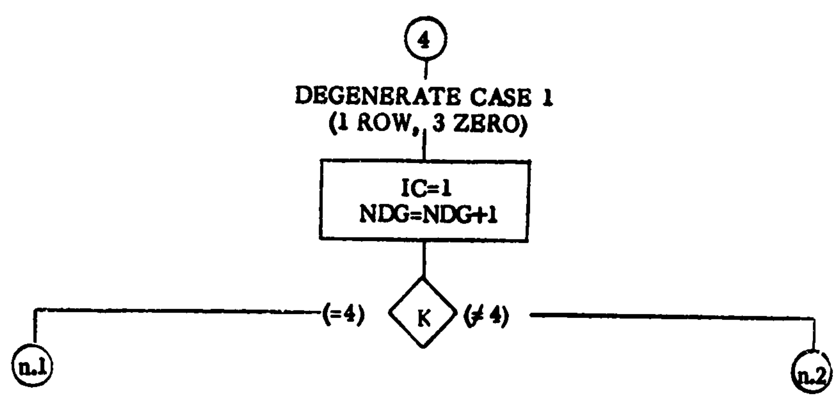
This case is characterized by a decoupling of the equations for $\beta_i^{(\ell)}$ such that two of the equations involve β_1 and β_3 only and one of the equations involves

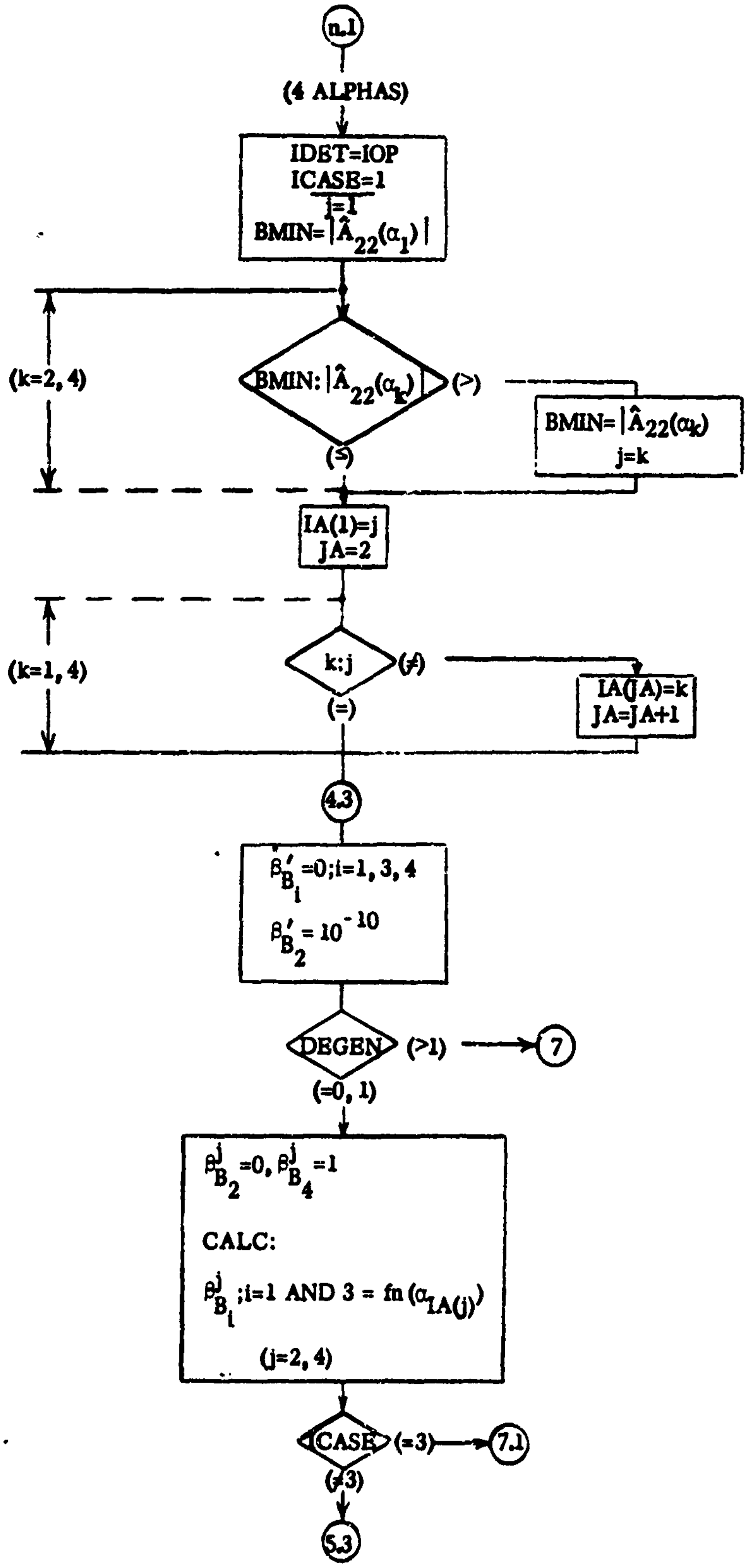

2
AC12 .AND. AC23
(.FALSE.)
(.TRUE.)
$\hat{A}_{12}=\hat{A}_{23}=0$
0 PIEZOELECTRIC .AND. K≠3
(.TRUE.)
m.l
(.FALSE.)
2.1
PIEZOELECTRIC
(NO)
6
(YES)
AC24
(.TRUE.)
$\hat{A}_{24}=0$
DEGENERATE CASE 1
(1 ROW, 3 ZERO)
4
(.FALSE.)
AC14 AND AC34
(.TRUE.)
$\hat{A}_{14}=0$
DEGENERATE CASE 2
(4 ROW, 2 ZERO)
5
(.FALSE.)
2.1
PIEZOELECTRIC .AND. AC34
(.TRUE.)
2.9
(.FALSE.)
NON-DEGENERATE
3


2.8
PRINT:
'NUMBER OF ALPHAS="K";-
CASE TERMINATED'
2.9
IALF=.TRUE.
RETURN

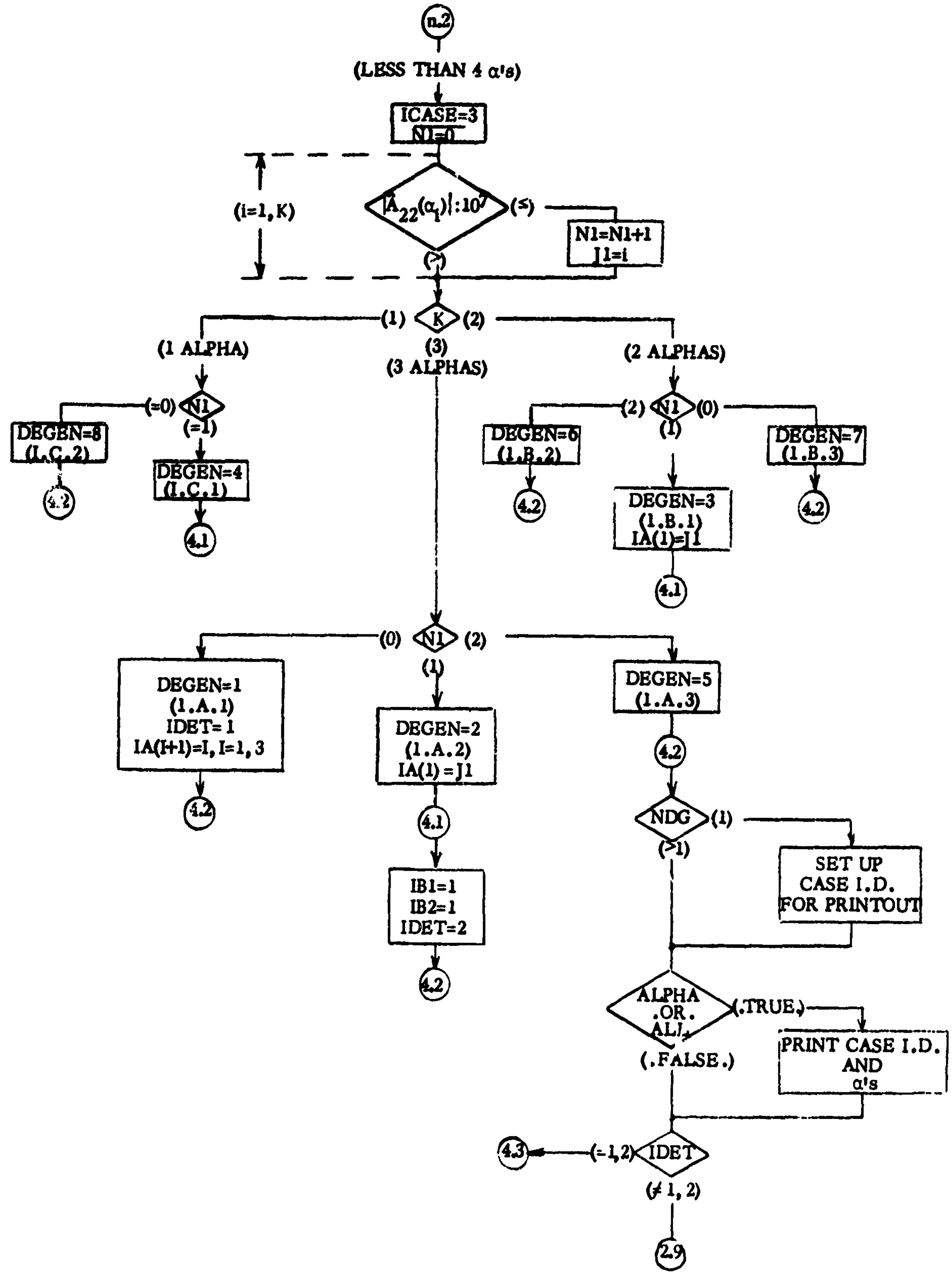


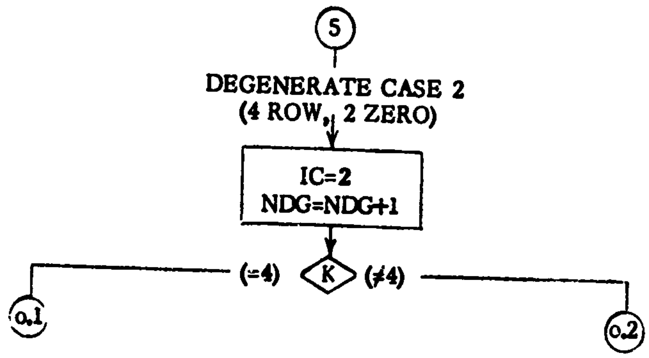
3
NON-DEGENERATE
PIEZOELECTRIC
(NO)
ALFAB(4)=0
(YES)
ROOTS .OR. ALL
(.TRUE.)
PRINT: ALFAB(I) I=1,4
ROOTS WITH POSITIVE REAL PART
(.FALSE.)
CALCULATE Â-MATRIX
PIEZOELECTRIC
(.FALSE.)
SET B'3=1, B'4=0; SOLVE FOR B'1,2
B'1Â11+B'2Â12=-B'3Â13
B'2Â21+B'2Â22=-B'3Â23
(.TRUE.)
k=1, K4 (K4=4, PIEZOELECTRIC; K4=3, 0-PIEZOELECTRIC)
SCALE Â-MATRIX
Âi,j = Âi,j · 10^-10
i=1,3; j=1,3
Â4,4 = Â4,4 · 10^10
HXAGNL
(.TRUE.)
SET B'1=1; SOLVE FOR B'i(i=2,4)
B'2Â22+B'3Â23+B'4Â24=-B'1Â21
B'2Â32+B'3Â33+B'4Â34=-B'1Â31
B'2Â42+B'3Â43+B'4Â44=-B'1Â41
(.FALSE.)
SET B'4=1; SOLVE FOR B'i(i=1,3)
B'1Â11+B'2Â12+B'3Â13=-B'4Â14
B'1Â21+B'2Â22+B'3Â23=-B'4Â24
B'1Â31+B'2Â32+B'3Â33=-B'4Â34
B^k_Bj = B'j · 10^-10; j=1,3
B^k_B4 = B'4
BETAB(I,k); I=1,4
7.1

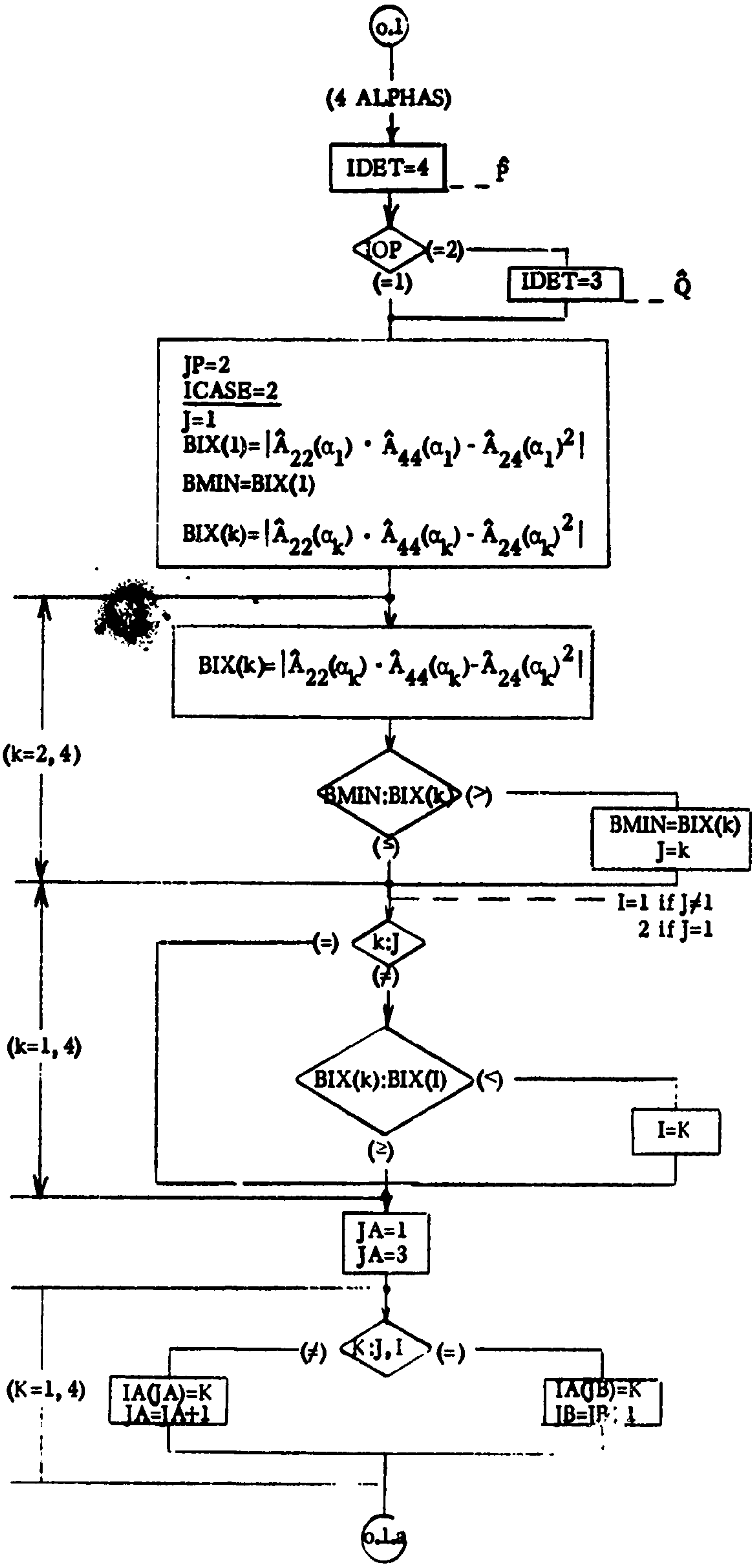

4
DEGENERATE CASE 1
(1 ROW, 3 ZERO)
IC=1
NDG=NDG+1
K
(=4)
(≠ 4)
n.1
n.2



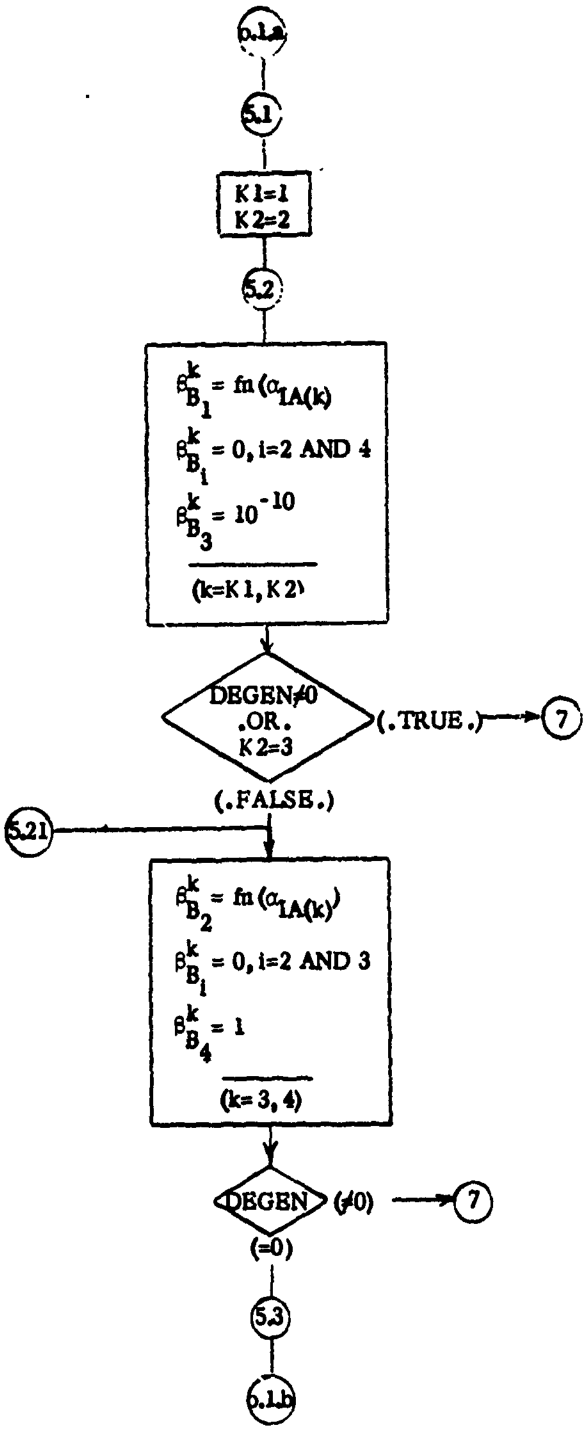
n.1
(4 ALPHAS)
IDET=IOP
ICASE=1
j=1
BMIN=|Â22(α1)|
(k=2, 4)
BMIN: |Â22(αk)|
(>)
(≤)
BMIN=|Â22(αk)
j=k
IA(1)=j
JA=2
(k=1, 4)
k:j
(≠)
(=)
IA(JA)=k
JA=JA+1
4.3
β'B_i =0;i=1, 3, 4
β'B_2 = 10^-10
DEGEN
(>1)
7
(=0, 1)
β^j B_2 =0, β^j B_4 =1
CALC:
β^j B_i ;i=1 AND 3 = fn (α_IA(j))
(j=2, 4)
ICASE
(=3)
7.1
(≠3)
5.3


n.2
(LESS THAN 4 α's)
ICASE=3
N1=0
(i=1, K)
$|A_{22}(\alpha_i)|:10^7$
(≤)
N1=N1+1
J1=i
(>)
(1) K (2)
(3)
(1 ALPHA)
(3 ALPHAS)
(2 ALPHAS)
(=0) N1
(=1)
DEGEN=8
(I.C.2)
4.2
DEGEN=4
(I.C.1)
4.1
(2) N1 (0)
(1)
DEGEN=6
(1.B.2)
4.2
DEGEN=7
(1.B.3)
4.2
DEGEN=3
(1.B.1)
IA(1)=J1
4.1
(0) N1 (2)
(1)
DEGEN=1
(1.A.1)
IDET=1
IA(I+1)=I, I=1, 3
4.2
DEGEN=2
(1.A.2)
IA(1)=J1
4.1
IB1=1
IB2=1
IDET=2
4.2
DEGEN=5
(1.A.3)
4.2
NDG (1)
(>1)
SET UP
CASE I.D.
FOR PRINTOUT
ALPHA
.OR.
ALL
(.TRUE.)
(.FALSE.)
PRINT CASE I.D.
AND
α's
4.3
(=1,2) IDET
(≠1, 2)
2.9

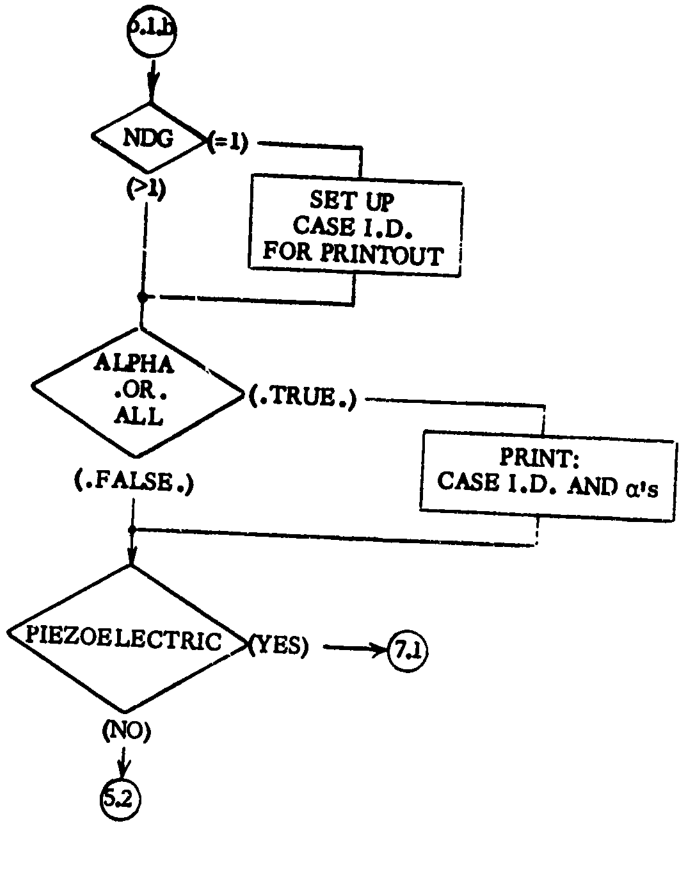

5
DEGENERATE CASE 2
(4 ROW, 2 ZERO)
IC=2
NDG=NDG+1
(=4)
K
(≠4)
0.1
0.2



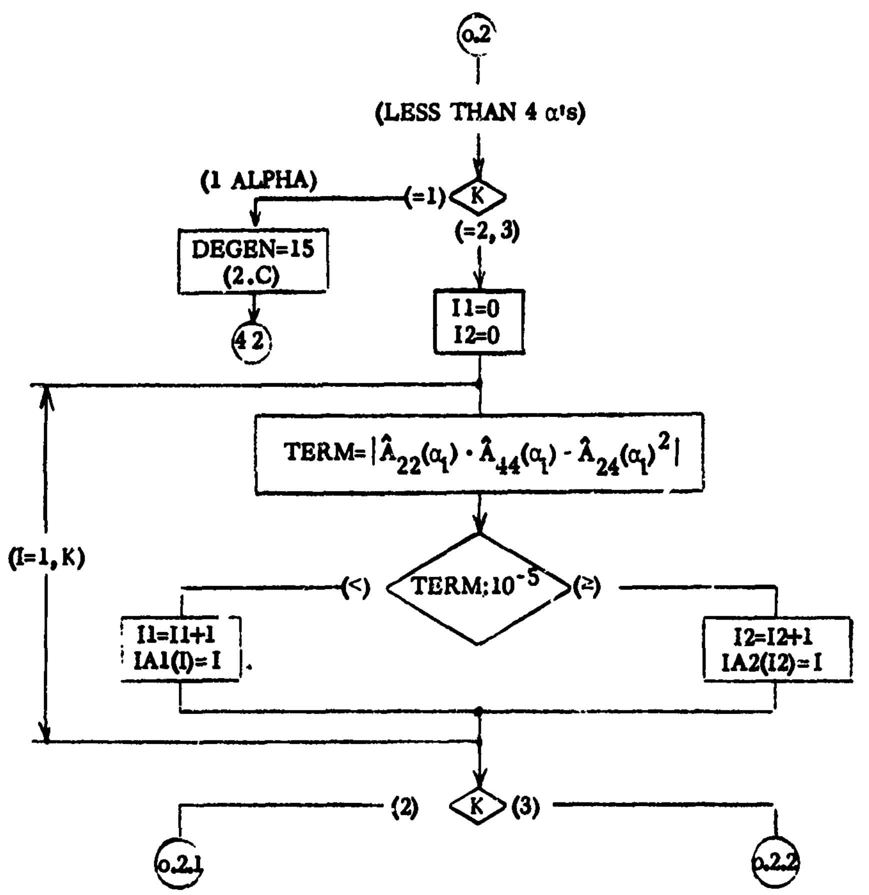
o.1
(4 ALPHAS)
IDET=4
P̂
IOP
(=2)
(=1)
IDET=3
Q̂
JP=2
ICASE=2
J=1
BIX(1)=|Â22(α1) · Â44(α1) - Â24(α1)²|
BMIN=BIX(1)
BIX(k)=|Â22(αk) · Â44(αk) - Â24(αk)²|
BIX(k)=|Â22(αk) · Â44(αk)-Â24(αk)²|
(k=2, 4)
BMIN:BIX(k)
(>)
BMIN=BIX(k)
J=k
(≤)
I=1 if J≠1
2 if J=1
(=)
k:J
(≠)
(k=1, 4)
BIX(k):BIX(I)
(<)
I=K
(≥)
JA=1
JA=3
(≠)
K:J, I
(=)
(K=1, 4)
IA(JA)=K
JA=JA+1
IA(JB)=K
o.1.a

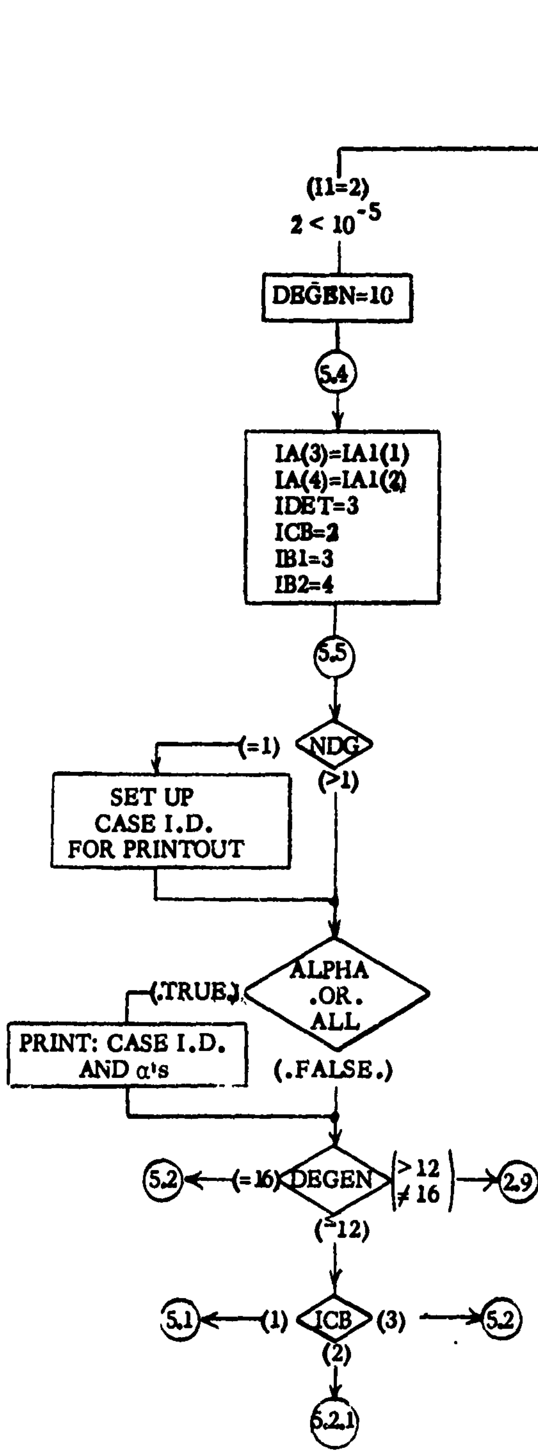


5.1a
5.1
K1=1
K2=2
5.2
$\beta^k_{B_1}$ = fn ($\alpha_{IA(k)}$
$\beta^k_{B_i}$ = 0, i=2 AND 4
$\beta^k_{B_3}$ = 10^{-10}
(k=K1, K2)
DEGEN≠0
.OR.
K2=3
(.TRUE.)
7
(.FALSE.)
5.21
$\beta^k_{B_2}$ = fn ($\alpha_{IA(k)}$)
$\beta^k_{B_i}$ = 0, i=2 AND 3
$\beta^k_{B_4}$ = 1
(k= 3, 4)
DEGEN
(≠0)
7
(=0)
5.3
5.1b

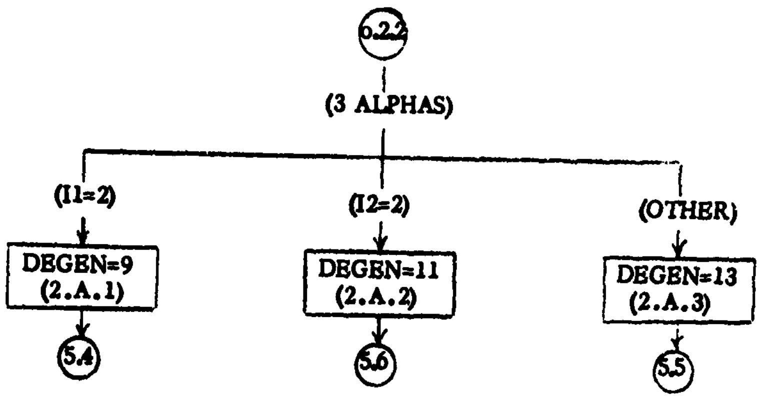


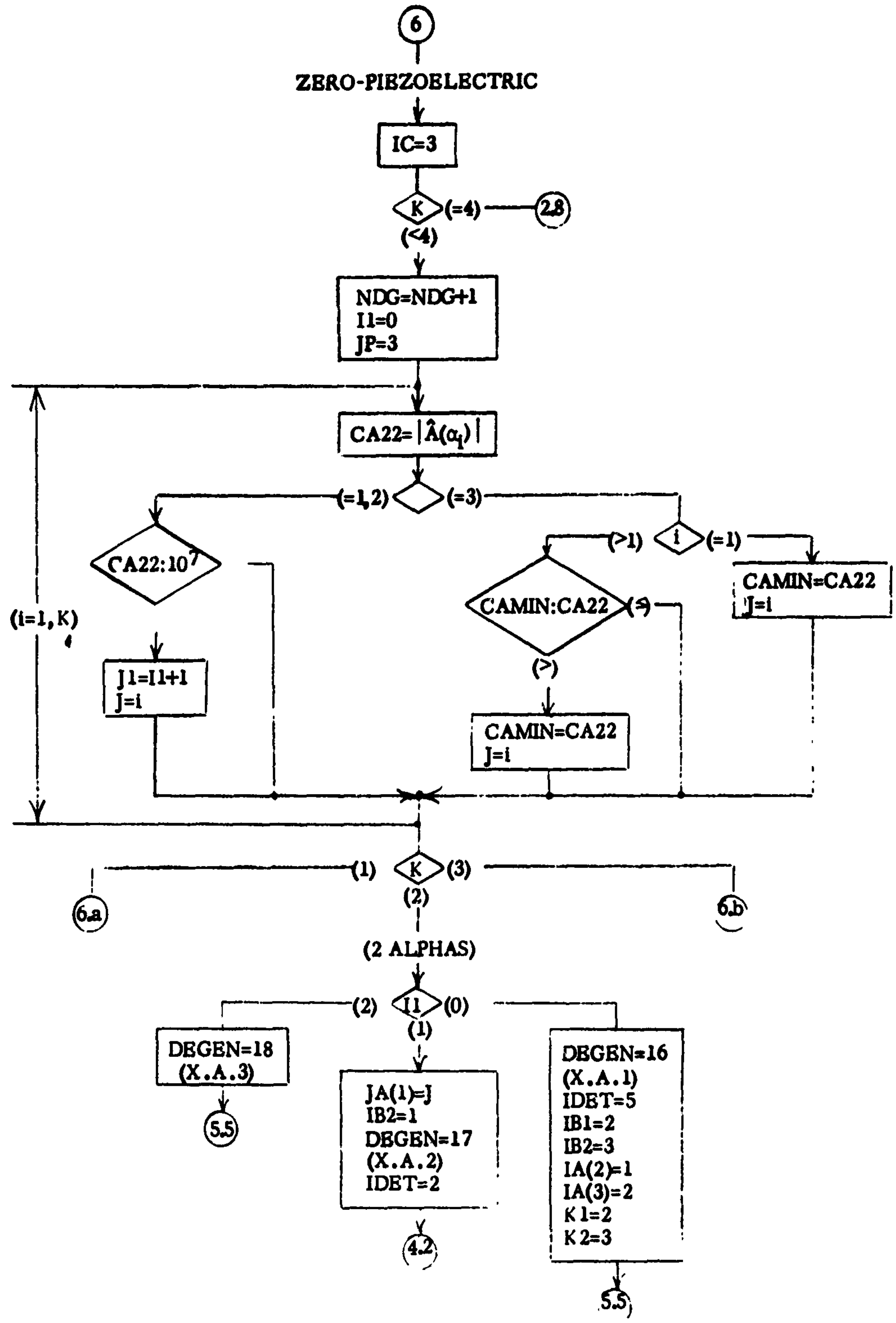
5.1.b
NDG
(=1)
(>1)
SET UP
CASE I.D.
FOR PRINTOUT
ALPHA
.OR.
ALL
(.TRUE.)
PRINT:
CASE I.D. AND α's
(.FALSE.)
PIEZOELECTRIC
(YES)
7.1
(NO)
5.2

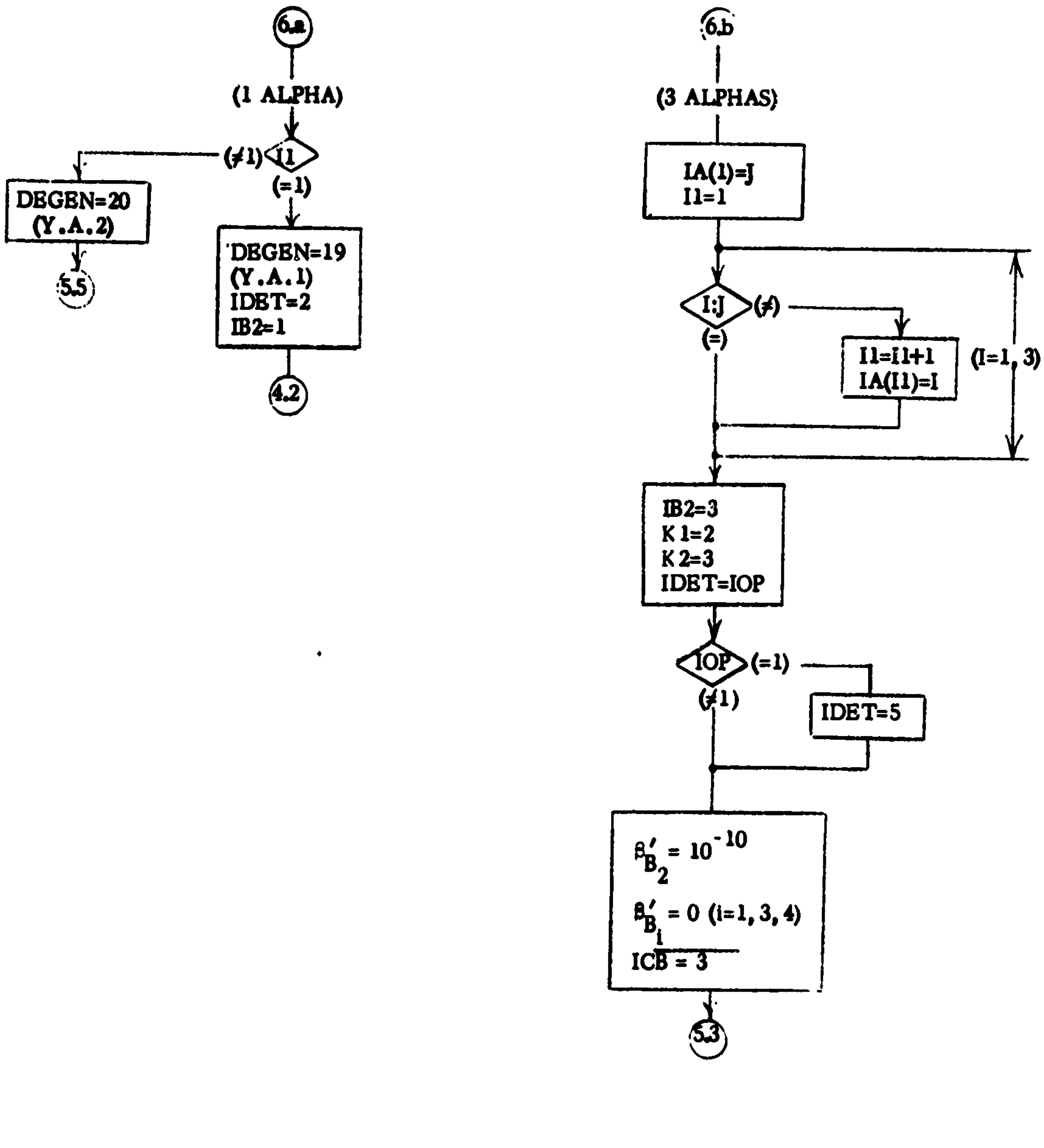

o.2
(LESS THAN 4 α's)
(1 ALPHA)
(=1)
K
(=2, 3)
DEGEN=15
(2.C)
4 2
I1=0
I2=0
TERM=$|\hat{A}_{22}(\alpha_I)\cdot\hat{A}_{44}(\alpha_I)-\hat{A}_{24}(\alpha_I)^2|$
(I=1, K)
(<)
TERM;10^{-5}
(≥)
I1=I1+1
IA1(I)=I
I2=I2+1
IA2(I2)=I
(2)
K
(3)
o.2.1
o.2.2



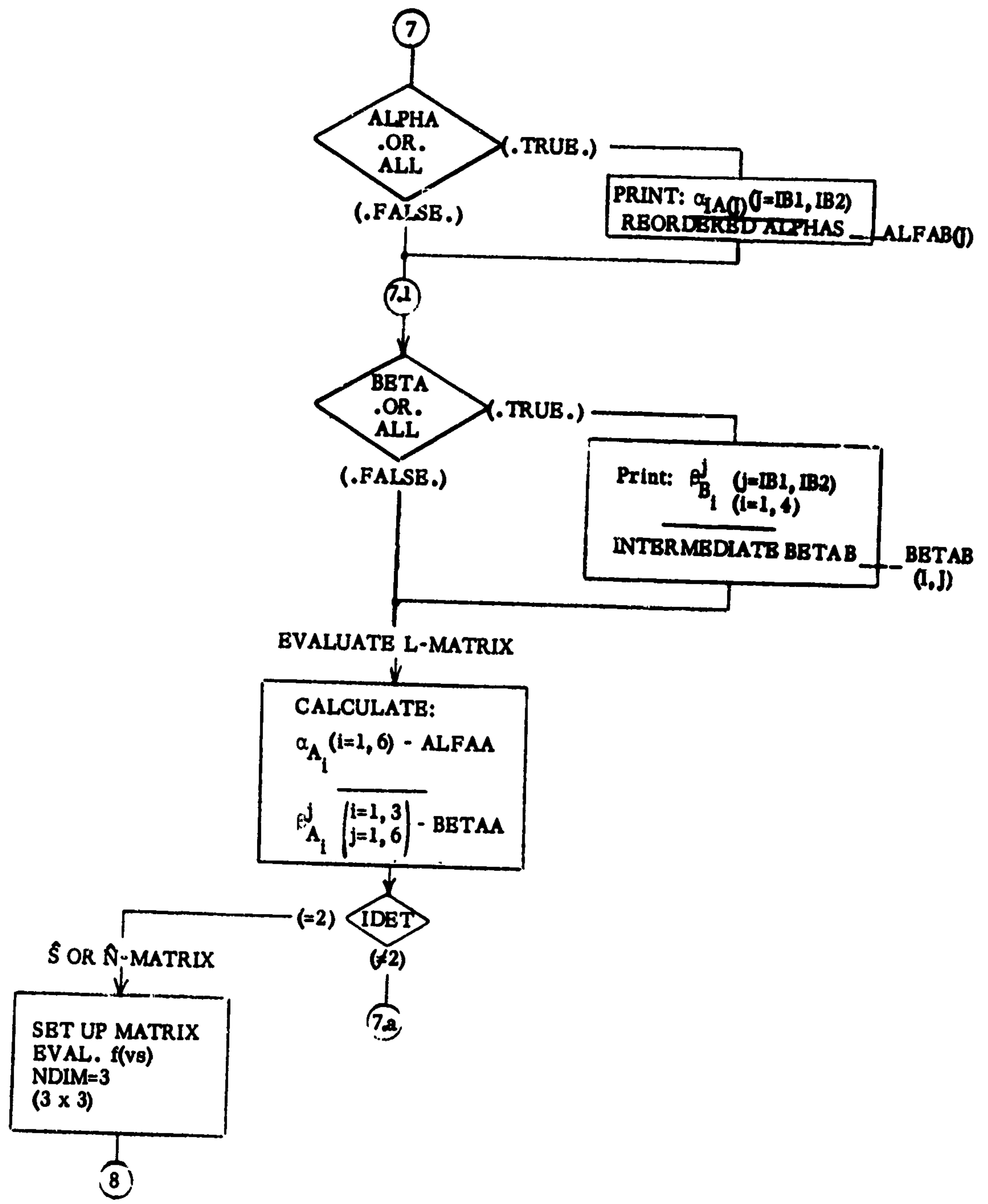
5.2.1
(2 ALPHAS)
(I1=2)
2 < 10^-5
(I2=2)
2 ≥ 10^-5
(OTHER)
DEGEN=10
DEGEN=12
(2.B.2)
DEGEN=14
(2.B.3)
5.4
5.6
5.5
IA(3)=IA1(1)
IA(4)=IA1(2)
IDET=3
ICB=2
IB1=3
IB2=4
IA(1)=IA2(1)
IA(2)=IA2(2)
IDET=4
ICB=1
IB1=1
IB2=2
5.5
5.5
NDG
(=1)
(>1)
SET UP
CASE I.D.
FOR PRINTOUT
ALPHA
.OR.
ALL
(.TRUE.)
PRINT: CASE I.D.
AND α's
(.FALSE.)
DEGEN
(=16)
5.2
> 12
≠ 16
2.9
(=12)
ICB
(1)
5.1
(3)
5.2
(2)
5.2.1


6.2.2
(3 ALPHAS)
(I1=2)
(I2=2)
(OTHER)
DEGEN=9
(2.A.1)
DEGEN=11
(2.A.2)
DEGEN=13
(2.A.3)
5.4
5.6
5.5

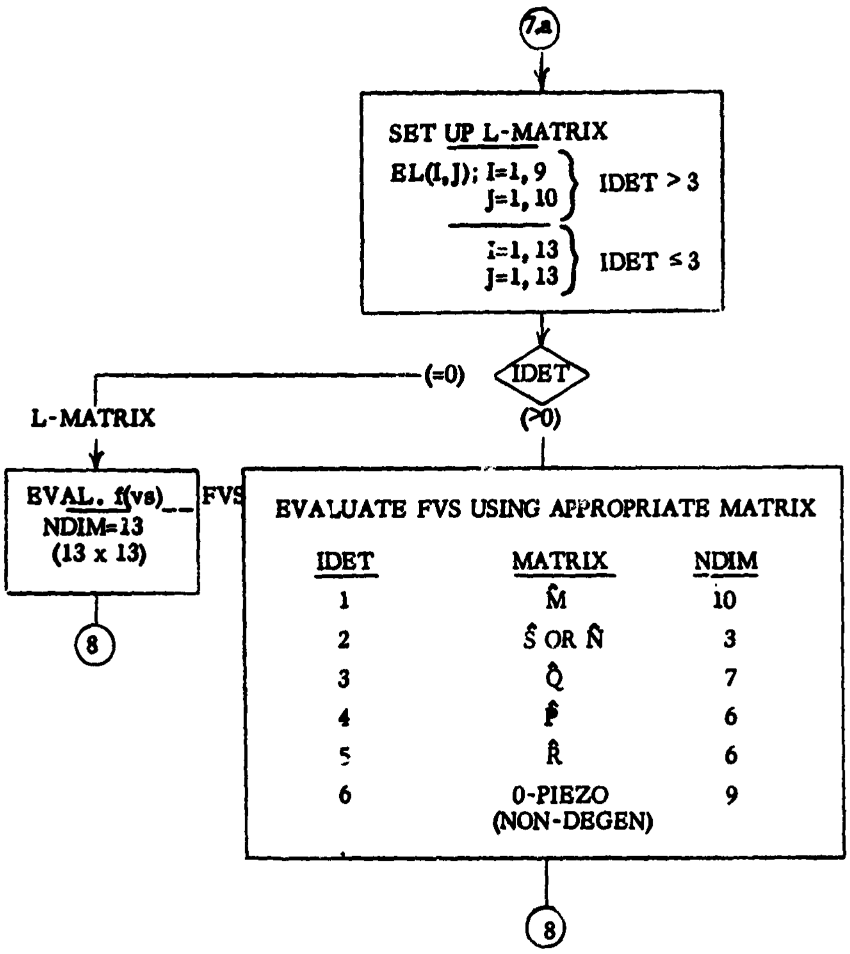

6
ZERO-PIEZOELECTRIC
IC=3
K
(=4)
2.8
(<4)
NDG=NDG+1
I1=0
JP=3
CA22=|Â(α_i)|
(=1,2)
(=3)
i
(>1)
(=1)
CA22:10^7
CAMIN=CA22
J=i
CAMIN:CA22
(≤)
(i=1, K)
(>)
I1=I1+1
J=i
CAMIN=CA22
J=i
K
(1)
(3)
(2)
6.a
6.b
(2 ALPHAS)
I1
(2)
(0)
(1)
DEGEN=18
(X.A.3)
5.5
JA(1)=J
IB2=1
DEGEN=17
(X.A.2)
IDET=2
4.2
DEGEN=16
(X.A.1)
IDET=5
IB1=2
IB2=3
IA(2)=1
IA(3)=2
K1=2
K2=3
5.5

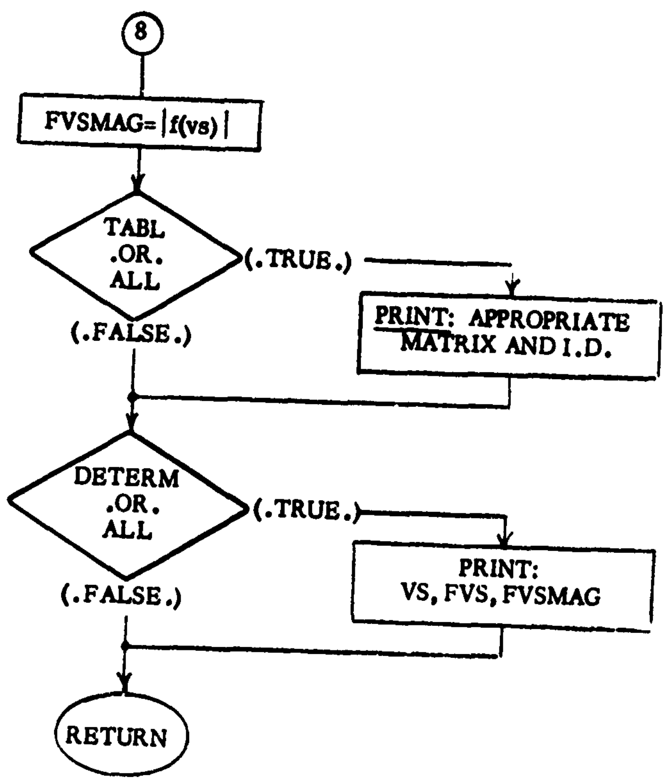


6.a
(1 ALPHA)
I1
(≠1)
(=1)
DEGEN=20
(Y.A.2)
5.5
DEGEN=19
(Y.A.1)
IDET=2
IB2=1
4.2
6.b
(3 ALPHAS)
IA(1)=J
I1=1
I:J
(≠)
(=)
I1=I1+1
IA(I1)=I
(I=1, 3)
IB2=3
K1=2
K2=3
IDET=IOP
IOP
(=1)
(≠1)
IDET=5
$\beta'_{B_2} = 10^{-10}$
$\beta'_{B_i} = 0$ (i=1, 3, 4)
ICB = 3
5.3



7
ALPHA .OR. ALL
(.TRUE.)
(.FALSE.)
PRINT: $\alpha_{IA(J)}$ (J=IB1, IB2)
REORDERED ALPHAS
ALFAB(J)
7.1
BETA .OR. ALL
(.TRUE.)
(.FALSE.)
Print: $\beta^j_{B_i}$ (j=IB1, IB2) (i=1, 4)
INTERMEDIATE BETAB
BETAB (I, J)
EVALUATE L-MATRIX
CALCULATE:
α_{A_i} (i=1, 6) - ALFAA
$\beta^j_{A_i}$ (i=1, 3; j=1, 6) - BETAA
IDET
(=2)
(≠2)
7.a
$\hat{S}$ OR $\hat{N}$-MATRIX
SET UP MATRIX
EVAL. f(vs)
NDIM=3
(3 x 3)
8


7a
SET UP L-MATRIX
EL(I,J); I=1, 9
J=1, 10
IDET > 3
I=1, 13
J=1, 13
IDET ≤ 3
IDET
(=0)
(>0)
L-MATRIX
EVAL. f(vs)
NDIM=13
(13 x 13)
FVS
8
EVALUATE FVS USING APPROPRIATE MATRIX
IDET
MATRIX
NDIM
1
M̂
10
2
Ŝ OR N̂
3
3
Q̂
7
4
P̂
6
5
R̂
6
6
0-PIEZO
(NON-DEGEN)
9
8


8
FVSMAG= |f(vs)|
TABL
.OR.
ALL
(.TRUE.)
(.FALSE.)
PRINT: APPROPRIATE
MATRIX AND I.D.
DETERM
.OR.
ALL
(.TRUE.)
(.FALSE.)
PRINT:
VS, FVS, FVSMAG
RETURN

APPENDIX I. ELEMENTS OF THE $\hat{L}$ MATRIX FOR THE ELASTIC-CONDUCTOR PIEZOELECTRIC SUBSTRATE PROBLEM

The subscript p corresponds to the piezoelectric medium while the subscript c corresponds to the conducting elastic medium. The C_{ij}'s and e_{ij}'s correspond to the piezoelectric medium while λ, μ correspond to the conductor (Lame's constants). The expressions for $\alpha_c^{(j)}$ are easily obtainable from equation (20) and are as follows

$$\alpha_c^{(1,2)} = \pm\sqrt{\frac{\mu - \rho_c v_s^2}{\mu}} = \alpha_c^{(5,6)}$$

$$\alpha_c^{(3,4)} = \pm\sqrt{\frac{2\mu + \lambda - \rho_c v_s^2}{2\mu + \lambda}} \quad .$$

The elements of the 10 x 10 boundary value determinant are as follows:

$$L_{i\ell} = \beta_{ci}^{(\ell)} \; [i = 1,2,3 \;\; \ell = 1,2,\ldots,6]$$

$$L_{i\ell} = -\beta_{pi}^{(\ell-6)} \; [i = 1,2,3 \qquad \ell = 7,8,9,10]$$

$$L_{4\ell} = \beta_{c1}^{(\ell)}\alpha_c^{(\ell)}\mu + j\,\beta_{c3}^{(\ell)}\mu \quad [\ell = 1,2,\ldots,6]$$

$$L_{4\ell} = -\beta_{p1}^{(\ell-6)}\,[j\,C_{15} + \alpha_p^{(\ell-6)}\,C_{55}] - \beta_{p2}^{(\ell-6)}\,[j\,C_{56} + \alpha_p^{(\ell-6)}\,C_{45}]$$

$$-\beta_{p3}^{(\ell-6)}\,[j\,C_{55} + \alpha_p^{(\ell-6)}\,C_{35}] - \beta_{p4}^{(\ell-6)}\,[j\,e_{15} + \alpha_p^{(\ell-6)}\,e_{35}] \quad [\ell = 7,8,9,10]$$

$$L_{5\ell} = \beta_{c2}^{(\ell)}\alpha_c^{(\ell)}\mu \; [\ell = 1,2,\ldots,6]$$

$$L_{5\ell} = -\beta_{p1}^{(\ell-6)}\,[j\,C_{14} + \alpha_p^{(\ell-6)}\,C_{45}] - \beta_{p2}^{(\ell-6)}\,[j\,C_{46} + \alpha_p^{(\ell-6)}\,C_{44}]$$

$$-\beta_{p3}^{(\ell-6)}\,[j\,C_{45} + \alpha_p^{(\ell-6)}\,C_{34}] - \beta_{p4}^{(\ell-6)}\,[j\,e_{14} + \alpha_p^{(\ell-6)}\,e_{34}] \quad [\ell = 7,8,9,10]$$

$$L_{6\ell} = j\,\beta_{c1}^{(\ell)}\,\lambda + \beta_{c3}^{(\ell)}\,\alpha_c^{(\ell)}\,(2\mu + \lambda) \qquad [\ell = 1, 2, \ldots, 6]$$

$$L_{6\ell} = -\,\beta_{p1}^{(\ell-6)}\,[j\,C_{13} + \alpha_p^{(\ell-6)}\,C_{35}] - \beta_{p2}^{(\ell-6)}\,[j\,C_{36} + \alpha_p^{(\ell-6)}\,C_{34}]$$

$$-\,\beta_{p3}^{(\ell-6)}\,[j\,C_{35} + \alpha_p^{(\ell-6)}\,C_{33}] - \beta_{p4}^{(\ell)}\,[j\,e_{13} + \alpha_p^{(\ell-6)}\,e_{33}] \qquad [\ell = 7, 8, 9, 10]$$

$$L_{7\ell} = L_{4\ell}\,e^{\alpha_c^{(\ell)}\,\omega h/v_s} \qquad [\ell = 1, 2, \ldots, 6]$$

$$L_{7\ell} = 0 \qquad [\ell = 7, 8, 9, 10]$$

$$L_{8\ell} = L_{5\ell}\,e^{\alpha_c^{(\ell)}\,\omega h/v_s} \qquad [\ell = 1, 2, \ldots, 6]$$

$$L_{8\ell} = 0 \qquad [\ell = 7, 8, 9, 10]$$

$$L_{9\ell} = L_{6\ell}\,e^{\alpha_c^{(\ell)}\,\omega h/v_s} \qquad [\ell = 1, 2, \ldots, 6]$$

$$L_{9\ell} = 0 \qquad [\ell = 7, 8, 9, 10]$$

$$L_{10\ell} = 0 \qquad [\ell = 1, 2, \ldots, 6]$$

$$L_{10\ell} = \beta_{p4}^{(\ell-6)} \qquad [\ell = 7, 8, 9, 10]$$

APPENDIX II. EXPLICIT FORMS OF THE ELEMENTS OF THE MATRIX M ASSOCIATED WITH THE FLUID MEDIUM PIEZOELECTRIC SUBSTRATE PROBLEM

$$M_{1\ell} = \beta_3^{(\ell)} \cdot 10^{10} , \qquad \ell = 1,2,3,4,$$

$$M_{15} = 0 ,$$

$$M_{16} = -1 ;$$

$$M_{2\ell} = \beta_4^{(\ell)} , \qquad \ell = 1,2,3,4$$

$$M_{25} = -1 ,$$

$$M_{26} = 0 ;$$

$$M_{3\ell} = [\beta_1^{(\ell)}(je_{31} + \alpha_c^{(\ell)} e_{35}) + \beta_2^{(\ell)} (je_{36} + \alpha_c^{(\ell)} e_{34}) + \beta_3^{(\ell)} (je_{35} + \alpha_c^{(\ell)} e_{33}) - \beta_4^{(\ell)} (j\varepsilon_{13} + \alpha_c^{(\ell)} \varepsilon_{33})] \cdot 10^{10} ,$$

$$\ell = 1,2,3,4$$

$$M_{35} = -\varepsilon_\ell \cdot 10^{10} ,$$

$$M_{36} = 0 ;$$

$$M_{4\ell} = \beta_1^{(\ell)}(jC_{13} + \alpha_c^{(\ell)} C_{35}) + \beta_2^{(\ell)} (jC_{36} + \alpha_c^{(\ell)} C_{34}) + \beta_3^{(\ell)} (jC_{35} + \alpha_c^{(\ell)} C_{33}) + \beta_4^{(\ell)} (je_{13} + \alpha_c^{(\ell)} e_{33}) ,$$

$$\ell = 1,2,3,4 ,$$

$$M_{45} = 0 ,$$

$$M_{46} = \frac{\rho_\ell v_s^2}{\alpha_\ell} \cdot 10^{-10} ;$$

$$M_{5\ell} = \beta_1^{(\ell)}(jC_{15} + \alpha_c^{(\ell)}C_{55}) + \beta_2^{(\ell)}(jC_{56} + \alpha_c^{(\ell)}C_{45})$$
$$+ \beta_3^{(\ell)}(jC_{55} + \alpha_c^{(\ell)}C_{35}) + \beta_4^{(\ell)}(je_{15} + \alpha_c^{(\ell)}e_{35}) \ ,$$
$$\ell = 1, 2, 3, 4 \ ,$$

$$M_{55} = M_{56} = 0 \ ;$$

$$M_{6\ell} = \beta_1^{(\ell)}(jC_{14} + \alpha_c^{(\ell)}C_{45}) + \beta_2^{(\ell)}(jC_{46} + \alpha_c^{(\ell)}C_{44})$$
$$+ \beta_3^{(\ell)}(jC_{45} + \alpha_c^{(\ell)}C_{34}) + \beta_4^{(\ell)}(je_{14} + \alpha_c^{(\ell)}e_{34}) \ ,$$
$$\ell = 1, 2, 3, 4 \ ,$$

$$M_{65} = M_{66} = 0 \ .$$

The factors 10^{10} and 10^{-10} are introduced to make the real and imaginary parts of all elements of the matrix on the order of unity.

APPENDIX III. ELEMENTS OF THE $\mathcal{L}$ MATRIX FOR THE ELASTIC LAYER PIEZOELECTRIC SUBSTRATE PROBLEM

$L_{i\ell} = \quad \beta_{di}^{(\ell)} \cdot 10^{10}$ [i=1, 2, 3 ℓ = 1, 2, ..., 6]

$L_{i\ell} = \quad -\beta_{ci}^{(\ell-6)} \cdot 10^{10}$ [i=1, 2, 3 ℓ=7, 8, 9, 10]

$L_{i\ell} = \quad 0$ [i=1, 2, 3 ℓ=11, 12, 13]

$L_{4\ell} = \quad \mu_d\, \alpha_d^{(\ell)}\, \beta_{d1}^{(\ell)} + j\, \mu_d\, \beta_{d3}^{(\ell)}$ [ℓ=1, 2, ..., 6]

$$L_{4\ell} = -\beta_{c1}^{(\ell-6)} [j\, c_{15} + \alpha_c^{(\ell-6)} c_{55}] - \beta_{c2}^{(\ell-6)} [j\, c_{56} + \alpha_c^{(\ell-6)} c_{45}]$$

$$-\beta_{c3}^{(\ell-6)} [j\, c_{55} + \alpha_c^{(\ell-6)} c_{35}] - \beta_{c4}^{(\ell-6)} [j\, e_{15} + \alpha_c^{(\ell-6)} e_{35}]$$

[ℓ = 7, 8, 9, 10]

$L_{4\ell} = \quad 0$ [ℓ = 11, 12, 13]

$L_{5\ell} = \quad \mu_d\, \alpha_d^{(\ell)}\, \beta_{d2}^{(\ell)}$ [ℓ = 1, 2, ..., 6]

$$L_{5\ell} = -\beta_{c1}^{(\ell-6)} [j\, c_{14} + \alpha_c^{(\ell-6)} c_{45}] - \beta_{c2}^{(\ell-6)} [j\, c_{46} + \alpha_c^{(\ell-6)} c_{44}]$$

$$-\beta_{c3}^{(\ell-6)} [j\, c_{45} + \alpha_c^{(\ell-6)} c_{34}] - \beta_{c4}^{(\ell-6)} [j\, e_{14} + \alpha_c^{(\ell-6)} e_{34}]$$

[ℓ = 7, 8, 9, 10]

$L_{5\ell} = \quad 0$ [ℓ = 11, 12, 13]

$L_{6\ell} = \quad j\, \lambda_d\, \beta_{d1}^{(\ell)} + (\lambda_d + 2\, \mu_d)\, \alpha_d^{(\ell)}\, \beta_{d3}^{(\ell)}$ [ℓ = 1, 2, ..., 6]

$$L_{6\ell} = -\beta_{c1}^{(\ell-6)} [j\, c_{13} + \alpha_c^{(\ell-6)} c_{35}] - \beta_{c2}^{(\ell-6)} [j\, c_{36} + \alpha_c^{(\ell-6)} c^{34}]$$

$$-\beta_{c3}^{(\ell-6)} [j\, c_{35} + \alpha_c^{(\ell-6)} c_{33}] - \beta_{c4}^{(\ell-6)} [j\, e_{13} + \alpha_c^{(\ell-6)} e_{33}]$$

[ℓ = 7, 8, 9, 10]

$$L_{6\ell} = 0 \quad [\ell = 11, 12, 13]$$

$$L_{7\ell} = L_{4\ell}\, e^{\alpha_d^{(\ell)}\, \omega h/v_s} \quad [\ell = 1, 2, \ldots, 6]$$

$$L_{7\ell} = 0 \quad [\ell = 7, 8, \ldots, 13]$$

$$L_{8\ell} = L_{5\ell}\, e^{\alpha_d^{(\ell)}\, \omega h/v_s} \quad [\ell = 1, 2, \ldots, 6]$$

$$L_{8\ell} = 0 \quad [\ell = 7, 8, \ldots, 13]$$

$$L_{9\ell} = L_{6\ell}\, e^{\alpha_d^{(\ell)}\, \omega h/v_s} \quad [\ell = 1, 2, \ldots, 6]$$

$$L_{9\ell} = 0 \quad [\ell = 7, 8, \ldots, 13]$$

$$L_{10\ell} = 0 \quad [\ell = 1, 2, \ldots, 6]$$

$$L_{10\ell} = \beta_{c4}^{(\ell-6)} \quad [\ell = 7, 8, 9, 10]$$

$$L_{10\ell} = -1 \quad [\ell = 11, 12]$$

$$L_{10\ell} = 0 \quad [\ell = 13]$$

$$L_{11\ell} = 0 \quad [\ell = 1, 2, \ldots, 10]$$

$$L_{11,11} = e^{-\omega h/v_s}$$

$$L_{11,12} = e^{\omega h/v_s}$$

$$L_{11,13} = -e^{-\omega h/v_s}$$

$$L_{12\ell} = 0 \qquad [\ell = 1, 2, \ldots, 6]$$

$$L_{12\ell} = \Big[\beta_{c1}^{(\ell-6)}\,[j\,e_{13} + \alpha_c^{(\ell-6)}\,e_{35}] + \beta_{c2}^{(\ell-6)}\,[j\,e_{36} + \alpha_c^{(\ell-6)}\,e_{34}]$$

$$+ \beta_{c3}^{(\ell-6)}\,[j\,e_{35} + \alpha_c^{(\ell-6)}\,e_{33}] - \beta_{c4}^{(\ell-6)}\,[j\,\varepsilon_{13} + \alpha_c^{(\ell-6)}\,\varepsilon_{33}]\Big] \cdot 10^{10}$$

$$\ell = 7, 8, 9, 10$$

$$L_{12,11} = -\epsilon_d \cdot 10^{10}$$

$$L_{12,12} = \epsilon_d \cdot 10^{10}$$

$$L_{12,13} = 0$$

$$L_{13,\ell} = 0 \qquad [\ell = 1, 2, \ldots, 10]$$

$$L_{13,11} = e^{-\omega h/v_s}$$

$$L_{13,12} = -e^{\omega h/v_s}$$

$$L_{13,13} = -\frac{\epsilon_o}{\epsilon_d}\, e^{-\omega h/v_s}$$

APPENDIX IV. EXPLICIT FORMS OF THE POLYNOMIAL COEFFICIENTS A_k

The elements of the matrix $\hat{A}$ have the general form $\hat{A}_{ik} = a_{ik}\alpha^2 + j\, b_{ik}\alpha + d_{ik}$ where a_{ik}, b_{ik}, and d_{ik} are easily deduced from equation (6). Therefore, the determinant of $\hat{A}$ can be expressed as the polynomial

$$A_1\alpha^8 + jA_2\alpha^7 + A_3\alpha^6 + jA_4\alpha^5 + A_5\alpha^4 + jA_6\alpha^3 + A_7\alpha^2 + jA_8\alpha + A_9$$

with coefficients

$$A_1 = \sum_{\{j,k,\ell,m\}} (-1)^h H_{jk\ell} S_m$$

$$A_2 = \sum_{\{j,k,\ell,m\}} (-1)^h [H_{jk\ell} \bar{U}_m + I_{jk\ell} S_m]$$

$$A_3 = \sum_{\{j,k,\ell,m\}} (-1)^h [H_{jk\ell} V_m - I_{jk\ell} \bar{U}_m + J_{jk\ell} S_m]$$

$$A_4 = \sum_{\{j,k,\ell,m\}} (-1)^h [I_{jk\ell} V_m + J_{jk\ell} \bar{U}_m + K_{jk\ell} S_m]$$

$$A_5 = \sum_{\{j,k,\ell,m\}} (-1)^h [J_{jk\ell} V_m - K_{jk\ell} \bar{U}_m + L_{jk\ell} S_m]$$

$$A_6 = \sum_{\{j,k,\ell,m\}} (-1)^h [K_{jk\ell} V_m + L_{jk\ell} \bar{U}_m + M_{jk\ell} S_m]$$

$$A_7 = \sum_{\{j,k,\ell,m\}} (-1)^h \, [L_{jk\ell} V_m - M_{jk\ell} \bar{U}_m + N_{jk\ell} S_m]$$

$$A_8 = \sum_{\{j,k,\ell,m\}} (-1)^h \, [M_{jk\ell} V_m + N_{jk\ell} \bar{U}_m]$$

$$A_9 = \sum_{\{j,k,\ell,m\}} (-1)^h \, N_{jk\ell} V_m \qquad .$$

$\sum\limits_{\{j,k,\ell,m\}}$ refers to a sum over all permutations of 1, 2, 3, 4. There are 24 terms in each sum. h is the number of interchanges for each term necessary to to return the indices to the order 1, 2, 3, 4; and

$$H_{jk\ell} = a_{1j} \, a_{2k} \, a_{3\ell}$$

$$I_{jk\ell} = a_{1j} \, a_{2k} \, b_{3\ell} + (a_{1j} \, b_{2k} + b_{1j} \, a_{2k}) \, a_{3\ell}$$

$$J_{jk\ell} = a_{1j} \, a_{2k} \, d_{3\ell} - (a_{1j} \, b_{2k} + b_{1j} \, a_{2k}) \, b_{3\ell} + (a_{1j} \, d_{2k} - b_{1j} \, b_{2k} + d_{1j} \, a_{2k}) \, a_{3\ell}$$

$$K_{jk\ell} = (a_{1j} \, b_{2k} + b_{1j} \, a_{2k}) \, d_{3\ell} - (a_{1j} \, b_{2k} + b_{1j} \, a_{2k}) \, b_{3\ell} + (b_{1j} \, d_{2k} + d_{1j} \, b_{2k}) \, a_{3\ell}$$

$$L_{jk\ell} = (a_{1j} \, d_{2k} - b_{1j} \, b_{2k} + d_{1j} \, a_{2k}) \, d_{3\ell} - (b_{1j} \, d_{2k} + d_{1j} \, b_{2k}) \, b_{3\ell} + d_{1j} \, d_{2k} \, a_{3\ell}$$

$$M_{jk\ell} = (b_{1j} \, d_{2k} + d_{1j} \, b_{2k}) \, d_{3\ell} + d_{1j} \, d_{2k} \, b_{3\ell}$$

$$N_{jk\ell} = d_{1j} \, d_{2k} \, d_{3\ell}$$

$$S_m = a_{4m}$$

$$\overline{U}_m = b_{4m}$$

$$V_n = d_{4m}$$

APPENDIX V. COMPUTER PROGRAMS

```
                        CONWAY      PHASE          N4.                      12/31/69    000109      PAGE  1

$ID    * 1614 N4CONWAY       PHASE            N4                 000109      112244      12/31/69          000109
$TCP              TIME=3,PAGES=20,DUMP
$SETUP LB4        CRTPLT
$ALL              CONTINUE
$IBSYS
 RETURNING TO IBSYS.
$IBJOB            DEBUG
$IBFTC LINBO3  DECK,DEBUG

CLINBC3                                                              MAIN0020
C*                 GOLD LITHIUM AND LITHIUM NIOBATE                  MAIN0030
C*                                                                   MAIN0040
C*****************************************************************   MAIN0050
                                                                     MAIN0060
      CIMENSION PANGLE(181)
      DIMENSION XX(2),YY(2)
      CIMENSION DELTA(181)
      DIMENSION RT11(181),RIT11(181),RU1(181),RIU1(181),RE1(181)
      CIMENSION REI1(181)
      CIMENSION CETRAY(100),DETIAY(100),VSARAY(100),AAAAA(200)
      CCMPLEX DISP(4)
      INTEGER BLIMIT,ELIMIT
      LOGICAL ROTATE
      LCGICAL GETOUT
      LCGICAL NEGAT
      LOGICAL MUIT
      LCGICAL ICHECK
       CCPMON /ROTAT/ ROTATE
      CCMMCN /GET/GETOUT
      CCMMON /PLOTS/ICHECK
      CATA XX/0.,10./,YY/5.,5./
      DIMENSION  DEG(181), TITLES(4)
      DIMENSION  VELOC(181), VELOCI(181), VEL(362)
      CCMMCN/OVR/KO,NI
      CCMMCN /ZZTZ/ CC(20),CE(17),CT(5)
      COMMON     /LINK/  ALFA(8), ALFAI(8), EL(100), ALFAA(6),       MAIN0080
     .                   ALFAB(4), BETAA(3,6), BETAB(4,4), EPSO,     MAIN0090
     .                   MUA, LAMDAA, RHOA, MUB, LAMCAB, NUB, RHOB,  MAIN0100
     .                   VS, KS, EPSLON, DIGIT, WH, WXA, WXB, KL,    MAIN0110
     .                   KM, ALL, ROOTS, ITER, COEFF, DETERM, POLY,  MAIN0120
     .                   ALPHA, BETA, MAX                            MAIN0130
      CCMMCN     /GPEPS/ G(21), P(18), EPS(9)                        MAIN0140
      COMMON     /FLAG/  ONCE    /BETAN/  NBETA                      MAIN0150
     .            /CSET/ CLIM, ELIM, TLIM                            MAIN0160
      CCMMON     /FROOT/ FVSMAG, NT, ICASE                           MAIN0170
     .             /COM/   ACAP, EPSR                                MAIN0180
     .             /CIA/   IA(4)                                     MAIN0190
     .           /ALESS/ IALF                                        MAIN0195
     .       /CIFL/ MXAGNL
      INTEGER    PLOTIT(6)
      CCMPLEX      XEL(3,3), FXL, XL(2,3), XET(2)                    MAIN0200
      COMPLEX    ALFA, FVS, EL, EA(6), EB(4), UA(3), UB(3), ALFAI,   MAIN0210
     .           ALFAA, ALFAB, BETAA, BETAB, ETA(10), PHIB           MAIN0220
      CCMPLEX      CXO, CX1                                          MAIN0230
      CCMPLEX    TFUN, P1FUN, U, EX(4), P1M, P2M,                    MAIN0240
     .           TW31, TW32, TW33, TW11, TW12, TW22, S11, S22, S33,  MAIN0250
     .           S12, S13, S23, C1, D2, D3, JIMAG, E1, E3            MAIN0260
      REAL       MUA, LAMDAA, MUB, LAMDAB, NUA, NUB,                 MAIN0270
     .           MAGU(4), PHASEU(4), NUMAX, TITLE(4)                 MAIN0280
      LOGICAL    ALL, ROOTS, COEFF, DETERM, PGLY, ALPHA, BETA,       MAIN0290
```

```
•            ICHECK, ACAP, IALF,                                        MAIN0300
• HXAGNL,
•            REPEAT, ONCE                                               MAIN0310
             LINB03   CONWAY     PHASE              K4                               12/31/69    000109     PAGE  1
      LINB03     -  EFN   SOURCE STATEMENT  -  IFN(S)  -

 EQUIVALENCE ( VEL(1), VELOC(1) ), ( VELOCI(1), VEL(102) )
 EQUIVALENCE ( TITLE, TITLES(1) )
 EQUIVALENCE (CC(1), C11), (CC(2), C13), (CC(3), C14), (CC(4),C15),MAIN0320
•            (CC(5), C33), (CC(6), C34), (CC(7), C35), (CC(8),C36),MAIN0330
•            (CC(9), C44), (CC(10),C45), (CC(11),C46),                  MAIN0340
•            (CC(12),C55), (CC(13),C56), (CC(14),C66),                  MAIN0350
•            (CC(15),C16), (CE(1), E11), (CE(2), E13),                  MAIN0360
•            (CE(3), E14), (CE(4), E15), (CE(5), E16),                  MAINC370
•            (CE(6), E31), (CE(7), E33), (CE(8), E34),                  MAIN0380
•            (CE(9), E35), (CE(10),E36), (CT(1), T11),                  MAIN0390
•            (CT(2), T13), (CT(3), T33)                                 MAIN0400
 EQUIVALENCE (CC(16),C12), (CC(17),C25), (CC(18),C26),                  MAIN0410
•            (CC(19),C24), (CC(20),C23), (CE(11),E12),                  MAIN0420
•            (CE(12),E32), (CE(13),E21), (CE(14),E23),                  MAIN0430
•            (CE(15),E24), (CE(16),E25), (CE(17),E26),                  MAIN0440
•            (CT(4), T21), (CT(5), T23)                                 MAIN0450
                                                                        MAIN0460
 NAMELIST   /INPUT/ MUA, LAMDAA, RHOA, MLB, LAMDAB, NUB, RHOB,          MAIN0470
•                   VS, KS, EPSLON, WH, WXA, WXB, EPSR,                 MAIN0480
• NEGAT,
• HXAGNL,
•                   KL, KM, ALL, ROOTS, COEFF, DETERM,                  MAIN0490
•                   POLY, ALPHA, BETA, MAX, EPSO, WX, REPEAT,           MAIN0500
•                   ICHECK, DVS, VSMAX, ACAP, CLIM, ELIM, TLIM,         MAIN0510
•                   DNU, NUMAX, DWX, WXMAX, TITLE                       MAIN0520
• ,MLIT,ROTATE
 NAMELIST   /CONST/ G, P, EPS                                           MAIN0530
                                                                        MAIN0540
                                                                        MAIN0550
 CATA         CX0,CX1/(0.,0.),(1.,0.)/                                  MAIN0560
 CATA        TT31,  TT32,  TT33,  TT11,  TT12,  TT22 /                  MAIN0570
•           3HT31, 3HT32, 3HT33, 3HT11, 3HT12, 3HT22 /                  MAIN0580
 CATA        SS11,  SS22,  SS33,  SS12,  SS13,  SS23 /                  MAIN0590
•           3HS11, 3HS22, 3HS33, 3HS12, 3HS13, 3HS23 /                  MAIN0600
 CATA        DD1,  DD2,  DD3,  UU1,  UU2,  UU3 /                        MAIN0610
•           2HD1, 2HD2, 2HD3, 2HU1, 2HU2, 2HU3 /                        MAIN0620
 CATA        PP1M, PP2M / 3HP1M, 3HP2M /                                MAIN0630
 CATA       EE1, EE3, JIMAG / 2HE1, 2HE3, (0.,1.) /                     MAIN0640
 DATA       TITLE(1) / 24HLITHIUM NIOBATE          /                    MAIN0650
                                                                        MAIN0660
                                                                        MAIN0670
 REAC(5,7773) JJJK                                                                1
7773 FCRMAT(I2)
 IF(JJJK.EQ.0) GO TO 7774
 CALL SKFILE(2,JJJK)                                                              6
7774 CCNTINUE
 CC 20 I=1,101
  20 CEG(I) = I-1
 CALL CRTPLT(1.0)                                                                 15
 CALL PLOT(0.,-.25,-3)                                                            17
 CLIM   = 1.E5                                                          MAIN0680
 ELIM   = 1.E-5                                                         MAIN0690
 TLIM   = 1.E-15                                                        MAIN0700
 EPSLON = 1.E-11                                                        MAIN0710
 EPSC   = 8.85E-12                                                      MAIN0720
 EPSR   = 1.                                                            MAIN0730
```

```
   LINB03      -  EFN   SOURCE STATEMENT  -  IFN(S)  -

                                                          MAIN0730
         RHOA   = 1.888E4                                 MAIN0740
         MUA    = 2.85E10                                 MAIN0750
         LAMDAA = 1.5E11                                  MAIN0760
         VSAVE  = 3000.                                   MAIN0770
         DVS    = 0.                                      MAIN0780
         VSMAX  = 0.                                      MAIN0790
         DIGIT  = 3.0                                     MAIN0800
         NUMAX  = 0.                                      MAIN0810
         DNU    = 0.                                      MAIN0820
         WXMAX  = 0.                                      MAIN0830
         DWX    = 0.                                      MAIN0840
                                                          MAIN0850
         KL     = 0                                       MAIN0860
         KM     = 0                                       MAIN0870
         MAX=25
         ALL    = .FALSE.                                 MAIN0890
         ROOTS  = .FALSE.                                 MAIN0900
         COEFF  = .FALSE.                                 MAIN0920
         DETERM = .FALSE.                                 MAIN0930
         NEGAT=.FALSE.
         POLY   = .FALSE.                                 MAIN0940
         ALPHA  = .FALSE.                                 MAIN0950
         BETA   = .FALSE.                                 MAIN0960
         ICHECK = .FALSE.                                 MAIN0970
         ACAP   = .FALSE.                                 MAIN0980
         HXAGNL=.FALSE.
         GETCUT =.FALSE.
         PUIT=.FALSE.
         ROTATE=.FALSE.
                                                          MAIN0990
C.....I N P U T    D A T A.....                           MAIN1000
  510 READ (5, CONST)                                     MAIN1020     22
    1 FCRMAT( 4A6 )
      REAC(5,1) ( TITLES(I), I=1,4 )                                   23
      KCUNT = 0
      KONT=0
      KNT1=0
      KNT2=0
      KNT3=0
      REPEAT = .FALSE.                                    MAIN1030
  520 VS = VSAVE                                          MAIN1040
      KTIME = 0                                           MAIN1050
      READ (5, INPUT)                                     MAIN1060     36
      SNU = NUB                                           MAIN1080
      SWX = WX                                            MAIN1090
      READ(5,7) (PLOTIT(I),I=1,6)                                      38
    7 FCRMAT(6I2)
  530 KTIME = KTIME + 1                                   MAIN1100
      IF (KTIME .LE. 2)  GC TO 535                        MAIN1110
      COA = (VS1 - VS2)/(AN1 - AN2)                       MAIN1120
      COB = VS1 - COA*AN1                                 MAIN1130
      VS  = COA*NUB + COB                                 MAIN1140
  535 VSAVE = VS                                          MAIN1150
      CNCE  = .TRUE.                                      MAIN1160
                                                          MAIN1170
```

```
              LINBO3    CONWAY       PHASE              N4                      12/31/69    000109      PAGE  3
         LINBO3     -  EFN   SOURCE STATEMENT  -  IFN(S)  -

C.....CALCULATE COEFFICIENTS.....                                          MAIN1180
      CALL SETCTE (MUB, NUB, LAMDAB, COEFF .OR. ALL)                       MAIN1190    53
      IF(ICHECK) GO TO 2
                                                                           MAIN1200
C.....CALCULATE ROOT AND OUTPUT RESULTS.....                               MAIN1210
      VSO = VS                                                             MAIN1220
      NT = 0                                                               MAIN1230
  550 CONTINUE                                                             MAIN1240
      CALL ROOT(VSO,VS,N,FVS,EPSLON,MAX,$550)                                          59
      IF(GETOUT) GO TO 17
      IF( PLOTIT(1) .EQ. 0 ) GO TO 32
      KCUNT = KOUNT + 1
      VEL(KOUNT) = VS
   32 CONTINUE
      VS1 = VS2                                                            MAIN1260
      VS2 = VS                                                             MAIN1270
      AN1 = AN2                                                            MAIN1280
      AN2 = NUB                                                            MAIN1290
      VSI = 1./VS                                                          MAIN1300
      DO 615 I = 1, 8                                                      MAIN1310
  615 ALFAI(I) = ALFA(I)/VS                                                MAIN1320
      WRITE  (6, 650)  KS, TITLE,  EPSLON,                                 MAIN1330
     .                 KL, KM, MAX, N, MUB, LAMDAB, NUB, LAMDAA, MUA,      MAIN1340
     .                 RHOA, RHOB, WH, VSO, VS, VSI, FVS,                  MAIN1350
     .                 (ALFA(I), ALFAI(I), I = 1, 8)                       MAIN1360    78
      PUNCH 1500,LAMDAB,MUB,NLB                                                        87
 1500 FORMAT(3F5.1)
      PUNCH 1502,VS,VSI,(ALFA(I),I=1,8)                                                88
      WRITE  (2, 650)  KS, TITLE,  EPSLON,                                 MAIN1330
     .                 KL, KM, MAX, N, MUB, LAMDAB, NUB, LAMDAA, MUA,      MAIN1340
     .                 RHOA, RHOB, WH, VSO, VS, VSI, FVS,                  MAIN1350
     .                 (ALFA(I), ALFAI(I), I = 1, 8)                       MAIN1360    95
  650 FORMAT (1H1,20X,4HKS =,I2,5X, 4A6                 /1H0,6X,           MAIN1370
     . 8HEPSLON =,E15.7,3X,32HCLOSENESS OF DETERMINANT TO ZERO/1H ,6X,     MAIN1380
     . 2HKL,5X,1H=,I3,15X,34H0 = COMPUTE FOURTH ROW OF L MATRIX/1H ,       MAIN1390
     . 32X,22H1 = SET FOURTH ROW = 1/1H ,6X,2HKM,5X,1H=,I3,15X,            MAIN1400
     . 25H0 = ELECTRIC FIELD (COTH)/1H ,32X,25H1 = MAGNETIC FIELD (TANH)MAIN1410
     . /1H ,6X,3HMAX,4X,1H=,I3,15X,28HMAXIMUM NUMBER OF ITERATIONS/        MAIN1420
     . 1H ,6X,1HN,6X,1H=,I3,15X,34HNUMBER OF ITERATIONS ACTUALLY USED/     MAIN1430
     . 1H ,6X,8HMU B   =,E15.7,7X,9HLAMDA B =,E15.7/1H ,6X,8HNU B   =,     MAIN1440
     . E15.7,7X,9HLAMDA A =,E15.7/1H ,6X,8HMU A   =,E15.7/1H ,6X,          MAIN1450
     . 8HRHO A  =,E15.7,7X,9HRHO B   =,E15.7/1H ,6X,2HWH,5X,1H=,E15.7/     MAIN1460
     . 1H , 6X,                                                            MAIN1470
     . 8HVS 0   =,E15.7,3X,16HINITIAL VELOCITY/1H0,6X,2HVS,5X,1H=,E15.7,MAIN1480
     . 3X,42HFINAL VELOCITY SUCH THAT F(VS) .LT. EPSLON/1H ,6X,            MAIN1490
     . 8H1/VS   =,                                                         MAIN1500
     . E15.7,3X,13HINVERSE OF VS/1H0,9X,21H..... DETERMINANT = (,E14.7,    MAIN1510
     . 1H,,E14.7,7H) ...../1H0,5X,25HFINAL ROOTS OF POLYNOMIAL,16X,        MAIN1520
     . 13HDIVIDED BY VS//(4H   (,E14.7,1H,,E14.7,1H),7X,1H(,E14.7,         MAIN1530
     . 1H,,E14.7,1H)))                                                     MAIN1540
                                                                           MAIN1550
      IF( IALF ) GO TO 1707                                                MAIN1555
      IF( .NOT. ICHECK ) GO TO 660                                         MAIN1560
      IF( VSO .GE. VSMAX ) GO TO 1708                                      MAIN1570
      WRITE(6,1705)                                                        MAIN1580    112
      VSO = VSO + DVS                                                      MAIN1590
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IF (A(4).EQ XCD)

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      GC TO 550                                                       MAIN1600
  660 CCNTINUE                                                        MAIN1610
      IF (KS .NE. 0)  GO TO 1240                                      MAIN1630
                                                                      MAIN1640
C.....CALCULATE FINAL RESULTS FOR LITHIUM NIOBATE AND OUTPUT RESULTS....MAIN1650
      IF( ICASE .EQ. 0 ) GO TO 1000                                   MAIN1660
      GO TO ( 50, 100, 60, 110, 105 ), ICASE                          MAIN1670
                                                                      MAIN1680
                                                                      MAIN1690
C.....1 ROW, 3 ZERO CASE.....                                          MAIN1700
                                                                      MAIN1710
C.....( 4 ALPHAS ).....                                                MAIN1720
   50 ETA(1) = CXO                                                    MAIN1730
      ETA(4) = CX1                                                    MAIN1740
      XL(1,1) = EL(5)                                                 MAIN1750
      XL(2,1) = EL(7)                                                 MAIN1760
      XL(1,2) = EL(9)                                                 MAIN1770
      XL(2,2) = EL(11)                                                MAIN1780
      XL(1,3) = - EL(13)                                              MAIN1790
      XL(2,3) = - EL(15)                                              MAIN1800
      CALL CMATS( XL, XET, 2, 1, $1740 )                              MAIN1810     123
      ETA(2) = XET(1)                                                 MAIN1820
      ETA(3) = XET(2)                                                 MAIN1830
      GC TO 1100                                                      MAIN1840
                                                                      MAIN1850
C.....( 3 ALPHAS ).....                                                MAIN1860
   60 ETA(3) = CX1                                                    MAIN1870
      ETA(4) = CXO                                                    MAIN1880
      XL(1,1) = EL(1)                                                 MAIN1890
      XL(2,1) = EL(3)                                                 MAIN1900
      XL(1,2) = EL(5)                                                 MAIN1910
      XL(2,2) = EL(7)                                                 MAIN1920
      XL(1,3) = -EL(9)                                                MAIN1930
      XL(2,3) = -EL(11)                                               MAIN1940
      CALL CMATS( XL, ETA, 2, 1, $1740 )                              MAIN1950     128
      GC TO 1100                                                      MAIN1960
                                                                      MAIN1970
                                                                      MAIN1980
C.....4 ROWS, 2 ZERO CASE.....                                         MAIN1990
  100 EL1 = CABS( EL(1)*EL(7) - EL(5)*EL(3) )                         MAIN2000     132
      EL2 = CABS( EL(10)*EL(16) - EL(14)*EL(12) )                     MAIN2010     133
      IF( EL1 .LE. EL2 ) GC TO 110                                    MAIN2020
  105 ETA(1) = CXO                                                    MAIN2030
      ETA(2) = CXO                                                    MAIN2040
      ETA(3) = - EL(14) / EL(10)                                      MAIN2050
      ETA(4) = CX1                                                    MAIN2060
      GC TO 1100                                                      MAIN2070
  110 ETA(1) = - EL(5) / EL(1)                                        MAIN2080
      ETA(2) = CX1                                                    MAIN2090
      ETA(3) = CXO                                                    MAIN2100
      ETA(4) = CXO                                                    MAIN2110
      GO TO 1100                                                      MAIN2120
                                                                      MAIN2130
 1000 IF (NBETA .EQ. 3)  GO TO 1010                                   MAIN2140
                                                                      MAIN2150
C.....ZERC - PIEZOELECTRIC CASE.....                                   MAIN2160
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         LINBO3   CONWAY      PHASE            N4                  12/31/69   000109     PAGE  5
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      EL(3) = EL(5)                                                MAIN2170
      EL(4) = EL(6)                                                MAIN2180
      EL(5) =-EL(9)                                                MAIN2190
      EL(6) =-EL(10)                                               MAIN2200
      GO TO 1095                                                   M.IN2210
                                                                   MAIN2220
 1010 EL(4)    = EL(5)                                             MAIN2230
      EL(5)    = EL(6)                                             MAIN2240
      EL(6)    = EL(7)                                             MAIN2250
      EL(7)    = EL(9)                                             MAIN2260
      EL(8)    = EL(10)                                            MAIN2270
      EL(9)    = EL(11)                                            MAIN2280
      EL(10)  =-EL(13)                                             MAIN2290
      EL(11)  =-EL(14)                                             MAIN2300
      EL(12)  =-EL(15)                                             MAIN2310
 1095 KGO = 12                                                     MAIN2320
      CALL CMATS  (EL, ETA, NBETA, 1, $1740)                       MAIN2330     148
      ETA(4)        = (0.,0.)                                      MAIN2340
      ETA(NBETA+1) = (1.,0.)                                       MAIN2350
                                                                   MAIN2360
 1100 CCNTINUE                                                     MAIN2370
      NBETA1 = NBETA + 1                                           MAIN2380
      WRITE  (6, 1120)  (I, ETA(I), I = 1, NBETA1)                 MAIN2390     153
      WRITE  (2, 1120)  (I, ETA(I), I = 1, NBETA1)                 MAIN2390     160
 1120 FORMAT (1H1/1H , 17X, 36H***  F I N A L    A N S W E R S  ***/  MAIN2400
     .          1H0, 30X,13HPARTIAL FIELD/1H ,27X,19HRELATIVE AMPLITUDES/ MAIN2410
     .                          /(1H , 17X, I2, 3H  (, E14.7, 1H,,  MAIN2420
     .          E14.7, 1H)))                                       MAIN2430
      PUNCH 1502, (ETA(I),I=1,4)                                                167
      GO TO 1800                                                   MAIN2440
                                                                   MAIN2450
......CALCULATE FINAL RESULTS FOR GOLD LITHIUM AND OUTPUT RESULTS..... MAIN2460
 1240 M1 = 1                                                       MAIN2470
      K  = 0                                                       MAIN2480
      CO 1310 I = 1, 9                                             MAIN2490
      M2 = M1 + 8                                                  MAIN2500
      DO 1300 J = M1, M2                                           MAIN2510
      K = K + 1                                                    MAIN2520
 1300 EL(K) = EL(J)                                                MAIN2530
 1310 M1 = M1 + 10                                                 MAIN2540
      CO 1340 I = 51, 99                                           MAIN2550
      K  = K + 1                                                   MAIN2560
 1340 EL(K) = -EL(I)                                               MAIN2570
      KGC = 90                                                     MAIN2580
      CALL CMATS  (EL, ETA, 9, 1, $1740)                           MAIN2590     201
      ETA(10) = (1.,0.)                                            MAIN2600
      CC 1490 I = 1, 3                                             MAIN2610
      UA(I) = (0.,0.)                                              MAIN2620
      UB(I) = (0.,0.)                                              MAIN2630
      CC 1480 J = 1, 6                                             MAIN2640
      IF (I .GT. 1)  GO TO 1460                                    MAIN2650
      EA(J) = CEXP(-ALFAA(J)*WXA/VS)                               MAIN2660     215
      IF (J .GT. 4)  GO TO 1460                                    MAIN2670
      EB(J) = CEXP(-ALFAB(J)*WXB/VS)                               MAIN2680     221
 1460 UA(I) = UA(I) + ETA(J)*BETAA(I,J)*EA(J)                      MAIN2690
      IF (J .GT. 4)  GO TO 1480                                    MAIN2700
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IJ    = IA(J)                          MAIN2675
EB(J)=CEXP(-ALFAB(IJ)*WXB/VS)          MAIN2680
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              LINB03    CONWAY       PHASE                N4                        12/31/69    000109      PAGE  6
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      UB(I) = UB(I) + ETA(J+6)*BETAA(I,J)*EB(J)                        MAIN2710
 1480 CCNTINUE                                                         MAIN2720
 1490 CCNTINUE                                                         MAIN2730
      PHIB = (0.,0.)                                                   MAIN2740
      CC 1520 J = 1, 4                                                 MAIN2750
 1520 PHIB = PHIB + ETA(J+6)*EB(J)                                     MAIN2760
      WRITE  (6, 1550)  (I, ETA(I), I = 1, 10),                        MAIN2770
     .                  (I, UA(I), UB(I), I = 1, 3), PHIB              MAIN2780     249
 1550 FORMAT (1HO/1H , 17X, 36H*** F I N A L   A N S W E R S ***/      MAIN2790
     .         1HO, 30X,13HPARTIAL FIELD/1H ,27X,19HRELATIVE AMPLITUDES/ MAIN2800
     .                    /10(1H , 17X, I2, 3H  (, E14.7, 1H,,         MAIN2810
     .         E14.7, 1H)/)/1H , 21X, 2HUA, 32X, 2HUB//                MAIN2820
     .         3(1H , I6, 3H  (, E14.7, 1H,, E14.7, 4H)  (, E14.7, 1H,, MAIN2830
     .         E14.7, 1H)/)/1H , 24X, 9HPHI B = (, E14.7, 1H,,         MAIN2840
     .         E14.7, 1H))                                             MAIN2850
 1708 WRITE  (6, 1705)                                                 MAIN2860     263
 1705 FCRMAT (1H1)                                                     MAIN2870
      IF(GETOUT) GO TO 17
      IF(NEGAT) MUB=-MUB
      IF(NEGAT) NUMAX=-NUMAX
      IF(NEGAT) NUB=-NUB
      IF( VSO .GE. VSMAX ) GO TO 1706                                  MAIN2880
      VSO = VSO + DVS                                                  MAIN2890
      GC TO 550                                                        MAIN2900
 1707 WRITE(6,1705)                                                    MAIN2905     277
 1706 IF (DNU .EQ. 0.)  GO TO 1710                                     MAIN2910
      IF(MUIT) GO TO 38
      NUB = NUB + DNU                                                  MAIN2920
      IF( NUB .GT. NUMAX ) GO TO 1709                                  MAIN2930
      GC TO 39
   38 MUB=MUB+DNU
      IF(NUB.GT.NUMAX) GO TO 1709
   39 CCNTINUE
      CALL OVERCK                                                                   293
      IF(NEGAT) MUB=-MUB
      IF(NEGAT) NUMAX=-NUMAX
      IF(NEGAT) NUB=-NUB
      IF( MAX .NE. 0 ) GO TO 530                                       MAIN2940
      VS = VSAVE                                                       MAIN2950
      GO TO 535                                                        MAIN2960
                                                                       MAIN2970
 1709 NUB = SNU                                                        MAIN2990
 1710 CCNTINUE

C.....INCLUDE PLOT ROUTINES HERE.....
      IF(PLOTIT(1).EQ.2) GC TC 34
      IF(PLOTIT(1).EQ.0) GO TO 34
      CALL SCALE(VEL,10.,KOUNT,1,10.,YMIN,DY)                                       313
      IF(.NOT. MUIT)
     1CALL AXIS(0.,0.,24HDIRECTION OF PROPAGATION,24,10.0,0.0,0.,18.0,
     1  10.0)                                                                       316
      IF(MUIT) CALL AXIS(0.,0.,25HDIRECTION OF PLATE NORMAL,25,10.,
     1 0.0,18.0,10.0)                                                               319
      CALL AXIS(C.0,0.0,21HSURFACE WAVE VELOCITY,21,10.,90.0,YMIN,DY,
     1  10.0)                                                                       321
      CALL AXIS(0.,10.,1H ,1,10.,0.,0.,18.,10.)                                     323
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             LINB03   CONWAY      PHASE              N4                          12/31/69    000109      PAGE  7
      LINB03     -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      CALL AXIS(10.,0.,1H ,1,10.,90.,YMIN,DY,10.)                                 325
      CALL LINE(DEG,VELOC,181,1,0,0,0.,18.,YMIN,DY)                               327
      IF(KCUNT.EQ.181) GO TO 36
      CALL LINE(DEG,VELOCI,181,1,0,0,0.,18.,YMIN,DY)                              332
   36 CALL PLOT(17.,0.,-3)                                                        334
      KCUNT=0
   34 CCNTINUE
      IF(PLOTIT(6).NE.3) GO TC 87
      CC 86 JK=1,181
   86 DELTA(JK)=(VEL(JK+181)-VEL(JK))/VEL(JK+181)
      CALL AXIS(0.,0.,24HDIRECTION OF PRORAGATION,24,10.,0.,0.,18.,10.)           350
      CALL SCALE(DELTA,10.,181,1,10.,DMIN,DY)                                     352
      CALL AXIS(0.,0.,11HDELTA V / V,11,10.,9C.,DMIN,DY,10.)                      354
      CALL AXIS(0.,10.,1H ,1,10.,0.,0.,18.,10.)                                   356
      CALL AXIS(10.,0.,1H ,1,10.,90.,DMIN,DY,10.)                                 358
      CALL LINE(DEG,DELTA,181,1,0,0,0.,18.,DMIN,DY)                               360
      CALL PLOT(17.0,0.,-3)                                                       362
   87 CCNTINUE
      IF(PLOTIT(2).EQ.0) GO TO 37
      CALL AXIS(0.,0.,24HDIRECTION OF PROPAGATION,24,10.0,0.0,0.,18.0,
     1  10.0)                                                                     367
      CALL AXIS.0.,0.,33HTIME AVERAGE POWER FLOW DIRECTION,33,10.,90.,
     1    -25.,5 .,10.)                                                           369
      CALL AXIS(C.,10.,1H ,1,10.,0.,0.,18.,10.)                                   371
      CALL AXIS(10.,0.,1H ,1,10.,90.,-25.,5.,10.)                                 373
      CALL LINE(XX,YY,2,1,0,0,0.,1.,0.,1.)                                        375
      CALL LINE(DEG,PANGLE,181,1,0,0,0.,18.,-25.,5.)                              377
      CALL PLOT(17.,0.,-3)                                                        379
      KCNT=0
   37 CCNTINUE
      IF(PLOTIT(3).EQ.0) GC TO 75
      CALL SCALE(RT11 ,10.,KNT1,1,10.,RTMIN,DR)                                   385
      CALL AXIS(0.,0.,24HDIRECTION OF PROPAGATION,24,10.0,0.0,0.,18.0,
     1  10.0)                                                                     387
      CALL AXIS(0.,0.,8HMAGN T11,8,10.,90.,RTMIN,DR,10.)                          389
      CALL LINE(DEG,RT11 ,181,1,0,0,0.,18.,RTMIN,DR)                              391
      CALL PLOT(17.,0.,-3)                                                        393
      KNT1=0
   75 CCNTINUE
      IF(PLOTIT(4).EQ.0) GO TO 76
      CALL SCALE(RU1 ,10.,KNT2,1,10.,RTMIN,DR)                                    399
      CALL AXIS(0.,0.,24HDIRECTION OF PROPAGATION,24,10.0,0.0,0.,18.0,
     1  10.0)                                                                     401
      CALL AXIS(C.,0.,6HMAG U3,6,10.,90.,RTMIN,DR,10.)                            403
      CALL LINE(DEG,RU1 ,181,1,0,0,0.,18.,RTMIN,DR)                               405
      CALL PLOT(17.,0.,-3)                                                        407
      CALL SCALE(RIU1,10.,KNT2,1,10.,RTMIN,DR)                                    409
      CALL AXIS(0.,0.,24HDIRECTION OF PROPAGATION,24,10.0,0.0,0.,18.0,
     1  10.0)                                                                     411
      CALL AXIS(C.,0.,6HPHASES,6,10.,90.,RTMIN,DR,10.)                            413
      CALL LINE(DEG,RIU1,181,1,0,0,0.,18.,RTMIN,DR)                               415
      CALL PLOT(17.,0.,-3)                                                        417
      KNT2=0
   76 CCNTINUE
      IF(PLOTIT(5).EQ.0) GO TO 74
      CALL SCALE(RE1 ,10.,KNT3,1,10.,RTMIN,DR)                                    423
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              LINBO3   CONWAY      PHASE             N4                      12/31/69    000109     PAGE  8
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      CALL AXIS(0.,0.,24HDIRECTION OF PROPAGATION,24,10.0,0.0,0.,10.0,
     1  10.0)                                                                 425
      CALL AXIS(0.,0.,7HMAGN D1,8,10.,90.,RTMIN,DR,1C.)                       427
      CALL LINE(DEG,RE1 ,181,1,0,0,0.,10.,RTMIN,DR)                           429
      CALL PLOT(17.,0.,-3)                                                    431
      KNT3=0
   74 CCNTINUE

      END FILE 2                                                              433
   17 CCNTINUE
      IF(CETOUT) ENC FILE 2                                                   435
      IF(EETOUT) KOUNT=0
      IF(GETOUT) KONT=0
      IF(GETOUT) KNT1=0
      IF(GETOUT) KNT2=0
      IF(GETOUT) KNT3=0
      GETCUT =.FALSE.
      IF (REPEAT)  GO TO 510                                     MAIN3000
      GO TO 520                                                  MAIN3010
                                                                 MAIN3020
C.....EPRCR - L MATRIX SINGULAR.....                              MAIN3030
 1740 WRITE  (6, 1750)  (EL(I), I = 1, KGO)                      MAIN3040     451
 1750 FCRMAT (42H1*** L MATRIX SINGULAR (OUTPUT BY COLUMNS)//    MAIN3050
     .        (1H , 6E18.7))                                     MAIN3060
      GC TO 1708                                                 MAIN3070
                                                                 MAIN3080
C.....CALCULATE ADDITIONAL PARAMETERS FOR LITHIUM NIOBATE.....    MAIN3090
 1790 WRITE(6,1705)                                              MAIN3100     459
 1800 DC 1810 I = 1, 4                                           MAIN3110
      J = IA(I)                                                  MAIN3120
 1810 EX(I) = CEXP(-ALFAB(J)*WX/VS)                              MAIN3130     467
      CG 1890 I = 1, 4                                           MAIN3140
      U = (0.,0.)                                                MAIN3150
      CC 1850 K = 1, 4                                           MAIN3160
 1850 U = U + ETA(K)*BETAB(I,K)*EX(K)                            MAIN3170
      MAGU(I) = CABS(U)                                          MAIN3180     481
      PHASEU(I) = 0.                                             MAIN3190
      IF (MAGU(I) .NE. 0.) PHASEU(I) = ATAN2(AIMAG(U),REAL(U))*57.295779MAIN3200     486
 1890 CCNTINUE                                                   MAIN3210
      E1 = U                                                     MAIN3220
                                                                 MAIN3230
C.....CCMPUTE TIME AVERAGE POWER FLOW.....                        MAIN3240
      P1M = P1FUN(ETA, C11, C15, C16, C14, C15, C13, E11, E31,   MAIN3250
     .                 C16, C56, C66, C46, C56, C36, E16, E36,   MAIN3260
     .                 C15, C55, C56, C45, C55, C35, E15, E35)   MAIN3270     492
      SPECL=SQRT(SQRT(REAL(P1M)**2+AIMAG(P1M)**2))                            493   494
      P2M = P1FUN (ETA, C16, C56, C66, C46, C56, C36, E16, E36,  MAIN3280
     .                  C12, C25, C26, C24, C25, C23, E12, E32,  MAIN3290
     .                  C14, C45, C46, C44, C45, C34, E14, E34)  MAIN3300     495
      IF(PLOTIT(2).EQ.0) GC TO 57
      KCNT=KCNT+1
      FANGLE(KONT)=ATAN(REAL(P2M)/REAL(P1M))*180./3.1415                      500
   57 CCNTINUE
                                                                 MAIN3310
C.....CALCULATE T S.....                                          MAIN3320
      TW31 = TFUN (ETA, WX, C15, C55, C56, C45, C55, C35, E15, E35)   MAIN3330     503
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          LINB03   CONWAY      PHASE              N4                   12/31/69    000109      PAGE  9
     LINB03     -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      TW32 = TFUN (ETA, WX, C14, C45, C46, C44, C45, C34, E14, E34)     MAIN3340    504
      TW33 = TFUN (ETA, WX, C13, C35, C36, C34, C35, C33, E13, E33)     MAIN3350    505
      TW11 = TFUN (ETA, WX, C11, C15, C16, C14, C15, C13, E11, E31)     MAIN3360    506
      TW12 = TFUN (ETA, WX, C16, C56, C66, C46, C56, C36, E16, E36)     MAIN3370    507
      TW22 = TFUN (ETA, WX, C12, C25, C26, C24, C25, C23, E12, E32)     MAIN3380    508
      IF(PLOTIT(3).EQ.0) GO TO 71
      KNT1=KNT1+1
      RT11(KNT1)=SQRT(REAL(TW11)**2+AIMAG(TW11)**2)/SPECL                           513
      RIT11(KNT1)=AIMAG(TW11)/SPECL
   71 CCNTINUE
      S11 = (0.,0.)                                                     MAIN3390
      S22 = (0.,C.)                                                     MAIN3400
      S33 = (0.,0.)                                                     MAIN3410
      S12 = (0.,0.)                                                     MAIN3420
      S13 = (0.,0.)                                                     MAIN3430
      S23 = (0.,0.)                                                     MAIN3440
      CO 2190 I = 1, 4                                                  MAIN3450
      J = IA(I)                                                         MAIN3460
      S11 = S11 - ETA(I)*BETAB(1,I)*EX(I)                               MAIN3470
      .33 = S33 - ALFAB(J)*ETA(I)*BETAB(3,I)*EX(I)                      MAIN3480
      S12 = S12 - ETA(I)*BET/B(2,I)*EX(I)                               MAIN3490
      S13 = S13 - ETA(I)*(BET.B(1,I)*ALFAB(J) + BETAB(3,I)*JIMAG)*EX(I) MAIN3500
 2190 S23 = S23 - ETA(I)*BETAB(2,I)*ALFAB(J)*EX(I)                      MAIN3510
      S11 = S11*JIMAG/VS                                                MAIN3520
      S33 = S33/VS                                                      MAIN3530
      S12 = S12*C.5*JIMAG/VS                                            MAIN3540
      S13 = 0.5*S13/VS                                                  MAIN3550
      S23 = 0.5*S23/VS                                                  MAIN3560
      C1 = TFUN(ETA, WX, E11, E15, E16, E14, E15, E13,-T11,-T13)        MAIN3570    544
      D2 = TFUN(ETA, WX, E21, E25, E26, E24, E25, E23,-T21,-T23)        MAIN3580    545
      C3 = TFUN(ETA, WX, E31, E35, E36, E34, E35, E33,-T13,-T33)        MAIN3590    546
      E1 = JIMAG*E1/VS                                                  MAIN3600
      E3 = (0.,0.)                                                      MAIN3610
      DC 2310 I = 1, 4                                                  MAIN3620
      J = IA(I)                                                         MAIN3630
 2310 E3 = E3 + ALFAB(J)*ETA(I)*EX(I)*BETAB(4,I)                        MAIN3640
      E3 = E3/VS                                                        MAIN3650
                                                                        MAIN3660
      IF(PLOTIT(5).EQ.0) GO TO 73
      KNT3=KNT3+1
      RE1(KNT3)=SCRT(REAL(D1)**2+AIMAG(D1)**2)/SPECL                                562
   73 CCNTINUE
C.....CUTPUT RESULTS.....                                               MAIN3670
      WRITE  (6, 2340)  WX                                              MAIN3680    564
      WRITE(2,2340) WX                                                              565
 2340 FCRMAT (10H0.....WX =, 1PE14.7)                                   MAIN3690
      WRITE  (6, 2440)  TT31, TW31, TT32, TW32, TT33, TW33,             MAIN3700
     .                  TT11, TW11, TT12, TW12, TT22, TW22,             MAIN3710
     .                  SS11, S11, SS22, S22, SS33, S33,                MAIN3720
     .                  SS12, S12, SS13, S13, SS23, S23,                MAIN3730
     .                  PP1M, P1M, PP2M, P2M,                           MAIN3740
     .                  DD1,  D1,   DD2,  D2,   DD3, D3,                MAIN3750
     .                  UU1, MAGU(1), PHASEU(1),                        MAIN3760
     .                  UU2, MAGU(2), PHASEU(2),                        MAIN3770
     .                  UU3, MAGU(3), PHASEU(3),                        MAIN3780
     .                  MAGU(4), PHASEU(4),                             MAIN3790
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          LINBO3    CONWAY       PHASE              N4                              12/31/69    000109     PAGE  10
   LINBO3     -   EFN   SOURCE STATEMENT  -  IFN(S)  -

      .                EE1, E1, EE3, E3                                   MAIN3800    566
      WRITE  (2, 2440)  TT31, TW31, TT32, TW32, TT33, TW33,               MAIN3700
      .                TT11, TW11, TT12, TW12, TT22, TW22,                MAIN3710
      .                SS11, S11, SS22, S22, SS33, S33,                   MAIN3720
      .                SS12, S12, SS13, S13, SS23, S23,                   MAIN3730
      .                PP1M, P1M, PP2M, P2M,                              MAIN3740
      .                DD1,  D1,   DD2,  D2,   DD3, D3,                   MAIN3750
      .                UU1, MAGU(1), PHASEU(1),                           MAIN3760
      .                UU2, MAGU(2), PHASEU(2),                           MAIN3770
      .                UU3, MAGU(3), PHASEU(3),                           MAIN3780
      .                MAGU(4), PHASEU(4),                                MAIN3790
      .                EE1, E1, EE3, E3                                   MAIN3800    567
 2440 FORMAT (1H0, 28X, 17HSTRESS COMPONENTS//6(1H , 18X, A3, 4H = (,     MAIN3810
      .        1PE14.7, 1H,, 1PE14.7, 1H)/)/1H , 28X,                     MAIN3820
      .        17HSTRAIN COMPONENTS//6(1H , 18X, A3, 4H = (,              MAIN3830
      .        1PE14.7, 1H,, 1PE14.7, 1H)/)/1H , 25X,                     MAIN3840
      .        23HTIME AVERAGE POWER FLOW//2(1H , 19X, A3, 4H = (,        MAIN3850
      .        1PE14.7, 1H,, 1PE14.7, 1H)/)/1H , 26X,                     MAIN3860
      .        21HELECTRIC DISPLACEMENT//3(1H , 19X, A2, 4H = (,          MAIN3870
      .        1PE14.7, 1H,, 1PE14.7, 1H)/)/1H , 25X,                     MAIN3880
      .        23HMECHANICAL DISPLACEMENT/1H , 24X, 9HMAGNITUDE,          MAIN3890
      .        7X, 5HPHASE/ 3(1H , 19X, A2, 3H = , 1PE14.7, 0PF10.3/)/    MAIN3900
      .        1H , 6X, 30HELECTRIC POTENTIAL MAGNITUDE =, 1PE15.7,       MAIN3910
      .        9H  PHASE =, 0PF9.3/1H0, 28X, 14HELECTRIC FIELD//          MAIN3920
      .        2(1H , 19X, A2, 4H = (, 1PE14.7, 1H,, 1PE14.7, 1H)/))      MAIN3930
 1502 FCRMAT(2E15.8)
      PUNCH 1502, P1M,P2M                                                             568
      PUNCH 1502, TW31,TW32,TW33,TW11,TW12,TW22                                       569
      PUNCH 1502,S11,S22,S33,S12,S13,S23                                              570
      PUNCH 1506, (MAGU(I),PHASEU(I),I=1,4)                                           571
 1506 FCRMAT(E15.8,F10.3)
      PUNCH 1502,E1,E3                                                                579
      PUNCH 1502, D1,D2,D3                                                            580
      DO 18 I=1,4
      RRRR=COS(PHASEU(I)*3.1415/180.)*MAGU(I)                                         585
      RIII=SIN(PHASEU(I)*3.1415/180.)*MAGU(I)                                         588
      DISP(I)=CMPLX(RRRR,RIII)
   18 CCNTINUE
      IF(PLOTIT(4).EQ.0) GO TO 72
      KNT2=KNT2+1
      RU1(KNT2)=SQRT(REAL(DISP(3))**2+AIMAG(DISP(3))**2)/SPECL                        598
      RIU1(KNT2)=PHASEU(1)-PHASEU(3)
   72 CCNTINUE
      WRITE(6,2441) (DISP(I),I=1,4)                                                   601
      WRITE(2,2441) (DISP(I),I=1,4)                                                   608
 2441 FORMAT(25X,23HMECHANICAL DISPLACEMENT,3(/23X,2E18.8),/,25X,
     1 20HELEC. POT. MAGNITUDE,/,23X,2E18.8)
      IF (DWX .EQ. 0.)  GO TO 1708                                        MAIN3940
      WX = WX + DWX                                                       MAIN3950
      IF (WX .LE. WXMAX)  GO TO 1790                                      MAIN3960
      WX = SWX                                                            MAIN3970
      GO TO 1708                                                          MAIN3980
    2 CCNTINUE

      READ(5,3) BLIMIT,ELIMIT,INCR                                                    622
    3 FCRMAT(3I5)
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            LINBO3   CONWAY      PHASE              N4                    12/31/69    000109     PAGE  11
      LINBO3    -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      CC 4 I=BLIMIT,ELIMIT,INCR
      K=(I-BLIMIT)/INCR+1
      VSARAY(K)=I
      VS=I
      VSO=VS
      CALL RUOT(VSO,VS,N,FVS,EPSLON,MAX)                                  631
      AAAAA(2*K-1)=AIMAG(FVS)
      AAAAA(2*K)=REAL(FVS)
      CETRAY(K)=REAL(FVS)
      CETIAY(K)=AIMAG(FVS)
      CEBUG K,DETRAY(K),DETIAY(K)                                         637
    4 CCNTINUE
      CC 5 JJ=1,K
      AAAAA(2*JJ-1)=AAAAA(2*JJ-1)*1.E10
      AAAAA(2*JJ)=AAAAA(2*JJ)*1.E10
      CETRAY(JJ)=DETRAY(JJ)*1.E10
      CETIAY(JJ)=DETIAY(JJ)*1.E10
      JJJ=2*JJ-1
      IF(AAAAA(JJJ).GT.1.) AAAAA(JJJ)=1.
      IF(AAAAA(JJJ).LT.-1.) AAAAA(JJJ)=-1.
      IF(AAAAA(2*JJ).GT.1.) AAAAA(2*JJ)=1.
      IF(AAAAA(2*JJ).LT.-1.) AAAAA(2*JJ)=-1.
      IF(DETRAY(JJ).GT.1.) DETRAY(JJ)=1.
      IF(DETRAY(JJ).LT.-1.) DETRAY(JJ)=-1.
      IF(DETIAY(JJ).GT.1.) DETIAY(JJ)=1.
      IF(DETIAY(JJ).LT.-1.) DETIAY(JJ)=-1.
    5 CCNTINUE
      YMIN=-1.
      CY=.2
      NNNNN=2*K
      CALL SCALE(VSARAY,15.0,K,1,10.,XMIN,DX)                             691
      CALL AXIS(0.,0.,2HVS,2,15.0,0.0,XMIN,DX,10.0)                       693
      CALL AXIS(0.,0.,6HDETERM,6,10.,90.0,YMIN,DY,10.)                    695
      CALL LINE(VSARAY,DETRAY,K,1,1,4,XMIN,DX,YMIN,DY)                    697
      CALL LINE(VSARAY,DETIAY,K,1,1,4,XMIN,DX,YMIN,DY)                    699
      CALL PLOT(17.,0.,-3)                                                701
      ICHECK=.FALSE.
      GC TO 17
      CALL ENDPLT                                                         704

ERROR MESSAGE NUMBER    1
      END                                                     MAIN3990

ERROR MESSAGE NUMBER    2
```

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$IBFTC ROCT..  DECK,DEBUG

      SUBROUTINE RCOT (VSO, VS, N, FVS, EPS, MAX)                       ROOT0030
CROOT                                                                   ROOT0020
                                                                        ROOT0040
      DIMENSION SUB(24)
      DATA SUB/1.,-2.,3.,-4.,5.,-6.,7.,-8.,9.,-10.,11.,1.,1.,1.,1.,1.,
     U 2.,2.,3.,5.,10.,50.,10C.,100./
      DATA KKKKK/0/
      DATA KICK/1/
      COMMON /GET/GETOUT
      COMMON/PLOTS/ICHECK
      LOGICAL GETCUT
      LOGICAL PREV
      DATA PREV/.FALSE./
      LOGICAL ICHECK
      COMMON     /FROOT/ FVSMAG, NT, ICASE                              ROOT0050
     .           /ALESS/ IALF                                           ROOT0060
                                                                        ROOT0070
      COMPLEX    F, FVS, FXO, FX1, FX2, FX3, G2, LAMDA3, ROUND,         ROOT0080
     .           T, T1, T2, EL                                          ROOT0090
      REAL       LAMDA2                                                 ROOT0100
      LOGICAL      IALF                                                 ROOT0110
                                                                        ROOT0120
      DATA         TEN10/1.E9/                                          ROOT0130
                                                                        ROOT0140
      KKKKK=KKKKK+1
      FACT1=1.01
      FACT2=1.005
      IF(KKKKK/2*2.NE.KKKKK) FACT1=.99
      IF(KKKKK/2*2.NE.KKKKK) FACT2=.995
      N = 0                                                             ROOT0150
      VS = VSO                                                          ROOT0160
      FVS    = ROUND(F(VS))                                             ROOT0170    8     9
      IF( IALF ) GO TO 600                                              ROOT0180
      IF(ICHECK) GO TO 530
      IF( MAX .EQ. 0 ) GO TO 530                                        ROOT0190
      IF( FVSMAG .LT. EPS ) GO TO 530                                   ROOT0200
      FXO = FVS                                                         ROOT0210
      VS=FACT1*VSC
      FVS = ROUND( F( VS ) )                                            ROOT0230   22    23
      IF( IALF ) GO TO 600                                              ROOT0240
      IF( FVSMAG .LT. EPS ) GO TO 530                                   ROOT0250
      FX1 = FVS                                                         ROOT0260
      X2=FACT2*VSC
      VS = X2                                                           ROOT0280
      FVS = ROUND( F( VS ) )                                            ROOT0290   30    31
      IF( IALF ) GO TO 600                                              ROOT0300
      IF( FVSMAG .LT. EPS ) GO TO 530                                   ROOT0310
      FX2 = FVS                                                         ROOT0320
      H2=(-1.)**(KKKKK-1)*.005*VSO                                                 38
      LAMDA2 = -0.5                                                     ROOT0340
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              LINBO3   CONWAY       PHASE              N4                     12/31/69     000109       PAGE  13
         ROOT..      -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      CELTA2 =  0.5                                                    ROOT0350
                                                                       ROOT0360
C.....B E G I N     I T E R A T I O N.....                             ROOT0370
  230 G2 = FXO*LAMCA2*LAMDA2 - FX1*DELTA2*DELTA2 + FX2*(LAMDA2 + DELTA2)ROOT0380
      T  = CSORT(G2*G2 - 4.*FX2*DELTA2*LAMDA2*(FXO*LAMDA2              ROOT0390
     .   - FX1*DELTA2 + FX2))                                          ROOT0400    40
      T1 = G2 + T                                                      ROOT0410
      T2 = G2 - T                                                      ROOT0420
      T = T1                                                           ROOT0430
      IF (CABS(T2) .GT. CABS(T1))  T = T2                              ROOT0440    41    42
      IF (CABS(T) .EQ. 0.)  GO TO 530                                  ROOT0450
                                                                       ROOT0460    45
  300 LAMCA3 = -2.*FX2*DELTA2/T                                        ROOT0470
      VS = X2 + REAL(LAMCA3*H2)                                        ROOT0480
      FVS = ROUND(F(VS))                                               ROOT0490    50    51
      IF( IALF ) GO TO 600                                             ROOT0500
      IF( FVSMAG .LT. EPS ) GC TO 530                                  ROOT0510
      N  = N + 1                                                       ROOT0520
      IF(N.GT.MAX) GO TO 6C0
      H3 = VS - X2                                                     ROOT0540
      LAMCA2 = H3/H2                                                   ROOT0550
      CELTA2 = 1. + LAMDA2                                             ROOT0560
      FXO     = FX1                                                    ROOT0570
      FX1     = FX2                                                    ROOT0580
      FX2     = FVS                                                    ROOT0590
      X2      = VS                                                     ROOT0600
      H2      = H3                                                     ROOT0610
      IF (H2 .NE. 0.)  GO TO 230                                       ROOT0620
                                                                       ROOT0630
C.....E N D    O F     I T E R A T I O N.....                          ROOT0640
  530 CCNTINUE
      IF(PREV) PREV=.FALSE.
      KKKKK=0
      KICK=1
      RETURN
  600 WRITE(6,1000) VS                                                 ROOT0660    70
 1000 FCRMAT( /// 51H .....LESS THAN 4 ALPHAS WITH A POSITIVE REAL PART ROOT0670
     *         /  32H .....CASE TERMINATED  ( VS = , 1PE14.7, 2H ) //    )ROOT0680
      IF(ICHECK) GO TO 530
      IF(KKKKK/2*2.NE.KKKKK) GO TO 998
      VSO=VSO-SUB(KICK)
      KICK=KICK+1
      IF(KICK.EG.25) GO TO 999
  998 RETURN 1
  999 CCNTINUE
      IF(PREV) GETOLT=.TRUE.
      PREV=.TRUE.
      KICK=1
      RETURN
      GC TC 530                                                        ROOT0690

ERROR MESSAGE NUMBER     1
      END                                                              ROOT0700
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$IBFTC SETCT.  DECK

CSETCTE     *** SETCTE COMPUTES THE C, T, E COEFFICIENTS FROM          CTE00020
C*              THE INPUT PARAMENTERS G, P, EPS, MU, LAMBDA, NU        CTE00030
      SUBROUTINE SETCTE (MU, NU, LAMBDA, COEFF)                         CTE00040
                                                                        CTE00050
      COMMON /ROTAT/ROTATE
      LOGICAL ROTATE
      DIMENSION CPR(6,6),EPR(3,6),EPSPR(3,3)
      LOGICAL    COEFF                                                  CTE00060
      LOGICAL    AC12, AC23, AC24, AC14, AC34                           CTE00070
      REAL     MU, NU, LAMBDA                                           CTE00080
      DOUBLE PRECISION GAMMA(3,3), D, Q, DR, RM, RN, RL, CM, SM, CN, SN,CTE00090
     *                 CL, SL, FF, R, TIJ                               CTE00100
                                                                        CTE00110
      DIMENSION    LABE(6), LABC(5), LABT(6)                            CTE00120
                                                                        CTE00130
      COMMON /ZZTZ/ C(20),E(17),T(5)
      COMMON   /FRT/  GAMMA, D(3,3,3,3), Q(3,3,3)                       CTE00150
      COMMON      /GPEPS/  G(21), P(18), EPSLON(3,3)                    CTE00160
     *             /CSET/ CLIM, ELIM, TLIM                              CTE00170
     *            /CSET1/ AC12, AC23, AC24, AC14, AC34                  CTE00180
     *            /BETAN/ NBETA                                         CTE00190
                                                                        CTE00200
      DATA     DR / 57.29577951308232 /                                 CTE00210
      DATA          LABE(1)/36HTRANSFORMED PIEZOELECTRIC CONSTANTS /,   CTE00220
     *              LABC(1)/30HTRANSFORMED ELASTIC CONSTANTS /,         CTE00230
     *              LABT(1)/36HTRANSFORMED DIELECTRIC CONSTANTS    /    CTE00240
                                                                        CTE00250
                                                                        CTE00260
      AC12 = .FALSE.                                                    CTE00270
      AC23 = .FALSE.                                                    CTE00280
      AC24 = .FALSE.                                                    CTE00290
      AC14 = .FALSE.                                                    CTE00300
      AC34 = .FALSE.                                                    CTE00310
      RM = MU/DR                                                        CTE00320
      RN = NU/DR                                                        CTE00330
      RL = LAMBDA/DR                                                    CTE00340
      CALL CSFUN (MU, RM, SM, CM)                                       CTE00350    2
      CALL CSFUN (NU, RN, SN, CN)                                       CTE00360    4
      CALL CSFUN (LAMBDA, RL, SL, CL)                                   CTE00370    6
      GAMMA(1,1) = CL*CN - SL*CM*SN                                     CTE00380
      GAMMA(1,2) = SL*CN + CL*CM*SN                                     CTE00390
      GAMMA(1,3) = SM*SN                                                CTE00400
      GAMMA(2,1) =-CL*SN - SL*CM*CN                                     CTE00410
      GAMMA(2,2) =-SL*SN + CL*CM*CN                                     CTE00420
      GAMMA(2,3) = SM*CN                                                CTE00430
      GAMMA(3,1) = SL*SM                                                CTE00440
      GAMMA(3,2) =-CL*SM                                                CTE00450
      GAMMA(3,3) = CM                                                   CTE00460
                                                                        CTE00470
C.....SET UP D AND Q MATRICES.....                                      CTE00480
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        LINBO3    CONWAY        PHASE              N4                           12/31/69     000109      PAGE  15
SETCT.         -  EFN   SOURCE STATEMENT  -  IFN(S)  -

C(1,1,1,1) = G(1)                                            CTE00490
C(2,2,2,2) = G(2)                                            CTE00500
C(3,3,3,3) = G(3)                                            CTE00510
C(1,1,2,2) = G(4)                                            CTE00520
C(2,2,1,1) = G(4)                                            CTE00530
C(1,1,3,3) = G(5)                                            CTE00540
C(3,3,1,1) = G(5)                                            CTE00550
C(1,1,2,3) = G(6)                                            CTE00560
C(1,1,3,2) = G(6)                                            CTE00570
C(2,3,1,1) = G(6)                                            CTE00580
C(3,2,1,1) = G(6)                                            CTE00590
C(1,1,1,3) = G(7)                                            CTE00600
C(1,1,3,1) = G(7)                                            CTE00610
C(1,3,1,1) = G(7)                                            CTE00620
C(3,1,1,1) = G(7)                                            CTE00630
C(1,1,1,2) = G(8)                                            CTE00640
C(1,1,2,1) = G(8)                                            CTE00650
C(1,2,1,1) = G(8)                                            CTE00660
C(2,1,1,1) = G(8)                                            CTE00670
C(2,2,3,3) = G(9)                                            CTE00680
C(3,3,2,2) = G(9)                                            CTE00690
C(2,2,2,3) = G(10)                                           CTE00700
C(2,2,3,2) = G(10)                                           CTE00710
C(2,3,2,2) = G(10)                                           CTE00720
C(3,2,2,2) = G(10)                                           CTE00730
C(2,2,1,3) = G(11)                                           CTE00740
C(2,2,3,1) = G(11)                                           CTE00750
C(1,3,2,2) = G(11)                                           CTE00760
C(3,1,2,2) = G(11)                                           CTE00770
C(2,2,1,2) = G(12)                                           CTE00780
C(2,2,2,1) = G(12)                                           CTE00790
C(1,2,2,2) = G(12)                                           CTE00800
C(2,1,2,2) = G(12)                                           CTE00810
C(3,3,2,3) = G(13)                                           CTE00820
C(3,3,3,2) = G(13)                                           CTE00830
C(2,3,3,3) = G(13)                                           CTE00840
C(3,2,3,3) = G(13)                                           CTE00850
C(3,3,1,3) = G(14)                                           CTE00860
C(3,3,3,1) = G(14)                                           CTE00870
C(1,3,3,3) = G(14)                                           CTE00880
C(3,1,3,3) = G(14)                                           CTE00890
C(3,3,1,2) = G(15)                                           CTE00900
C(3,3,2,1) = G(15)                                           CTE00910
C(1,2,3,3) = G(15)                                           CTE00920
C(2,1,3,3) = G(15)                                           CTE00930
C(2,3,2,3) = G(16)                                           CTE00940
C(2,3,3,2) = G(16)                                           CTE00950
C(3,2,2,3) = G(16)                                           CTE00960
C(3,2,3,2) = G(16)                                           CTE00970
C(2,3,1,3) = G(17)                                           CTE00980
C(2,3,3,1) = G(17)                                           CTE00990
C(3,2,1,3) = G(17)                                           CTE01000
C(3,2,3,1) = G(17)                                           CTE01010
C(1,3,2,3) = G(17)                                           CTE01020
C(3,1,2,3) = G(17)                                           CTE01030
C(1,3,3,2) = G(17)                                           CTE01040
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SETCT.      -  EFN   SOURCE STATEMENT  -  IFN(S)  -

C(3,1,3,2) = G(17)                                  CTE01050
D(2,3,1,2) = G(18)                                  CTE01060
C(2,3,2,1) = G(18)                                  CTE01070
D(3,2,1,2) = G(18)                                  CTE01080
D(3,2,2,1) = G(18)                                  CTE01090
C(1,2,2,3) = G(18)                                  CTE01100
C(2,1,2,3) = G(18)                                  CTE01110
C(1,2,3,2) = G(18)                                  CTE01120
C(2,1,3,2) = G(18)                                  CTE01130
D(1,3,1,3) = G(19)                                  CTE01140
C(1,3,3,1) = G(19)                                  CTE01150
D(3,1,1,3) = G(19)                                  CTE01160
C(3,1,3,1) = G(19)                                  CTE01170
D(1,3,1,2) = G(20)                                  CTE01180
C(1,3,2,1) = G(20)                                  CTE01190
C(3,1,1,2) = G(20)                                  CTE01200
C(3,1,2,1) = G(20)                                  CTE01210
C(1,2,1,3) = G(20)                                  CTE01220
C(2,1,1,3) = G(20)                                  CTE01230
C(1,2,3,1) = G(20)                                  CTE01240
D(2,1,3,1) = G(20)                                  CTE01250
C(1,2,1,2) = G(21)                                  CTE01260
C(1,2,2,1) = G(21)                                  CTE01270
C(2,1,1,2) = G(21)                                  CTE01280
C(2,1,2,1) = G(21)                                  CTE01290
C(1,1,1) = P(1)                                     CTE01300
C(1,2,2) = P(2)                                     CTE01310
C(1,3,3) = P(3)                                     CTE0132C
C(1,2,3) = P(4)                                     CTE01330
C(1,3,2) = P(4)                                     CTE01340
C(1,1,3) = P(5)                                     CTE01350
C(1,3,1) = P(5)                                     CTE01360
C(1,1,2) = P(6)                                     CTE01370
C(1,2,1) = F(6)                                     CTE01380
C(2,1,1) = P(7)                                     CTE01390
C(2,2,2) = P(8)                                     CTE01400
C(2,3,3) = P(9)                                     CTE01410
C(2,2,3) = P(10)                                    CTE01420
C(2,3,2) = P(10)                                    CTE01430
C(2,1,3) = P(11)                                    CTE01440
C(2,3,1) = P(11)                                    CTE01450
C(2,1,2) = P(12)                                    CTE01460
C(2,2,1) = P(12)                                    CTE01470
C(3,1,1) = P(13)                                    CTE01480
C(3,2,2) = P(14)                                    CTE01490
C(3,3,3) = P(15)                                    CTE01500
C(3,2,3) = P(16)                                    CTE01510
C(3,3,2) = P(16)                                    CTE01520
C(3,1,3) = P(17)                                    CTE01530
C(3,3,1) = P(17)                                    CTE01540
C(3,1,2) = P(18)                                    CTE01550
C(3,2,1) = P(18)                                    CTE01560
C( 1) =FF(1,1,1,1)                                  CTE01570    8
C( 2) =FF(1,1,3,3)                                  CTE01580    9
C( 3) =FF(1,1,2,3)                                  CTE01590    10
C( 4) =FF(1,1,1,3)                                  CTE01600    11
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        LIN003    CONWAY       PHASE             N4                        12/31/69    000109      PAGE  17
   SETCT.      -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      C( 5) =FF(3,3,3,3)                                          CTE01610    12
      C( 6) =FF(3,3,2,3)                                          CTE01620    13
      C( 7) =FF(3,3,1,3)                                          CTE01630    14
      C( 8) =FF(3,3,1,2)                                          CTE01640    15
      C( 9) =FF(2,3,2,3)                                          CTE01650    16
      C(10) =FF(2,3,1,3)                                          CTE01660    17
      C(11) =FF(2,3,1,2)                                          CTE01670    18
      C(12) =FF(1,3,1,3)                                          CTE01680    19
      C(13) =FF(1,3,1,2)                                          CTE01690    20
      C(14) =FF(1,2,1,2)                                          CTE01700    21
      C(15) =FF(1,1,1,2)                                          CTE01710    22
      C(16) =FF(1,1,2,2)                                          CTE01720    23
      C(17) =FF(2,2,1,3)                                          CTE01730    24
      C(18) =FF(2,2,1,2)                                          CTE01740    25
      C(19) =FF(2,2,2,3)                                          CTE01750    26
      C(20) =FF(2,2,3,3)                                          CTE01760    27
      IF(.NOT. RCTATE) GO TO 1601
      CPR(1,1)=FF(1,1,1,1)                                                    31
      CPR(1,2)=FF(1,1,2,2)                                                    32
      CPR(1,3)=FF(' 1,3,3)                                                    33
      CPR(1,4)=FF(1,1,2,3)                                                    34
      CPR(1,5)=FF(1,1,1,3)                                                    35
      CPR(1,6)=FF(1,1,1,2)                                                    36
      CPR(2,2)=FF(2,2,2,2)                                                    37
      CPR(2,3)=FF(2,2,3,3)                                                    38
      CPR(2,4)=FF(2,2,2,3)                                                    39
      CPR(2,5)=FF(2,2,1,3)                                                    40
      CPR(2,6)=FF(2,2,1,2)                                                    41
      CPR(3,3)=FF(3,3,3,3)                                                    42
      CPR(3,4)=FF(3,3,2,3)                                                    43
      CPR(3,5)=FF(3,3,1,3)                                                    44
      CPR(3,6)=FF(3,3,1,2)                                                    45
      CPR(4,4)=FF(2,3,2,3)                                                    46
      CPR(4,5)=FF(2,3,1,3)                                                    47
      CPR(4,6)=FF(2,3,1,2)                                                    48
      CPR(5,5)=FF(1,3,1,3)                                                    49
      CPR(5,6)=FF(1,3,1,2)                                                    50
      CPR(6,6)=FF(1,2,1,2)                                                    51
 1601 CCNTINUE
      CO 10 I=1,20                                                CTE01770
      IF( ABS( C(I) ) .LT. CLIM ) C(I) = 0.                       CTE01780
   10 CONTINUE                                                    CTE01790
      E( 1) = R(1,1,1)                                            CTE01800    63
      E( 2) = R(1,3,3)                                            CTE01810    64
      E( 3) = R(1,2,3)                                            CTE01820    65
      E( 4) = R(1,1,3)                                            CTE01830    66
      E( 5) = R(1,1,2)                                            CTE01840    67
      E( 6) = R(3,1,1)                                            CTE01850    68
      E( 7) = R(3,3,3)                                            CTE01860    69
      E( 8) = R(3,2,3)                                            CTE01870    70
      E( 9) = R(3,1,3)                                            CTE01880    71
      E(10) = R(3,1,2)                                            CTE01890    72
      E(11) = R(1,2,2)                                            CTE01900    73
      E(12) = R(3,2,2)                                            CTE01910    74
      E(13) = R(2,1,1)                                            CTE01920    75
      E(14) = R(2,3,3)                                            CTE01930    76
```

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              LINBO3   CONWAY      PHASE              N4                    12/31/69    000109      PAGE  18
         SETCT.      -  EFN   SOURCE STATEMENT  -  IFN(S)  -
              E(15) = R(2,2,3)                                      CTE01940    77
              E(16) = R(2,1,3)                                      CTE01950    78
              E(17) = R(2,1,2)                                      CTE01960    79
              IF(.NOT. RCTATE) GO TO 1602
              EPR(1,1)=R(1,1,1)                                                 83
              EPR(1,2)=R(1,2,2)                                                 84
              EPR(1,3)=R(1,3,3)                                                 85
               EPR(1,4)=R(1,2,3)                                                86
              EPR(1,5)=R(1,1,3)                                                 87
              EPR(1,6)=R(1,1,2)                                                 88
              EPR(2,1)=R(2,1,1)                                                 89
              EPR(2,2)=R(2,2,2)                                                 90
              EPR(2,3)=R(2,3,3)                                                 91
              EPR(2,4)=R(2,2,3)                                                 92
              EPR(2,5)=R(2,1,3)                                                 93
              EPR(2,6)=R(2,1,2)                                                 94
              EPR(3,1)=R(3,1,1)                                                 95
              EPR(3,2)=R(3,2,2)                                                 96
              EPR(3,3)=R(3,3,3)                                                 97
              EPR(3,4)=R(3,2,3)                                                 98
              EPR(3,5)=R(3,1,3)                                                 99
              EPR(3,6)=R(3,1,2)                                                 100
         1602 CCNTINUE
              CO 20 I=1,17                                          CTE01970
              IF( ABS( E(I) ) .LT. ELIM ) E(I) = 0.                 CTE01980
           20 CCNTINUE                                              CTE01990
              EPSLCN(3,2) = EPSLON(2,3)                             CTE02000
              EPSLCN(2,1) = EPSLON(1,2)                             CTE02010
              EPSLON(3,1) = EPSLCN(1,3)                             CTE02020
              T(1) = TIJ(1,1)                                       CTE02030    112
              T(2) = TIJ(1,3)                                       CTE02040    113
              T(3) = TIJ(3,3)                                       CTE02050    114
              T(4) = TIJ(2,1)                                       CTE02060    115
              T(5) = TIJ(2,3)                                       CTE02070    116
              IF(.NOT. ROTATE) GO TO 1603
              EPSPR(1,1)=TIJ(1,1)                                               120
              EPSPR(1,2)=TIJ(1,2)                                               121
              EPSPR(1,3)=TIJ(1,3)                                               122
              EPSPR(2,1)=TIJ(2,1)                                               123
              EPSPR(2,2)=TIJ(2,2)                                               124
              EPSPR(2,3)=TIJ(2,3)                                               125
              EPSPR(3,1)=TIJ(3,1)                                               126
              EPSPR(3,2)=TIJ(3,2)                                               127
              EPSPR(3,3)=TIJ(3,3)                                               128
         1603 CCNTINUE
              CC 30 I=1,5                                           CTE02080
              IF( ABS( T(I) ) .LT. TLIM ) T(I) = 0.                 CTE02090
           30 CCNTINUE                                              CTE02100
                                                                    CTE02110
        C.....SEE IF PIEZOELECTRIC COEFFICIENTS ARE ALL ZERO.....   CTE02120
              NBETA = 3                                             CTE02130
              CC 470 K = 1, 18                                      CTE02140
              IF (P(K) .NE. 0.)  GC TO 490                          CTE02150
          470 CCNTINUE                                              CTE02160
              NBETA = 2                                             CTE02170
          490 CCNTINUE                                              CTE02180
```

```
      IF( C(10).EQ.0. .AND. C(3).EQ.0. .AND. C(13).EQ.0. .AND.          CTE02190
     .    C(15).EQ.0. ) AC12 = .TRUE.                                   CTE02200
      IF( C(6).EQ.0. .AND. C(8).EQ.0. .AND. C(10).EQ.0. .AND.           CTE02210
     .    C(13).EQ.0. ) AC23 = .TRUE.                                   CTE02220
      IF( E(8).EC.0. .AND. E(3).EQ.0. .AND. E(10).EQ.0. .AND.           CTE02230
     .    E(5).EQ.0. ) AC24 = .TRUE.                                    CTE02240
      IF( E(9).EC.0. .AND. E(4).EQ.0. .AND. E(6).EQ.0. .AND.            CTE02250
     .    E(1).EC.0. ) AC14 = .TRUE.                                    CTE02260
      IF( E(7).EC.0. .AND. E(2).EQ.0. .AND. E(5).EQ.0. .AND.            CTE02270
     .    E(4).EQ.0. ) AC34 = .TRUE.                                    CTE02280
      IF( COEFF ) WRITE(6,1599) LABC, ( C(I), I=1,20 ),                 CTE02290
     .            LABE, ( E(I), I=1,17 ), LABT, ( T(I), I=1,5 )         CTE02300    162
      IF(.NOT. ROTATE) GO TO 1604
      WRITE(6,1600) CPR(1,1),CPR(2,2),CPR(3,3),CPR(1,2),
     1 ((CPR(I,J),J=3,6),I=1,2),
     U   ((CPR(I,J),J=4,6),I=3,4),CPR(5,5),CPR(5,6),CPR(6,6),
     1((EPR(I,J),J=1,6),I=1,3),((EPSPR(I,J),J=1,3),I=1,3)                           187
 1600 FCRMAT(5H C    ,21(/E18.7),/,5H  E   ,18(/E18.7),/,5H  EPS,9(/E18.7))
     1  )
 1604 CCNTINUE
 1599 FORMAT( 1H0,4X,5A6, 18H  ( C(I), I=1,20 ) // 20( 1H ,1PE18.7/ ) / CTE02310
     .        1H0,4X,6A6, 18H  ( E(I), I=1,17 ) // 17( 1H ,1PE18.7/ ) / CTE02320
     .        1H0,4X,6A6, 17H  ( T(I), I=1,5 ) // 5( 1H ,1PE18.7/ ) )   CTE02330
      RETURN                                                            CTE02340
      END                                                               CTE02350
```

```
$IBFTC RCUND.  CECK

CRCUNC
      CCMPLEX FUNCTION ROUND (F)                                        ROUND020
      REAL   I, F(2)                                                    ROUND030
      R = F(1)                                                          ROUND040
      I = F(2)                                                          ROUND050
      IF (R .EQ. 0. .OR. I .EQ. 0.)  GO TO 10C                          ROUND060
      IF (ABS(I/R) .LT. 1.E5) GO TO 50                                  ROUND070
      R = 0.                                                            ROUND080
      GC TO 100                                                         ROUND090
   50 IF (ABS(R/I) .GE. 1.E5)  I = 0.                                   ROUND100
  100 RCUND = CMPLX(R, I)                                               ROUND110
      RETURN                                                            ROUND120
      END                                                               ROUND130
                                                                        ROUND140
```

```
$IBFTC CSFUN.  DECK

CCSFUN                                                                  CSFUN020
      SUBROUTINE CSFUN (Y, RX, SX, CX)                                  CSFUN030
      DOUBLE PRECISION RX, SX, CX                                       CSFUN040
      X = Y                                                             CSFUN050
      IF (X .LT. 0.)  X = X + 360.                                      CSFUN060
      IF (X .EQ. 0. .OR. X .EQ. 180.)  GO TO 150                        CSFUN070
      IF (X .EQ. 90. .OR. X .EQ. 270.)  GO TO 200                       CSFUN080
      SX = DSIN(RX)                                                     CSFUN090   10
      CX = DCOS(RX)                                                     CSFUN100   11
  100 RETURN                                                            CSFUN110
  150 CX = 1.                                                           CSFUN120
      SX = 0.                                                           CSFUN130
      IF (X .EQ. 180.)  CX = -1.                                        CSFUN140
      GO TO 100                                                         CSFUN150
  200 SX = 1.                                                           CSFUN160
      CX = 0.                                                           CSFUN170
      IF (X .EQ. 270.)  SX = -1.                                        CSFUN180
      GO TO  100                                                        CSFUN190
      END                                                               CSFUN200
```

```
$IBFTC FF....  DECK

CFF                                                              FF000020
      CCUBLE PRECISICN FUNCTION FF (I, J, K, L)                  FF000030
      CCMMCN    /FRT/  GAMMA(3,3), D(3,3,3,3), Q(3,3,3)          FF000040
      DOUBLE PRECISION  GAMMA, D, Q                              FF000050
      INTEGER   R, S, T, U                                       FF000060
      FF= 0.                                                     FF000070
      CC 50 R = 1, 3                                             FF000080
      CC 50 S = 1, 3                                             FF000090
      CC 50 T = 1, 3                                             FF000100
      CC 50 U = 1, 3                                             FF000110
   50 FF=FF + GAMMA(I,R)*GAMMA(J,S)*GAMMA(K,T)*GAMMA(L,U)*D(R,S,T,U)  FF000120
      RETURN                                                     FF000130
      FAD                                                        FF000140
```

```
$IBFTC R.      DECK
CR
      DOUBLE PRECISION FUNCTION R (I, J, K)
      COMMON   /FRT/  GAMMA(3,3), D(3,3,3,3), Q(3,3,3)
      DOUBLE PRECISION GAMMA, D, Q
      INTEGER  S, T, U
      R = 0.
      DO 50 S = 1, 3
      DO 50 T = 1, 3
      DO 50 U = 1, 3
   50 R = R + GAMMA(I,S)*GAMMA(J,T)*GAMMA(K,U)*Q(S,T,U)
      RETURN
      END
```

```
$IBFTC TIJ.    DECK
CTIJ
      DOUBLE PRECISION FUNCTION TIJ (I, J)
      CCMMCN   /FRT/  GAMMA(3,3), JUNK(108)
      COMMON    /GPEPS/  G(21), P(18), EPSLCN(3,3)
      CCUBLE PRECISION GAMMA, JUNK
      INTEGER  R, S
      TIJ = 0.
      CC 50 R = 1, 3
      CC 50 S = 1, 3
   50 TIJ = TIJ + GAMMA(I,R)*GAMMA(J,S)*EPSLON(R,S)
      RETURN
      FND
```

```
$IBFTC F.....  DECK

CF     THIS FUNCTICN EVALUATES THE L DETERMINANT FOR A VALUE OF VS      F0000020
C*     FOR EITHER THE LITHIUM OR GOLD LITHIUM CASE                      F0000030
C*                                                                      F0000040
C************************************************************           F0000050
                                                                        F0000060
      CCMPLEX FUNCTION F (VS)                                           F0000070
                                                                        F0000080
      CCMMCN /ZZTZ/ CC,CE,CT
      CCMMCN     /FROOT/ FVSMAG, NT, ICASE                              F0000100
     .           /BETAN/ NBETA                                          F0000110
     .           /CSET1/ AC12, AC23, AC24, AC14, AC34                   F0000120
      CCMMCN     /LINK / ALFA(8), ALFAI(8), EL(100), ALFAA(6),          F0000130
     .                   ALFAB(4), BETAA(3,6), BETAB(4,4), EPSO,        F0000140
     .                   MUA, LAMDAA, RHOA, MUB, LAMCAB, NUB, RHOB,     F0000150
     .                   VSX,KS, EPSLON, DIGIT, WH, WXA, WXB, KL,       F0000160
     .                   KM, ALL, ROCTS, ITER, COEFF, DETERM, POLYN,    F0000170
     .                   ALPHA, BETA, MAX                               F0000180
     .           /CIA/ IA(4)                                            F0000190
     .           /CCM  / ACAP, EPSR                                     F0000200
     .           /GPEPS/ XG(21), XP(18), XEPS(9)                        F0000210
     .           /ALESS/ IALF                                           F0000220
     )       /CIFL/FXAGNL
                                                                        F0000230
      CCMPLEX    ALFA, FVS, EL, EA(6), EB(4), UA(3), UB(3), ALFAI,      F0000240
     .           ALFAA, ALFAB, BETAA, BETAB, POLY(9), JIMAG,            F0000250
     .           B1(4,4), CA, DB, DC, ELL(1G,10), ALF(8), AOLD(9)       F0000260
      CCMPLEX    ALF1, ALF2, XEL(100), XXEL(3,3)                        F0000270
      CCMPLEX      B11, B12, B13, B14, B22, B23, B24, B33, B34, B44, AK,F0000280
     .     BB(3,4),BB1(3,3),BETAB1(4),
     .     B1A(2,3),BETAX(2),CX0,CX1,B1B(4,4)
      REAL       MUA, LAMCAA, MUB, LAMCAB, NUB, CA(4,4), CB(4,4),       F0000300
     .           CD(4,4), CC(20), CE(17), CT(5), B1D(2,4,4)             F0000310
      LOGICAL    ALL, ROOTS, ITER, COEFF, DETERM, POLYN, ALPHA, BETA,   F0000320
     .   FXAGNL,
     .           ACAP                                                   F0000330
      LCGICAL    AC12, AC23, AC24, AC14, AC34, IALF                     F0000340
                                                                        F0000350
      CIMENSION    ALAB(3,2), IA1(3), IA2(3), LAB(3)                    F0000360
      CIMENSION  B1X(4), NB(4)                                          F0000370
      EQUIVALENCE  (EL, ELL), ( B1, B1D ),                              F0000380
     .             (CC(1), C11), (CC(2), C13), (CC(3), C14), (CC(4),C15),F0000390
     .             (CC(5), C33), (CC(6), C34), (CC(7), C35), (CC(8),C36),F0000400
     .             (CC(9), C44), (CC(10),C45), (CC(11),C46),            F0000410
     .             (CC(12),C55), (CC(13),C56), (CC(14),C66),            F0000420
     .             (CC(15),C16), (CE(1), E11), (CE(2), E13),            F0000430
     .             (CE(3), E14), (CE(4), E15), (CE(5), E16),            F0000440
     .             (CE(6), E31), (CE(7), E33), (CE(8), E34),            F0000450
     .             (CE(9), E35), (CE(10),E36), (CT(1), T11),            F0000460
     .             (CT(2), T13), (CT(3), T33)                           F0000470
                                                                        F0000480
```

```
F.....     -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      CATA         LAB(1)/18H (1,3) (2,4) CASE /
      CATA          JIMAG / (0.,1.) /, CXO,CX1/(0.,0.),(1.,0.)/
      CATA         ALAB(1,1)/54H1 ROW, 3 ZERO CASE4 ROW, 2 ZERO CASEZERO
     .-PIEZUELECTRIC/,  TEN,TEN10/1.E-10,1.E1C/

      B11( AK ) =  AK * ( C55*AK + CMPLX(0.,(C15+C15)) ) - C11 + RVS
      B12( AK ) =  AK * ( C45*AK + CMPLX(0.,(C14+C56)) ) - C16
      B13( AK ) =  AK * ( C35*AK + CMPLX(0.,(C13+C55)) ) - C15
      B14( AK ) = -AK * ( E35*AK + CMPLX(0.,(E15+E31)) ) + E11
      B22( AK ) =  AK * ( C44*AK + CMPLX(0.,(C46+C46)) ) - C66 + RVS
      B23( AK ) =  AK * ( C34*AK + CMPLX(0.,(C36+C45)) ) - C56
      B24( AK ) = -AK * ( E34*AK + CMPLX(0.,(E14+E36)) ) + E16
      B33( AK ) =  AK * ( C33*AK + CMPLX(0.,(C35+C35)) ) - C55 + RVS
      B34( AK ) = -AK * ( E33*AK + CMPLX(0.,(E13+E35)) ) + E15
      B44( AK ) = +AK * ( T33*AK + CMPLX(0.,(T13+T13)) ) - T11

      IALF = .FALSE.
      ICASE = 0
      ICB = 0
      K1 = 1
      K2 = 4
      L1 = 1
      L2 = 16
      IB1 = 1
      IB2 = 4

C.....CALCULATE COEFFICIENTS OF POLYNOMIAL.....
  490 RVS   = RHOB*VS*VS
      CALL STRIP  (POLY, VS, RHOB, ALL, POLYN)                               30
      CALL CROOT (POLY, AOLD, NT, ALFA)                                      32
      IF (NBETA .EQ. 3)  GO TC 520
      CC = T33 + T33
      DA = CMPLX(0., -T13 - T13)
      CB = CSORT(CMPLX(-4.*T13*T13 + 4.*T11*T33, 0.))                        37
      ALF1 = (DA + CB)/DC
      ALF2 = (DA - CB)/DC
      CC 510 I = 1, 8
      IF (CABS(ALFA(I) - ALF1) .LE. 1.E-5)  ALFA(I) = (-10.,-10.)            42
      IF (CABS(ALFA(I) - ALF2) .LE. 1.E-5)  ALFA(I) = (-10.,-10.)            47
  510 CCNTINUE
  520 CC 525 I = 1, 8
  525 ALF(I) = ALFA(I)
      IF( ROOTS .OR. ALL ) WRITE(6,528) ( ALF(I), I=1,8 )                    63
  528 FCRMAT (33H0INTERMEDIATE ROOTS OF POLYNCMIAL//(1H , 1P8E13.5))

C.....SELECT POSITIVE REAL ROCTS.....
      CC 789 K=1,4
  789 IA(K) = K
      K = 0
      CC 630 I = 1, 8
      RA = REAL(ALFA(I))
      IF( RA .GT. 0. ) GO TO 610
      GC TO 630
  610 K = K + 1
      ALFAB(K) = ALFA(I)
      IF( K .EQ. 4 ) GO TO 64C
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              LIN803   CONWAY      PHASE           N4                    12/31/69    000109     PAGE  27
         F.....     -  EFN   SOURCE STATEMENT  -  IFN(S)  -

  630 CCNTINUE                                                      F0001050
      IF( K .LE. 1 ) GO TO 637                                      F0001060
      IF( AC12 .AND. AC23 ) GO TO 631                               F0001070
      IF( NBETA.EC.2 .AND. K.EQ.3 ) GO TO 640                       F0001072
      GO TC 634                                                     F0001074
  631 IF( NBETA .EC. 2 ) GC TO 400                                  F0001080
      IF( .NOT. AC24 ) GO TO 636                                    F0001090
      IC = 1                                                        F0001100
      IF( K .EQ. 3 ) GO TO 140                                      F0001110
  632 WRITE(6,633) ( ALAB(I,IC), I=1,3 ), K                         F0001120    117
  633 FCRMAT( /// 16H *** CEGENERATE ,3A6,3H  (,I2,9H ALPHAS ) )    F0001130
  634 IALF = .TRUE.                                                 F0001140
      GC TO 1357                                                    F0001150
  636 IF( .NCT. ( AC14.AND.AC34 ) ) GC TO 634                       F0001160
      GC TO 280                                                     F0001170
  637 WRITE(6,638) K                                                F0001180    130
  638 FCRMAT( /// 23H *** NUMBER OF ALPHAS =,I2,                    F0001190
     .              20H  -  CASE TERMINATED /            )          F0001200
      CC TO 634                                                     F0001210
                                                                    F0001220
  640 IF (NBETA .EQ. 2)  ALFAB(4) = (C.,0.)                         F0001230
      IF( ROOTS .OR. ALL ) WRITE(6,788 ) ( ALFAB(K), K=1,4 )        F0001240    135
  788 FCRMAT (28H0INTERMEDIATE POSITIVE ROOTS//(1H , 1P8E13.5))     F0001250
                                                                    F0001260
C.....CALCULATE EETA B.....                                         F0001270
      K4 = NBETA + 1                                                F0001280
  810 CC 980 K = 1, K4                                              F0001290
      B1(1,1) =  B11( ALFAB(K) )                                    F0001300
      B1(1,2) =  B12( ALFAB(K) )                                    F0001310
      B1(1,3) =  B13( ALFAB(K) )                                    F0001320
      B1(1,4) =  B14( ALFAB(K) )                                    F0001330
      B1(2,2) =  B22( ALFAB(K) )                                    F0001340
      B1(2,3) =  B23( ALFAB(K) )                                    F0001350
      B1(2,4) =  B24( ALFAB(K) )                                    F0001360
      B1(3,3) =  B33( ALFAB(K) )                                    F0001370
      B1(3,4) =  B34( ALFAB(K) )                                    F0001380
      B1(2,1) = B1(1,2)                                             F0001390
      B1(3,1) = B1(1,3)                                             F0001400
      B1(3,2) = B1(2,3)                                             F0001410
      IF(NBETA.EC.2) GO TO 94C
      B1(4,1) =-B1(1,4)                                             F0001460
      B1(4,2) =-B1(2,4)                                             F0001470
      B1(4,3) =-B1(3,4)                                             F0001480
      B1(4,4) = B44( ALFAB(K) )                                     F0001490
      IF(K.GT.1) GO TO 830
                                                                    F0001440
C.....CHECK FOR DEGENERATE CASES.....                               F0001450
      CC 820 I=1,4                                                  F0001500
      NB(I) = 0                                                     F0001510
      CC 820 J=1,4                                                  F0001520
      IF( B1D(1,I,J).EQ.0. .AND. B1D(2,I,J).EC.0. ) NB(I) = NB(I) + 1  F0001530
  820 CCNTINUE                                                      F0001540
      IF( NB(2) .EC. 3 ) GC TO 100                                  F0001550
      IF( NB(1).NE.2 .OR. NB(2).NE.2 .OR. NB(3).NE.2 .OR. NB(4).NE.2 )  F0001560
     .          GC TC 830
      GC TO 200                                                     F0001580
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        F.....       -  EFN   SOURCE STATEMENT  -  IFN(S)  -

C.....PIEZOELECTRIC.....
  830 IF( ACAP ) WRITE(6,975) K, ( ( B1(I,J), J=1,4 ), I=1,4 )                  185
      CC 850 I=1,3
      CC 840 J=1,3
  840 B1B(J,I)=B1(J,I)*TEN
      B1B(4,I)=B1(4,I)
  850 B1B(I,4)=B1(I,4)
      B1B(4,4)=B1(4,4)*TEN10
      KK=4
      IF(HXAGNL) GC TO 9C0
  860 CC 870 J=1,4
      CO 870 I=1,3
  870 BB(I,J)=B1B(I,J)
      IF( ACAP ) WRITE(6,875) K, ( ( BB(I,J), J=1,4 ), I=1-3 )                  222
  875 FORMAT(4HOBB(,I1,1H),//(1H,1P8E13.5))
      CALL CMATS(BB,BETAB1(1),3,1,$1960)                                        231
  880 BETAB1(KK)=CX1
      BETAB(4,K)=BETAB1(4)
      CO 890 J=1,3
  890 BETAB(J,K)=BETAB1(J)*TEN
      GC TC 980
C.....HEXAGONAL CRYSTAL.....
  900 CC 910 J=1,3
      CC 910 I=1,3
  910 BB1(I,J)=B1B(I,J)
      CALL CDET(BB1,3,FVS,KEXP)                                                 253
      FVSMAG=CABS(FVS)                                                          255
      IF(FVSMAG.GT.1.E-5) CO TO 860
      KK=1
      CO 930 I=1,3
      CO 920 J=1,3
  920 BB(I,J)=B1B(I+1,J+1)
      BB(I,3)=-BB(I,3)
  930 BB(I,4)=-B1B(I+1,1)
      IF( ACAP ) WRITE(6,875) K, ( ( BB(I,J), J=1,4 ), I=1,3 )                  274
      CALL CMATS(BB,BETAB1(2),3,1,$1960)                                        283
      GO TO 880
C.....ZERC-PIEZOELECTRIC CASE.....
  940 IF( ACAP ) WRITE(6,974) K, ( ( B1(I,J), J=1,3 ), I=1,2 )                  287
  974 FCRMAT(6HOACAP(,I1,1H) // (1H ,1P6E13.5))
  975 FCRMAT(6HOACAP(,I1,1H) // (1H ,1P8E13.5))
      B1(3,1)=B1(1,2)
      B1(4,1)=B1(2,2)
      B1(1,2)=-B1(1,3)
      B1(2,2)=-B1(2,3)
      K2=3
      CALL CMATS(B1,BETAB(1,K),NBETA,1,$196C )                                  298
      BETAB(4,K)=CX0
      BETAB(3,K)=CX1
  980 CCNTINUE
      IF (NBETA .EG. 3)  GC TO 983                                     F0001750
                                                                       F0001760
C.....ZERC - PIEZCELECTRIC CASE.....                                   F0001770
      CC 982 I = 1, 4                                                  F0001780
      BETAB(I,4) = (0.,0.)                                             F0001790
      CC 982 J = 1, 3                                                  F0001800
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           LINBO3   CONWAY      PHASE             N4                    12/31/69    000109      PAGE  29
        F.....     -  EFN   SOURCE STATEMENT  -  IFN(S)  -

  982 BETAB(I,J) = BETAB(I,J)*1.E-10                                F0001810
      GO TO 983                                                     F0001820
                                                                    F0001830
                                                                    F0001840
C.....1 ROW, 3 ZERO CASE.....                                       F0001850
                                                                    F0001860
C.....( 4 ALPHAS ).....                                             F0001870
  100 CONTINUE                                                      F0001880
      ICASE = 1                                                     F0001890
      J = 1                                                         F0001900
      BMIN = CABS( B1(2,2) )                                        F0001910    325
      DO 110 K=2,4                                                  F0001920
      B1X = CABS( B22( ALFAB( K ) ) )                               F0001930    330
      IF( B1X .GE. BMIN ) GO TO 110                                 F0001940
      J = K                                                         F0001950
      BMIN = B1X                                                    F0001960
  110 CONTINUE                                                      F0001970
      IA(1) = J                                                     F0001980
      JA = 2                                                        F0001990
      DO 120 K=1,4                                                  F0002000
      IF( K .EQ. J ) GO TO 120                                      F0002010
      IA(JA) = K                                                    F0002020
      JA = JA + 1                                                   F0002030
  120 CONTINUE                                                      F0002040
      BETAB(2,1) = 1.E-10                                           F0002050
      BETAB(1,1) = CXO                                              F0002060
      BETAB(3,1) = CXO                                              F0002070
      BETAB(4,1) = CXO                                              F0002080
      J1 = 2                                                        F0002090
      J2 = 4                                                        F0002100
  125 DO 130 J=J1,J2                                                F0002110
      K = IA(J)                                                     F0002120
      B1A(1,1) =  B11( ALFAB(K) )                                   F0002130
      B1A(2,1) =  B13( ALFAB(K) )                                   F0002140
      B1A(2,2) =  B33( ALFAB(K) )                                   F0002160
      B1A(1,3) = -B14( ALFAB(K) )                                   F0002170
      B1A(2,3) = -B34( ALFAB(K) )                                   F0002180
      BETAX = B1A(1,1)*B1A(2,2) - B1A(2,1)**2                       F0002190    361
      BETAB(1,J) = ( B1A(2,1)*B1A(2,3) - B1A(1,3)*B1A(2,2) ) / BETAX   F0002200
      BETAB(2,J) = CXO                                              F0002210
      BETAB(3,J) = ( B1A(2,1)*B1A(1,3) - B1A(1,1)*B1A(2,3) ) / BETAX   F0002220
  130 BETAB(4,J) = CX1                                              F0002230
      IF( ICASE .EQ. 3 ) GO TO 983                                  F0002240
      GO TO 270                                                     F0002250
                                                                    F0002260
C.....( 3 ALPHAS ).....                                             F0002270
  140 ICASE = 3                                                     F0002280
      N1 = 0                                                        F0002290
      DO 150 I=1,K                                                  F0002300
      IF( CABS( B22( ALFAB(I) ) ) .GT. 1.E7 ) N1 = N1 + 1           F0002310    380
  150 CONTINUE                                                      F0002320
      IF( N1 .LT. 3 ) GO TO 632                                     F0002330
      J1 = 1                                                        F0002340
      J2 = 3                                                        F0002350
      K2 = 3                                                        F0002360
      IB2 = 3                                                       F0002370
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          LINBO3   CONWAY      PHASE           N4                       12/31/69    000109      PAGE  30
      F.....    -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      L2 = 12                                                          F0002380
      ICB = 3                                                          F0002390
      WRITE(6,160) ( ALAB(I,1), I=1,3 ), ( I, ALFAB(I), I=1,K )        F0002400    394
  160 FORMAT( /// 16H *** DEGENERATE ,3A6,14H  ( 3 ALPHAS ) /          F0002410
     *              ( 11H     ALPHA(,I1,3H) =, 1P2E13.5 / )     )      F0002420
      GO TO 125                                                        F0002430
                                                                       F0002440
                                                                       F0002450
C.....4 ROW, 2 ZERO CASE.....                                          F0002460
                                                                       F0002470
C.....( 4 ALPHAS ).....                                                F0002480
  200 CONTINUE                                                         F0002490
      ICASE = 2                                                        F0002500
      B1X(1) = CABS( B1(1,1)*B1(3,3) - B1(1,3)**2 )                    F0002510    410   411
      J = 1                                                            F0002520
      BMIN = B1X(1)                                                    F0002530
      DO 210 K=2,4                                                     F0002540
      B1X(K) = CABS( B11(ALFAB(K))*B33(ALFAB(K))-B13(ALFAB(K))**2 )    F0002550    419   420
      IF( B1X(K) .GE. BMIN ) GO TO 210                                 F0002560
      J = K                                                            F0002570
      BMIN = B1X(K)                                                    F0002580
  210 CONTINUE                                                         F0002590
      I = 1                                                            F0002600
      IF( J .EQ. 1 ) I = 2                                             F0002610
      DO 220 K=1,4                                                     F0002620
      IF( K .EQ. J ) GO TO 220                                         F0002630
      IF( B1X(K) .GE. B1X(I) ) GO TO 220                               F0002640
      I = K                                                            F0002650
  220 CONTINUE                                                         F0002660
  230 CONTINUE                                                         F0002670
      JA = 3                                                           F0002680
      JB = 1                                                           F0002690
      DO 240 K=1,4                                                     F0002700
      IF( K.EQ.J .OR. K.EQ.I ) GO TO 235                               F0002710
      IA(JA) = K                                                       F0002720
      JA = JA + 1                                                      F0002730
      GO TO 240                                                        F0002732
  235 IA(JB) = K                                                       F0002734
      JB = JB + 1                                                      F0002736
  240 CONTINUE                                                         F0002740
  245 DO 250 K=1,2                                                     F0002750
      I = IA(K)                                                        F0002760
      BETAB(1,K) = - B13(ALFAB(I)) / B11(ALFAB(I)) * 1.E-10            F0002770
      BETAB(2,K) = CX0                                                 F0002780
      BETAB(3,K) = 1.E-10                                              F0002790
  250 BETAB(4,K) = CX0                                                 F0002800
      IF( ICB .EQ. 1 ) GO TO 983                                       F0002810
  255 DO 260 K=3,4                                                     F0002820
      I = IA(K)                                                        F0002830
      BETAB(1,K) = CX0                                                 F0002840
      BETAB(2,K) = B24(ALFAB(I)) / B22(ALFAB(I))                       F0002850
      BETAB(3,K) = CX0                                                 F0002860
  260 BETAB(4,K) = CX1                                                 F0002870
      IF( ICB .EQ. 2 ) GO TO 983                                       F0002880
      GO TO 270                                                        F0002890
                                                                       F0002900
```

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        F.....      -  EFN   SOURCE STATEMENT  -  IFN(S)  -

C......( LESS THAN 4 ALPHAS )......
  280 IC = 2
      I1 = 0
      I2 = 0
      CC 310 I=1,K
      TERM = CABS( - B22(ALFAB(I))*B44(ALFAB(I)) - B24(ALFAB(I))**2 )        511   512
      IF( TERM .GE. 1.E-5 ) GO TO 300
      I1 = I1 + 1
      IA1(I1) = I
      GC TO 310
  300 I2 = I2 + 1
      IA2(I2) = I
  310 CONTINUE
      IF( I2 .EQ. 2 ) GO TO 340
      IF( I1 .EQ. 2 ) GO TO 320
      GO TO 632

C......( 2,4 CASE )......
  320 IA(3) = IA1(1)
      IA(4) = IA1(2)
      ICASE = 5
      IB1 = 3
      ICB = 2
      L1 = 9
      K1 = 3
      GO TO 360

C......( 1,3 CASE )......
  340 IA(1) = IA2(1)
      IA(2) = IA2(2)
      ICB = 1
  350 ICASE = 4
      IB2 = 2
      L2 = 8
      K2 = 2
  360 J = IA(IB1)
      K = IA(IB2)
      WRITE(6,370) ( ALAB(I,IC),I=1,3 ), LAB(ICB), IB1,IB2,IB1,ALFAB(J),
     .                                 IB2, ALFAB(K)                          547
  370 FCRMAT( /// 16H *** DEGENERATE ,4A6 /
     .               43H     CALCULATE BETA(I,K) AND L(I,K) FOR K =,I2,
     .               4H AND,I2,8H (I=1,4) //
     .               2(11H     ALPHA(,I1,3H) =,1P2E13.5 / ) /            )
      IF( ICB .EQ. 3 ) ICB = 1
      GC TO ( 245, 255 ), ICB

C......ZERC - PIEZOELECTRIC CASE......
  400 IC = 3
      I1 = 0
      CO 430 I=1,K
      CA22 = CABS( B22( ALFAB(I) ) )                                          567
      IF( K .EQ. 3 ) GO TO 410
      IF( CA22 .GT. 1.E7 ) I1 = I1 + 1
      GC TO 430
  410 IF( I .GT. 1 ) GO TO 420
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             LINBO3   CONWAY      PHASE             N4                      12/31/69    000109      PAGE  32
        F.....     -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      CAMIN = CA22
      J = I
      GC TC 430
  420 IF( CAMIN .LE. CA22 ) GO TO 430
      CAMIN = CA22
      J = I
  430 CCNTINUE
      IF( K .EQ. 2 ) GO TO 450
      I1 = 0
      CC 440 I=1,3
      IF( I .EQ. J ) GO TO 440
      I1 = I1 + 1
      IA(I1) = I
  440 CCNTINUE
      GO TO 460
  450 IF( I1 .NE. 2 ) GC TO 632
  460 ICB = 3
      GC TC 350


  270 IF( .NOT. ( RCOTS .OR. ALL ) ) GO TO 983
      II1 = IA(1)
      II2 = IA(2)
      II3 = IA(3)
      II4 = IA(4)
      WRITE(6,984) ( ALAB(I,ICASE), I=1,3 ), ALFAB(II1), ALFAB(II2),
     .                                   ALFAB(II3), ALFAB(II4)          611
  984 FCRMAT( 21HORE-ORDERED ALPHAS ( , 3A6, 2H ) // ( 1H ,1P8E13.5 ) )
  983 IF( BETA .OR. ALL ) WRITE(6,985 ) ( ( BETAB(I,J),
     .                             J=IB1,IB2 ), I=1,4 )                   623
  985 FCRMAT (20HOINTERMEDIATE BETA 8//(1H , 1P8E13.5))
      IF (KS .NE. 0)  GO TC 1390

C.....EVALUATE LITHIUM NIOBATE EQUATICNS.....
      L = L1
      CO 1330 K=K1,K2
      J = IA(K)
      EL(L)   = BETAB(1,K)*(CMPLX(C., C15) + ALFAB(J)*C55)
     .         + BETAB(2,K)*(CMPLX(0., C56) + ALFAB(J)*C45)
     .         + BETAB(3,K)*(CMPLX(0., C55) + ALFAB(J)*C35)
     .         + BETAB(4,K)*(CMPLX(0., E15) + ALFAB(J)*E35)
      EL(L+1) = BETAB(1,K)*(CMPLX(0., C14) + ALFAB(J)*C45)
     .         + BETAB(2,K)*(CMPLX(0., C46) + ALFAB(J)*C44)
     .         + BETAB(3,K)*(CMPLX(0., C45) + ALFAB(J)*C34)
     .         + BETAB(4,K)*(CMPLX(0., E14) + ALFAB(J)*E34)
      EL(L+2) = BETAB(1,K)*(CMPLX(0., C13) + ALFAB(J)*C35)
     .         + BETAB(2,K)*(CMPLX(0., C36) + ALFAB(J)*C34)
     .         + BETAB(3,K)*(CMPLX(0., C33) + ALFAB(J)*C33)
     .         + BETAB(4,K)*(CMPLX(0., E13) + ALFAB(J)*E33)
      IF (KL .EC. 0)  GO TO 1190
 1170 EL(L+3) = BETAB(4,K)
      GO TO 1330
 1190 IF (WH .EQ. 0. .AND. KM .EQ. 0)  GO TO 1170
      IF (WH .LE. 1.E10)  GO TO 1220
      ST = EPSR
      GC TO 1290
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F.....     -  EFN   SOURCE STATEMENT  -  IFN(S)  -

 1220 IF (WH .NE. 0. .OR. KM .NE. 1)  GO TO 1250                       F0004030
      ST = 0.                                                          F0004040
      GC TO 1290                                                       F0004050
 1250 IF (KM .NE. 0 .OR. WH .EQ. 0. .OR. WH .GE. 1.E10)  GO TO 1280    F0004060
      ST=COSH(WH/VS)/SINH(WH/VS)*EPSR                                             691   692
      CO TO 1290
 1280 ST = TANH(WH/VS) * EPSR                                          F0004090   695
 1290 EL(L+3) = BETAB(1,K)*(CMPLX(0., E31) + ALFAB(J)*E35)             F0004100
     .          + BETAB(2,K)*(CMPLX(0., E36) + ALFAB(J)*E34)           F0004110
     .          + BETAB(3,K)*(CMPLX(0., E35) + ALFAB(J)*E33)           F0004120
     .          -(CMPLX(0., T13) + ALFAB(J)*T33 + EPSO*ST)*BETAB(4,K)  F0004130
      EL(L+3) = EL(L+3) * 1.E+10                                       F0004140
 1330 L = L + 4                                                        F0004150
      IF (NBETA .EQ. 3)  GO TC 1340                                    F0004160
                                                                       F0004170
C.....ZERC - PIEZCELECTRIC CASE.....                                   F0004180
      EL(4) = (0.,0.)                                                  F0004190
      EL(8) = (0.,0.)                                                  F0004200
      EL(12)= (0.,0.)                                                  F0004210
      EL(13)= (0.,0.)                                                  F0004220
      EL(14)= (0.,0.)                                                  F0004230
      EL(15)= (0.,0.)                                                  F0004240
      EL(16) = CX1                                                     F0004250
                                                                       F0004260
 1340 IF( DETERM .OR. ALL ) WRITE(6,1335) ( EL(I), I=L1,L2 )           F0004270   716
 1335 FORMAT (33H0INTERMEDIATE L MATRIX BY COLUMNS//                   F0004280
     .         (1H ,1P8E13.5))                                         F0004290
      ICB1 = ICB + 1                                                   F0004300
      GC TO ( 1343, 1360, 137C, 1380 ), ICB1                           F0004310
 1343 CC 1345 I=1,16                                                   F0004320
 1345 XEL(I) = EL(I)                                                   F0004330
      CALL CDET (XEL, 4, FVS, KEXP)                                    F0004340   734
 1350 F = FVS                                                          F0004350
      FVSMAG = CABS( FVS )                                             F0004360   736
      IF (DETERM .OR. ALL)  WRITE (6, 1355)  VS, FVS, FVSMAG           F0004370   737
 1355 FCRMAT (5H0VS =, E15.7, 5X, 7HF(VS) =, 2E15.7,5X,5HMAG = E15.7 ) F0004380
 1357 RETURN                                                           F0004390
 1360 FVS = EL(1)*EL(7) - EL(5)*EL(3)                                  F0004400
      GC TO 1350                                                       F0004410
 1370 FVS = EL(10)*EL(16) - EL(14)*EL(12)                              F0004420
      GC TO 1350                                                       F0004430
 1380 J = 1                                                            F0004440
      CC 1385 K=1,3                                                    F0004450
      XXEL(1,K) = EL(J)                                                F0004460
      XXEL(2,K) = EL(J+2)                                              F0004470
      XXEL(3,K) = EL(J+3)                                              F0004480
 1385 J = J + 4                                                        F0004490
      CALL CDET( XXEL, 3, FVS, KEXP )                                  F0004500   759
      GC TO 1350                                                       F0004510
                                                                       F0004520
C.....EVALUATE GOLD LITHIUM NIOBATE EQUATIONS.....                     F0004530
1390  CD = MUA + MUA + LAMCAA                                          F0004540
      CA = MUA*DC/1.E20                                                F0004550
      RVS=RHOA*VS*VS                                                   F0004560
      CB =(RVS*(MUA + DD) - 2.*MUA*DC)/1.E20                           F0004570
      CC = (RVS - DD)*(RVS - MUA)/1.E20                                F0004580
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      FVS = CSQRT(CE*DB - 4.*CA*DC)                                     F0004590    762
      ALFAA(1) = CSCRT((-DB + FVS)/(DA + DA))                           F0004600    763
      ALFAA(2) = -ALFAA(1)                                              F0004610
      ALFAA(3) = CSCRT((-DB - FVS)/(DA + DA))                           F0004620    764
      ALFAA(4) = -ALFAA(3)                                              F0004630
      ALFAA(5) = CSCRT(CMPLX((MUA - RVS)/MUA, 0.))                      F0004640    765
      ALFAA(6) = -ALFAA(5)                                              F0004650
      IF( ALPHA .CR. ALL ) WRITE(6,1505) ( ALFAA(I), I=1,6 )            F0004660    766
 1505 FCRMAT (21HOINTERMEDIATE ALPHA A//(1H , 1P6E13.5))                F0004670
      CC 1550 K = 1, 4                                                  F0004680
      BETAA(1,K) = CMPLX(0., -LAMDAA - MUA)*ALFAA(K)/                   F0004690
     .             ((MUA*ALFAA(K)*ALFAA(K) - DD + RVS)*1.E10)           F0004700
      BETAA(2,K) = (0.,0.)                                              F0004710
 1550 BETAA(3,K) = (1.E-10,0.)                                          F0004720
      BETAA(2,5) = (1.E-10,0.)                                          F0004730
      BETAA(2,6) = (1.E-10,0.)                                          F0004740
      BETAA(3,5) = (0.,0.)                                              F0004750
      BETAA(3,6) = (0.,0.)                                              F0004760
      IF( BETA .OR. ALL ) WRITE(6,1595) ( ( BETAA(I,J), I=1,3 ), J=1,6 )F0004770
     .   ((BETAA(I,J), I = 1, 3), J = 1, 6)                             F0004780    786
 1595 FCRMAT (31HOINTERMECIATE BETA A BY COLUMNS//(1H , 1P6E13.5))      F0004790
      CC 1620 J = 1, 6                                                  F0004800
      CO 1620 I = 1, 3                                                  F0004810
 1620 ELL(I,J) = BETAA(I,J)                                             F0004820
      CO 1650 J = 7, 10                                                 F0004830
      CO 1650 I = 1, 3                                                  F0004840
 1650 ELL(I,J) = -BETAB(I,J-6)                                          F0004850
      CC 1740 J = 1, 6                                                  F0004860
      ELL(4,J) = BETAA(1,J)*ALFAA(J)*MUA + CMPLX(0.,MUA)*BETAA(3,J)     F0004870
      ELL(5,J) = BETAA(2,J)*ALFAA(J)*MUA                                F0004880
      ELL(6,J) = CMPLX(0.,LAMDAA)*BETAA(1,J) + BETAA(3,J)*ALFAA(J)*DD   F0004890
      FVS = CEXP(ALFAA(J)*WH/VS)                                        F0004900    843
      ELL(7,J) = ELL(4,J)*FVS                                           F0004910
      ELL(8,J) = ELL(5,J)*FVS                                           F0004920
      ELL(9,J) = ELL(6,J)*FVS                                           F0004930
 1740 ELL(10,J)= (0.,0.)                                                F0004940
      CO 1910 J = 7, 10                                                 F0004950
      I = IA(J-6)                                                       F0004960
      ELL(4,J) =-BETAB(1,J-6)*(CMPLX(0., C15) + ALFAB(I)*C55)           F0004970
     .         - BETAB(2,J-6)*(CMPLX(0., C56) + ALFAB(I)*C45)           F0004980
     .         - BETAB(3,J-6)*(CMPLX(0., C55) + ALFAB(I)*C35)           F0004990
     .         - BETAB(4,J-6)*(CMPLX(0., E15) + ALFAB(I)*E35)           F0005000
      ELL(5,J) = -BETAB(1,J-6)*(CMPLX(0., C14) + ALFAB(I)*C45)          F0005010
     .         -  BETAB(2,J-6)*(CMPLX(0., C46) + ALFAB(I)*C44)          F0005020
     .         -  BETAB(3,J-6)*(CMPLX(0., C45) + ALFAB(I)*C34)          F0005030
     .         - BETAB(4,J-6)*(CMPLX(0., E14) + ALFAB(I)*E34)           F0005040
      ELL(6,J) = -BETAB(1,J-6)*(CMPLX(0., C13) + ALFAB(I)*C35)          F0005050
     .           -BETAB(2,J-6)*(CMPLX(0.,C36)+ALFAB(I)*C34  )
     .         -  BETAB(3,J-6)*(CMPLX(0., C35) + ALFAB(I)*C33)          F0005070
     .         - BETAB(4,J-6)*(CMPLX(0., E13) + ALFAB(I)*E33)           F0005080
      ELL(7,J) =  (0.,0.)                                               F0005090
      ELL(8,J) =  (0.,0.)                                               F0005100
      ELL(9,J) =  (0.,0.)                                               F0005110
 1910 ELL(10,J)=  BETAB(4,J-6)                                          F0005120
      CO 1915 I = 1, 3                                                  F0005130
      DO 1915 J = 1, 10                                                 F0005140
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       F.....    -  EFN   SOURCE STATEMENT  -  IFN(S)  -

 1915 ELL(I,J) = ELL(I,J)*1.E10                                          F0005150
      IF( DETERM .OR. ALL ) WRITE(6,1930) (( ELL(I,J), J=1,10 ), I=1,10)F0005160
     .   ((ELL(I,J), J = 1, 10), I = 1, 10)                              F0005170    905
 1930 FORMAT (17H1*** L MATRIX ***//(1H , 1P8E15.6))                     F0005180
      DO 1940 I=1,100                                                    F0005190
 1940 XEL(I) = ELL(I,1)                                                  F0005200
      CALL CDET  (XEL, 10, FVS, KEXP)                                    F0005210    933
      GO TO 1350                                                         F0005220
                                                                         F0005230
C.....ERROR IN MATRIX INVERSION.....                                     F0005240
 1960 WRITE  (6, 1970)                                                   F0005250    935
 1970 FORMAT (30H0***SINGULAR MATRIX IN BETA***)                         F0005260
      GO TO 1350                                                         F0005270
      END                                                                F0005280
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$IBFTC STRIP.  DECK

CSTRIP     *** ARRAY OF CONDUCTING STRIPS OVER PIEZOELECTRIC MEDIUM ***
C*GV012
C*                           G.V. ROBERTS
C*                             03.15.68
C*********************************************************************
      SUBROUTINE STRIP  (A, VS, RHO, ALL, COEFF)

      COMMON /ZZTZ/ CC(20),CE(17),CT(5)
      COMMON     /FLAG/  ONCE
      COMPLEX    A(9), ZERO, ONE, JIMAG
      REAL       ONES(5), CA(4,4), CB(4,4), CC(4,4)
      INTEGER    P
      LOGICAL    ALL, COEFF, ONCE
      EQUIVALENCE (CC(1), C11), (CC(2), C13), (CC(3), C14), (CC(4),C15),
     .            (CC(5), C33), (CC(6), C34), (CC(7), C35), (CC(8),C36),
     .            (CC(9), C44), (CC(10),C45), (CC(11),C46),
     .            (CC(12),C55), (CC(13),C56), (CC(14),C66),
     .            (CC(15),C16), (CE(1), E11), (CE(2), E13),
     .            (CE(3), E14), (CE(4), E15), (CE(5), E16),
     .            (CE(6), E31), (CE(7), E33), (CE(8), E34),
     .            (CE(9), E35), (CE(10),E36), (CT(1), T11),
     .            (CT(2), T13), (CT(3), T33)

      DATA    TPI, ZERO, ONE, ONES, JIMAG / 6.2831853, (0.,0.), (1.,0.),
     .         1., -1., -1., 1., 1., (0.,1.) /

C.....SET UP MATRICES.....
      IF (.NOT. ONCE)  GO TO 670
      ONCE = .FALSE.
      CA(1,1) = C55
      CA(1,2) = C45
      CA(2,1) = C45
      CA(1,3) = C35
      CA(3,1) = C35
      CA(1,4) = E35
      CA(4,1) = E35
      CA(2,2) = C44
      CA(2,3) = C34
      CA(3,2) = C34
      CA(2,4) = E34
      CA(4,2) = E34
      CA(3,3) = C33
      CA(3,4) = E33
      CA(4,3) = E33
      CA(4,4) = -T33
      CB(1,1) = C15 + C15
      CB(1,2) = C14 + C56
      CB(2,1) = CB(1,2)
      CB(1,3) = C13 + C55
      CB(3,1) = CB(1,3)
```

```
      CB(1,4) = E15 + E31
      CB(4,1) = CB(1,4)
      CB(2,2) = C46 + C46
      CB(2,3) = C36 + C45
      CB(3,2) = CB(2,3)
      CB(2,4) = E14 + E36
      CB(4,2) = CB(2,4)
      CB(3,3) = C35 + C35
      CB(3,4) = E13 + E35
      CB(4,3) = CB(3,4)
      CB(4,4) = -T13 - T13
      CD(1,2) = -C16
      CD(2,1) = -C16
      CD(1,3) = -C15
      CD(3,1) = -C15
      CD(1,4) = -E11
      CD(4,1) = -E11
      CD(2,3) = -C56
      CD(3,2) = -C56
      CD(2,4) = -E16
      CD(4,2) = -E16
      CC(3,4) = -E15
      CC(4,3) = -E15
      CD(4,4) =  T11

C.....P F E W.....
  670 DO 680 I = 1, 9
  680 A(I) = ZERO

C.....PREPARE FOR GAMMA(N) LOOP.....
      RGAM = RHO*VS*VS
      CD(1,1) = RGAM - C11
      CD(2,2) = RGAM - C66
      CD(3,3) = RGAM - C55

C.....COMPUTE COEFFICIENTS OF EIGHT DEGREE PCLYNOMIAL.....
      P = 0
      DO 1030 J = 1, 4
      DO 1020 K = 1, 4
      IF (K .EQ. J)  GO TO 1020
      DO 1010 L = 1, 4
      IF (L .EQ. K .OR. L .EQ. J)  GO TO 1010
      M = 10 - J - K - L
      P = P + 1
      IF (P .GT. 5)  P = 2
      T1 = CA(1,J)*CB(2,K) + CB(1,J)*CA(2,K)
      T2 = CA(1,J)*CD(2,K) - CB(1,J)*CB(2,K) + CD(1,J)*CA(2,K)
      T3 = CB(1,J)*CD(2,K) + CD(1,J)*CB(2,K)
      ZH = CA(1,J)*CA(2,K)*CA(3,L)
      ZI = CA(1,J)*CA(2,K)*CB(3,L) + T1*CA(3,L)
      ZJ = CA(1,J)*CA(2,K)*CD(3,L) - T1*CB(3,L) + T2*CA(3,L)
      ZK = CD(3,L)*T1 + CB(3,L)*T2 + CA(3,L)*T3
      ZL = T2*CD(3,L) - T3*CB(3,L) + CD(1,J)*CD(2,K)*CA(3,L)
      ZM = T3*CD(3,L) + CD(1,J)*CD(2,K)*CB(3,L)
      ZN = CD(1,J)*CD(2,K)*CD(3,L)
      ZS = CA(4,M)
```

```
     STRIP.     -  EFN   SOURCE STATEMENT  -  IFN(S)  -

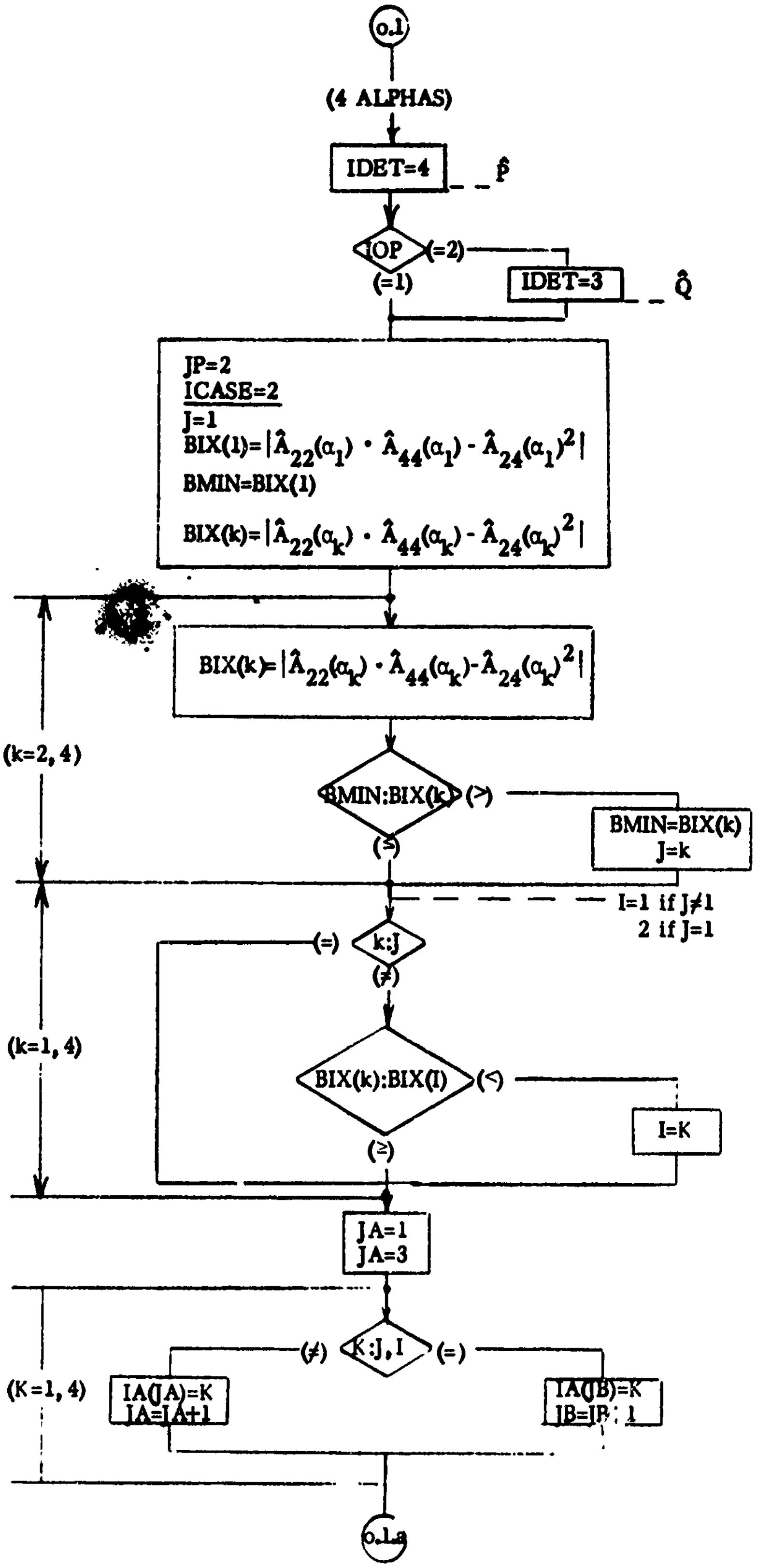
      ZU = CB(4,M)                                              STRP1080
      ZV = CD(4,M)                                              STRP1090
      A(1) = A(1) + ONES(P)*ZH*ZS                               STRP1100
      A(2) = A(2) + ONES(P)*(ZH*ZU + ZI*ZS)                     STRP1110
      A(3) = A(3) + ONES(P)*(ZH*ZV - ZI*ZU + ZJ*ZS)             STRP1120
      A(4) = A(4) + ONES(P)*(ZI*ZV + ZJ*ZU + ZK*ZS)             STRP1130
      A(5) = A(5) + ONES(P)*(ZJ*ZV - ZK*ZU + ZL*ZS)             STRP1140
      A(6) = A(6) + ONES(P)*(ZK*ZV + ZL*ZU + ZM*ZS)             STRP1150
      A(7) = A(7) + ONES(P)*(ZL*ZV - ZM*ZU + ZN*ZS)             STRP1160
      A(8) = A(8) + ONES(P)*(ZM*ZV + ZN*ZU)                     STRP1170
      A(9) = A(9) + ONES(P)*ZN*ZV                               STRP1180
 1010 CCNTINUE                                                  STRP1190
 1020 CCNTINUE                                                  STRP1200
 1030 CCNTINUE                                                  STRP1210
      DC 1C35 I = 2, 8, 2                                       STRP1220
 1035 A(I) = JIMAG*A(I)                                         STRP1230
      IF (ALL .OR. COEFF)  WRITE (6, 1050) (A(I), I = 1, 9)     STRP1240    99
 1050 FORMAT (27HOCOEFFICIENTS OF POLYNOMIAL/(1H , 2E18.7))     STRP1250
                                                                STRP1260
C.....NCRMALIZE COEFFICIENTS TO LARGEST VALUE.....               STRP1270
      ANORM =CABS(A)                                            STRP1280    1C7
      CC 1155 I = 2, 9                                          STRP1290
 1155 ANORM = AMAX1 (ANORM,CABS(A(I)))                          STRP1300    113
      CO 1157 I = 1, 9                                          STRP1310
      A(I) = A(I)/ANORM                                         STRP1320
      IF (CABS(A(I)) .LT. 1.E-6)  A(I) = ZERO                   STRP1330    122
 1157 CCNTINUE                                                  STRP1340
      IF (ALL .OR. COEFF)  WRITE (6, 1050)  (A(I), I = 1, 9)    STRP1350    129
      RETURN                                                    STRP1360
      END                                                       STRP1370
```

```
$IBFTC CROOT.  DECK

CCROOT                                                                  CROOT020
C*    CROOT CALCULATES THE ROOTS OF AN EIGHTH DEGREE POLYNOMIAL         CROOT030
      SUBROUTINE CROOT (A, XO, KODE, X)                                 CROOT040
      COMMON      /CRDER/  C, K, M                                      CROOT050
      COMPLEX     A(9), C(9), XO(8), X(8), X1,ROUND, CXO                CROOT060
      EXTERNAL    POL                                                   CROOT070
      DATA        CXO/(0.,0.)/                                          CROOT075
                                                                        CROOT080
      K = 1                                                             CROOT090
      DO 100 I = 1, 9                                                   CROOT100
  100 C(I) = A(I)                                                       CROOT110
  110 IF (CABS(C(1)).NE. 0.)  GO TO 130                                 CROOT120    12
      X(K) = (0.,0.)                                                    CROOT130
      K = K + 1                                                         CROOT140
      IF (K .GT. 8)  GO TO 280                                          CROOT150
      M = 10 - K                                                        CROOT160
      DO 120 I = 1, M                                                   CROOT170
  120 C(I) = C(I+1)                                                     CROOT180
      GO TO 110                                                         CROOT190
                                                                        CROOT200
C.....C O M P U T E   K-TH   R O O T.....                               CROOT210
  130 X1 = (.5,.5)                                                      CROOT220
      IF (KODE .NE. 0)  X1 = XO(K)                                      CROOT230
                                                                        CROOT240
C.....X1 = INTIAL GUESS, USE MULLERS METHOD TC REFINE IT.....            CROOT250
      M = 10 - K                                                        CROOT260
      CALL MULLER (POL, X1, 20, 1.E-8, JUNK, X(K), N)                   CROOT270    36
      X1 = (.5, -.5)                                                    CROOT280
      IF (N .GE. 20)  CALL MULLER (POL, X1, 20, 1.E-8, JUNK, X(K), N)   CROOT290    40
      X(K) = ROUNC(X(K))                                                CROOT300    43
      K = K + 1                                                         CROOT310
      IF (K .GT. 8)  GO TO 280                                          CROOT320
                                                                        CROOT330
C.....R E D U C E   C O E F F I C I E N T S.....                        CROOT340
      DO 240 I = 2, M                                                   CROOT350
  240 C(I) = C(I) + X(K-1)*C(I-1)                                       CROOT360
      IF (K .NE. 8)  GO TO 130                                          CROOT370
      IF (CABS(C(1)) .NE. 0.)  GO TO 250                                CROOT380    62
      X(8) = CXO                                                        CROOT390
      GC TO 280                                                         CROOT400
  250 X(8) = -C(2)/C(1)                                                 CROOT410
      X(8) = ROUND( X(8) )                                              CROOT415
                                                                        CROOT420
C.....E X I T.....                                                       CROOT430    68
  280 DO 290 I = 1, 8                                                   CROOT440
  290 XO(I) = X(I)                                                      CROOT450
      KODE = 1                                                          CROOT460
      RETURN                                                            CROOT470
      END                                                               CROOT480
```

```
$IBFTC MULER.  DECK

CMULLER     ***** MULLERS METHOD FOR RCOT FINCING  ****             MULER020
C*                                                                  MULER030
C*                          G.V. ROBERTS                            MULER040
C*                           04.24.68                               MULER050
C******************************************************             MULER060
      SUBROUTINE MULLER (F, XC, MAX, EPSLON, KODE, X, N)            MULER070
                                                                    MULER080
      CCMPLEX   F, X, XO, X1, X2, X3, H2, L2, D2, G, FX, FX1, FX2, D,  MULER090
     .          TWO, ONE, GPD, GMD, L3, FX3, H1                     MULER100
      DATA      TWC, ONE / (0.02,0.), (0.01,0.) /                   MULER110
                                                                    MULER120
      FX   = F(XO)                                                  MULER130    2
      X2   = XO                                                     MULER140
      KCDE = 0                                                      MULER150
      IF (CABS(X) .EC. 0.)  GO TO 190                               MULER160    4
      FX1 =  F(1.C2*XO)                                             MULER170    8
      FX2  = F(1.01*XO)                                             MULER180    9
      H2   = -0.C1*XO                                               MULER190
      GC TO 220                                                     MULER200
  190 FX1  = F(TWO)                                                 MULER210    12
      FX2  = F(ONE)                                                 MULER220    13
      H2   = -0.01                                                  MULER230
  220 L2   = -0.50                                                  MULER240
      D2   = 0.50                                                   MULER250
                                                                    MULER260
C.....BEGIN ITERATICNS.....                                         MULER270
      DC 540 K = 1, MAX                                             MULER280
      N    = K                                                      MULER290
      G    = FX*L2*L2 - FX1*D2*D2 + FX2*(L2 + D2)                   MULER30C
      CALL OVERCK                                                               19
      D    = CSORT (G*G - 4.*FX2*D2*L2*(FX*L2 - FX1*D2 + FX2))      MULER310     1
      GPD = G + D                                                   MULER320
      GMD = G - D                                                   MULER330
      CGPD = CABS(GPD)                                              MULER340    22
      CGMD = CABS(GMD)                                              MULER350    23
      IF (CGPD .GT. CGMD) GO TO 360                                 MULER360
      IF (CGMD .EQ. 0.)  GO TO 580                                  MULER370
      L3  = -2.*FX2*D2/GMD                                          MULER380
      GC TO 370                                                     MULER390
  360 IF (CGPD .EC. 0.)  GC TO 580                                  MULER400
      L3  = -2.*FX2*D2/GPD                                          MULER410
  370 X3 = X2 + L3*H2                                               MULER420
      FX3 = F(X3)                                                   MULER430    35
      IF (CABS(X2) .EQ. 0.)  GO TO 420                              MULER440    36
      IF (CABS((X3 - X2)/X2) .GT. EPSLON)  GO TO 450                MULER450    40
      CALL OVERCK                                                               44
  390 X = X3                                                        MULER460
      GC TO 570                                                     MULER470
  420 IF (CABS(FX3) .LE. EPSLCN)  GO TO 390                         MULER480
                                                                    MULER490
```


o.1
(4 ALPHAS)
IDET=4
P̂
IOP
(=2)
(=1)
IDET=3
Q̂
JP=2
ICASE=2
J=1
BIX(1)=|Â22(α1) · Â44(α1) - Â24(α1)^2|
BMIN=BIX(1)
BIX(k)=|Â22(αk) · Â44(αk) - Â24(αk)^2|
BIX(k)=|Â22(αk) · Â44(αk)-Â24(αk)^2|
(k=2, 4)
BMIN:BIX(k)
(>)
(≤)
BMIN=BIX(k)
J=k
I=1 if J≠1
2 if J=1
k:J
(=)
(≠)
(k=1, 4)
BIX(k):BIX(I)
(<)
(≥)
I=K
JA=1
JA=3
K:J, I
(≠)
(=)
(K=1, 4)
IA(JA)=K
JA=JA+1
IA(JB)=K
o.1.a

```
              LINB03   CONWAY      PHASE             N4                       12/31/65    000109      PAGE  41
        MULER.      -  EFN   SOURCE STATEMENT  -  IFN(S)  -

C.....NOT CONVERGENT YET.....                                        MULER500   48
  450 FX  = FX1                                                      MULER510
      FX1 = FX2                                                      MULER520
      FX2 = FX3                                                      MULER530
      X = X1                                                         MULER540
      X1  = X2                                                       MULER550
      X2  = X3                                                       MULER560
      H1  = H2                                                       MULER570
      H2 = X2 - X1                                                   MULER580
      L2 = H2/H1                                                     MULER590
  540 D2  = 1. + L2                                                  MULER600
      KCDE= 2                                                        MULER610
      X   = X2                                                       MULER620
  570 RETURN                                                         MULER630
  580 X = X2                                                         MULER640
      GC TO 570                                                      MULER650
      END                                                            MULER660
```

```
$IBFTC POL...  DECK
CPOL                                                         POL00020
      COMPLEX FUNCTION POL (X)                               POL00030
      COMMON /ORDER/  C, K, M                                POL00040
      COMPLEX T, C(9), X                                     POL00050
      POL = (0.,0.)                                          POL00060
      T   = (1.,0.)                                          POL00070
      J   = M                                                POL00080
      DO 100 I = 1, M                                        POL00090
      POL = POL + C(J)*T                                     POL00100
      CALL OVERCK                                                      ?
      T   = T*X                                              POL00110
  100 J   = J - 1                                            POL00120
      RETURN                                                 POL00130
      END                                                    POL00140
```

```
$IBFTC TFUN..  DECK

CTFUN
      COMPLEX FUNCTION TFUN (ETA, WX, C1, C2, C3, C4, C5, C6, C7, C8)
      COMMON    /LINK/   JUNK1(244), ALFAB(4), JUNK2(36), BETAB(4,4),
     .                   JUNK3(8), VS, JUNK4(17)
     .          /CIA /   IA(4)
      COMPLEX   ALFAB, BETAB, ETA(4)

      CEBUG ETA,WX,C1,C2,C3,C4,C5,C6,C7,C8
      TFUN = (0.,0.)
      DO 80 K = 1, 4
      I = IA(K)
      CEBUG I
      CEBUG TFUN
   80 TFUN = TFUN + ETA(K)*(
     .      - BETAB(1,K)*(CMPLX(0.,C1) + ALFAB(I)*C2)/VS
     .      - BETAB(2,K)*(CMPLX(0.,C3) + ALFAB(I)*C4)/VS
     .      - BETAB(3,K)*(CMPLX(0.,C5) + ALFAB(I)*C6)/VS
     .      - BETAB(4,K)*(CMPLX(0.,C7) + ALFAB(I)*C8)/VS )
     .      *              CEXP(-ALFAB(I)*WX/VS)                      17
      CEBUG TFUN
      RETURN
      END
```

```
$IBFTC P1FUN.  DECK

CP1FUN
      COMPLEX FUNCTION P1FUN (ETA, C1,  C2,  C3,  C4,  C5,  C6,  C7,
     .                       C8,  C9,  C10, C11, C12, C13, C14, C15,
     .                       C16, C17, C18, C19, C20, C21, C22, C23,
     .                       C24)
      COMMON  /LINK/ JUNK1(244), ALFAB(4), JUNK2(36), BETAB(4,4),
     .               JUNK3(26)     /BETAN/  NBETA
     .        /CIA / IA(4)
      COMPLEX ALFAB, BETAB, ETA(4), J
      DATA    J / (0.,1.) /

      NK4 = NBETA + 1
      P1FUN = (0.,0.)
      DO 230 I = 1, 4
      L = IA(I)
      DO 230 K = 1, NK4
      M = IA(K)
      P1FUN=P1FUN + (ETA(I)*CONJG(ETA(K))/(ALFAB(L) + CONJG(ALFAB(M))))
     .       *(CONJG(BETAB(1,K))*(BETAB(1,I)*(C1 - J*C2*ALFAB(L))
     .       + BETAB(2,I)*(C3 - J*ALFAB(L)*C4)
     .       + BETAB(3,I)*(C5 - J*ALFAB(L)*C6)
     .       + BETAB(4,I)*(C7 - J*ALFAB(L)*C8))
     .       + CONJG(BETAB(2,K))*(BETAB(1,I)*(C9 - J*ALFAB(L)*C10)
     .       + BETAB(2,I)*(C11 - J*ALFAB(L)*C12)
     .       + BETAB(3,I)*(C13 - J*ALFAB(L)*C14)
     .       + BETAB(4,I)*(C15 - J*ALFAB(L)*C16))
     .       + CONJG(BETAB(3,K))*(BETAB(1,I)*(C17 - J*ALFAB(L)*C18)
     .       + BETAB(2,I)*(C19 - J*ALFAB(L)*C20)
     .       + BETAB(3,I)*(C21 - J*ALFAB(L)*C22)
     .       + BETAB(4,I)*(C23 - J*ALFAB(L)*C24)))
  230 CONTINUE
      P1FUN = P1FUN/2.
      RETURN
      END
```

```
$IBFTC CDET..  DECK

CCDET          SUBROUTINE CDET          6-12-68                  CDET0000
C          DETERMINANT OF COMPLEX MATRIX.     DET(A) = F * 10**M   CDET0001
      SUBROUTINE CDET(A,N,F,M)                                   0001
      DATA PI/3.14159265/                                        0002
      DIMENSION A(N,N),S(2)                                      0003
      COMPLEX A,F,B                                              0004
      EQUIVALENCE (S,PHI,B),(S(2),SUM)                           0005
      ALOGB2(X)=ALOG(X)/ALOG(2.)                                       2     3
      SGN = 1.                                                   0006
 6    DO 15 I=2,N                                                0007
      II=I-1                                                     0008
      DO 15 J=1,II                                               0009
 8    IF(REAL(A(I,J)).EQ.0..AND.AIMAG(A(I,J)).EQ.0.) GO TO 15    0010
 9    IF (REAL(CABS(A(J,J))) - REAL(CABS(A(I,J)))) 11,10,10      0011  20    22
 11   DO 12 K=J,N                                                0012
      B = A(J,K)                                                 0013
      A(J,K)=A(I,K)                                              0014
 12   A(I,K) = B                                                 0015
      SGN = - SGN                                                0016
      IF (REAL(A(I,J)).EQ.0..AND.AIMAG(A(I,J)).EQ.0.) GO TO 15   0017
 10   B = A(I,J)/A(J,J)                                          0018
      JJ=J+1                                                     0019
      DO 14 L=JJ,N                                               0020
 14   A(I,L) = A(I,L) - B*A(J,L)                                 0021
 15   CONTINUE                                                   0022
 16   SUM=0.                                                     0023
      PHI=0.                                                     0024
      DO 20 I=1,N                                                0025
      IF(REAL(A(I,I)).EQ.0..AND.AIMAG(A(I,I)).EQ.0.) GO TO 18    0026
      SUM = SUM + ALOGB2(REAL(CABS(A(I,I))))                     0027  63
 20   PHI=PHI+ATAN2(AIMAG(A(I,I)), REAL(A(I,I)))/PI              0028  67
      SUM = SUM*.301029996                                       0029
      PHI = PI*PHI                                               0030
      F=CMPLX(COS(PHI),SIN(PHI))                                 0031  70    71
      S = ABS(SUM) - 37.                                         0032
      IF (S.LE.0.)  GO TO 100                                    0033
      M = SIGN(S,SUM)                                            0034
      F = F*10.**(SUM-FLOAT(M))                                  0035  76
      GO TO 200                                                  0036
 100  M = 0                                                      0037
      F = F*10.**SUM                                             0038  80
 200  IF (SGN.GT.0.) RETURN                                      0039
      F = - F                                                    0040
      RETURN                                                     0041
 18   M = 0                                                      0042
      F=(0.,0.)                                                  0043
      RETURN                                                     0044
      END                                                        0045
```

```
$IBFTC CMATS.  DECK

CCMATS          SUBRCUTINE CMATS          6-12-68                        CMATS020
C             SOLUTICN OF COPPLEX LINEAR EQUATICNS                       CMATS030
       SUBROUTINE CMATS(A,X,N1,M1,*)                                     CMATS040
       DIMENSION A(N1,50),X(N1,M1)                                       CMATS050
      CCMPLEX          A,X,R                                             CMATS060
       M=M1                                                              CMATS070
       N=N1                                                              CMATS080
      IF (M)  199,199,5                                                  CMATS090
 5    NM1 = N - 1                                                        CMATS100
      IF (NM1)  199,160,6                                                CMATS110
 6    MM=N+M                                                             CMATS120
      II = 1                                                             CMATS130
      DC 155 I=2,N                                                       CMATS140
      DC15 J=1,II                                                        CMATS150
      IF (REAL(A(I,J)) .EQ. 0. .AND. AIMAG(A(I,J)) .EQ. 0.)  GO TO 15    CMATS160
      IF (     CABS(A(J,J)) -      CABS(A(I,J)))   11,10,10              CMATS170    22    24
 11   DC 12 K=J,MM                                                       CMATS180
      R = A(J,K)                                                         CMATS190
      A(J,K)=A(I,K)                                                      CMATS200
 12   A(I,K) = R                                                         CMATS210
      IF (REAL(A(I,J)) .EQ. 0. .AND. AIMAG(A(I,J)) .EQ. 0.)  GO TO 15    CMATS220
 10   R = -A(I,J)/A(J,J)                                                 CMATS230
      JJ = J + 1                                                         CMATS240
      DC14 K=JJ,MM                                                       CMATS250
 14   A(I,K) = A(I,K) + R*A(J,K)                                         CMATS260
      A(I,J) = (0.,0.)                                                   CMATS270
  15 CCNTINUE                                                            CMATS280
 155  II = I                                                             CMATS290
 160  DC 166 I=1,N                                                       CMATS300
 166  IF (REAL(A(I,I)) .EQ.0. .AND. AIMAG(A(I,I)) .EQ. 0.)  RETURN 1     CMATS310
      DC 28 J=1,M                                                        CMATS320
      KK=N+J                                                             CMATS330
      X(N,J)=A(N,KK)/A(N,N)                                              CMATS340
      IF (NM1)  287,28,287                                               CMATS350
 287  JJ = N                                                             CMATS360
      DC 289 I=2,N                                                       CMATS370
      II = JJ                                                            CMATS380
      JJ = JJ - 1                                                        CMATS390
      R = (0.,0.)                                                        CMATS400
      DO 25 K=II,N                                                       CMATS410
 25   R = R + A(JJ,K)*X(K,J)                                             CMATS420
 289  X(JJ,J) = (A(JJ,KK) - R)/A(JJ,JJ)                                  CMATS430
  28  CCNTINUE                                                           CMATS440
      RETURN                                                             CMATS450
 199  RETURN 1                                                           CMATS460
      END                                                                CMATS470
```

```
$IBFTC CVRDK   DECK
       SUBRCUTINE OVERCK
       CCMMON/OVR/KO,NI
       ECUIVALENCE (IAND,XAND)
       CCMMCN KOUNT(1)
       CATA MASK/C000000077777/
       KC=0
       XANC=AND(MASK,KOUNT(11))
       KNTFPT=IAND+239
       KATSAV=KOUNT(KNTFPT)
       KNT1=KNTSAV/32768
       KAT2=KNTSAV-32768*KNT1
       IF(KNT2.LE.1)RETURN
       KCUNT(KNTFPT)=32768*KNT1+1
       KO=1
       RETURN
       END
```

```
      LINBO3   CONWAY      PHASE            N4                        12/31/69    000109      PAGE  48
   LINBO3                                STORAGE  MAP

                                          MAIN PROGRAM

                                      COMMCN   VARIABLES

              CCMMON BLOCK      ROTAT            ORIGIN      00001          LENGTH      00001

SYMBOL      LOCATION     TYPE          SYMBOL     LOCATION      TYPE        SYMBOL      LOCATION      TYPE
RCTATE       000C0        L

              CCMMON BLOCK      GET              ORIGIN      00002          LENGTH      00001
GETOUT       00000        L

              CCMMON BLOCK      PLOTS            CRIGIN      00003          LENGTH      00001
ICHECK       00000        L

              CCMMON BLOCK      OVR              CRIGIN      00004          LENGTH      00002
KO           000C0        I            NI          00001         I

              CCMMON BLOCK      ZZTZ             ORIGIN      00006          LENGTH      00052
CC           00000        R            CE          00024         R          CT          00045         R
C11          000CC        R            C13         0C001         R          C14         00002         R
C15          00003        R            C33         00004         R          C34         00005         R
C35          000C6        R            C36         000C7         R          C44         00010         R
C45          00011        R            C46         000C12        R          C55         00013         R
C56          00014        R            C66         00015         R          C16         00016         R
C12          00017        R            C25         00020         R          C26         00021         R
C24          00022        R            C23         00023         R          E11         00024         R
E13          00025        R            E14         00026         R          E15         00027         R
E16          00030        R            E31         00C31         R          E33         00032         R
E34          00033        R            E35         00C34         R          E36         00035         R
E12          00036        R            E32         00037         R          E21         00040         R
E23          00041        R            E24         00042         R          E25         00043         R
E26          00044        R            T11         00C45         R          T13         00046         R
T33          00047        R            T21         00050         R          T23         00051         R

              CCMMON BLOCK      LINK             ORIGIN      00060          LENGTH      00532
ALFA         00000        C            ALFAI       00020         C          EL          00040         C
ALFAA        00350        C            ALFAB       00364         C          BETAA       00374         C
BETAB        00440        C            EPSC        00500         R          MUA         00501         R
LAMCAA       005C2        R            RHOA        00503         R          MUB         00504         R
LAMCAB       005C5        R            NUB         0C506         R          RHOB        00507         R
VS           00510        R            KS          00511         I          EPSLON      00512         R
DIGIT        00513        R            WH          00514         R          WXA         00515         R
WXB          00516        R            KL          00517         I          KM          00520         I
ALL          00521        L            ROOTS       00522         L          ITER        00523         I
CCEFF        00524        L            DETERM      00525         L          POLY        00526         L
ALPHA        00527        L            BETA        00530         L          MAX         00531         I

              CCMMON BLOCK      GPEPS            ORIGIN      00612          LENGTH      00060
G            000CC        R            P           00025         R          EPS         00047         R

              CCMMON BLOCK      FLAG             ORIGIN      00672          LENGTH      00001
 ICE         000CC        L

              CCMMON BLOCK      P TAN            ORIGIN      00673          LENGTH      00001
```

```
                LINB03    CONWAY     PHASE            H4                       12/31/69    000109      PAGE  49
          LINB03                                  STORAGE  MAP

NBETA       00000           I

                 CCMMON BLOCK     CSET            CRIGIN       00674           LENGTH      00003
CLIM        00000           R              ELIP     00001          R           TLIM        00002          R

                 CCMMON BLCCK     FROOT           ORIGIN       00677           LENGTH      00003
FVSMAG      00000           R              NT       00001          I           ICASE       00002          I

                 CCMMON BLOCK     CGM             ORIGIN       00702           LENGTH      00002
ACAP        00000           L              EPSR     00001          R

                 CCMMON BLOCK     CIA             ORIGIN       00704           LENGTH      00004
IA          000CC           I

                 CCMMON BLOCK     ALESS           CRIGIN       00710           LENGTH      00001
IALF        00000           L

                 CCMMON BLOCK     CIFL            ORIGIN       00711           LENGTH      00001
HXAGNL      00000           L
```

DIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
PANGLE	01470	R	XX	01755	R	YY	01757	R
DELTA	01761	R	RTI1	02246	R	RITI1	02533	R
RU1	03020	R	RIU1	03305	R	REI	03572	R
REI1	04057	R	DETRAY	04344	R	DETIAY	04510	R
VSARAY	04654	R	AAAAA	05020	R	DISP	05330	C
DEG	05340	R	TITLES	01464	R	VELOC	00712	R
VELOCI	01177	R	VEL	00712	R	PLOTIT	05625	I
XEL	05633	C	XL	05655	C	XET	05671	C
EA	05675	C	EB	05711	C	UA	05721	C
UB	05727	C	ETA	05735	C	EX	05761	C
MAGU	05771	R	PHASEU	05775	R	TITLE	01464	R

UNDIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
BLIMIT	06001	I	ELIPIT	06002	I	NEGAT	06003	L
MUIT	06004	L	FXL	06005	C	FVS	06007	C
PHIB	06011	C	CXO	06013	C	CX1	06015	C
U	06017	C	PIM	06021	C	P2M	06023	C
TW31	06025	C	TW32	06027	C	TW33	06031	C
TW11	06033	C	TW12	06035	C	TW22	06037	C
S11	06041	C	S22	06043	C	S33	06045	C
S12	06047	C	S13	06051	C	S23	06053	C
D1	06055	C	D2	06057	C	D3	06061	C
JIMAG	06063	C	E1	06065	C	E3	06067	C
NUA	06071	R	NUMAX	06072	R	REPEAT	06073	L
WX	06074	R	DVS	06075	R	VSMAX	06076	R
DNU	06077	R	DWX	06100	R	WXMAX	06101	R
JJJK	06102	I	I	06103	I	VSAVE	06104	R
KOUNT	06105	I	KCNT	06106	I	KNT1	06107	I
KNT2	06110	I	KNT3	06111	I	KTIME	06112	I

LINBO3 STORAGE MAP

SNU	06113	R	SWX	06114	R	COA	06115	R
VS1	06116	R	VS2	06117	R	AN1	06120	R
AN2	06121	R	COB	06122	R	VSO	06123	R
N	06124	I	VSI	06125	R	EL1	06126	R
EL2	06127	R	KGO	06130	I	NBETA1	06131	I
M1	06132	I	K	06133	I	M2	06134	I
J	06135	I	YMIN	06136	R	DY	06137	R
DMIN	06140	R	RTMIN	06141	R	DR	06142	R
SPECL	06143	R	TT31	06144	R	TT32	06145	R
TT33	06146	R	TT11	06147	R	TT12	06150	R
TT22	06151	R	SS11	06152	R	SS22	06153	R
SS33	06154	R	SS12	06155	R	SS13	06156	R
SS23	06157	R	PP1P	06160	R	PP2M	06161	R
DD1	06162	R	DD2	06163	R	DD3	06164	R
UU1	06165	R	UU2	06166	R	UU3	06167	R
EE1	06170	R	EE3	06171	R	RRRR	06172	R
RIII	06173	R	INCR	06174	I	VSO	06175	R
JJ	06176	I	JJJ	06177	I	NNNNN	06200	I
XMIN	06201	R	DX	06202	R			

ENTRY POINTS

......	SECTION	17

SUBROUTINES CALLED

TFUN	SECTION	18	P1FUN	SECTION	19	.FRDD.	SECTION	20
SKFILE	SECTION	21	CRTPLT	SECTION	22	PLOT	SECTION	23
.FRCU.	SECTION	24	SETCTE	SECTION	25	ROOT	SECTION	26
.CFCP.	SECTION	27	.FWRD.	SECTION	28	.FSLO.	SECTION	29
.FPUN.	SECTION	30	CMATS	SECTION	31	CABS	SECTION	32
.CFMP.	SECTION	33	CEXP	SECTION	34	OVERCK	SECTION	35
SCALE	SECTION	36	AXIS	SECTION	37	LINE	SECTION	38
.FEFT.	SECTION	39	ATAN2	SECTION	40	SQRT	SECTION	41
ATAN	SECTION	42	COS	SECTION	43	SIN	SECTION	44
ENDPLT	SECTION	45	.FXEM.	SECTION	46	.UN05.	SECTION	47
.FRTN.	SECTION	48	.FCNV.	SECTION	49	.UN06.	SECTION	50
.FFIL.	SECTION	51	.UN02.	SECTION	52	E.1	SECTION	53
E.2	SECTION	54	E.3	SECTION	55	E.4	SECTION	56
CC.1	SECTION	57	CC.2	SECTION	58	CC.3	SECTION	59
CC.4	SECTION	60	SYSLOC	SECTION	61			

EFN IFN CORRESPONDENCE

EFN	IFN	LOCATION	EFN	IFN	LOCATION	EFN	IFN	LOCATION
7773	FORMAT	06427	7774	7A	07565	20	11A	07570
510	22A	07677	1	FORMAT	06430	520	35A	07725
7	FORMAT	06432	530	45A	07754	535	50A	10006
2	622A	15561	550	58A	10036	17	434A	12754
32	70A	10065	615	74A	10102	650	FORMAT	06435
1500	FORMAT	06433	1502	FORMAT	07342	1707	277A	11630
660	114A	10404	1708	263A	11572	1705	FORMAT	07102
1240	175A	11120	1000	141A	10713	50	122A	10426

LINBO3 LINBO3 CONWAY PHASE N4 STORAGE MAP 12/31/69 000109 PAGE 51

100	131A	10534	60	127A	10473	110	139A	10666
105	137A	10641	174C	451A	13006	1100	152A	11017
1010	145A	10740	1095	146A	10776	1120	FORMAT	06732
1800	460A	13040	1310	188A	11155	1300	184A	11143
1340	195A	11167	149C	239A	11463	1480	237A	11457
1460	223A	11360	152C	245A	11472	1550	FORMAT	0677[illegible]
1706	278A	11636	1710	306A	11716	38	288A	11656
1709	305A	11714	39	292A	11666	34	336A	12106
36	333A	12075	87	363A	12240	86	343A	12113
37	381A	12375	75	3[illegible]5A	12471	76	419A	12654
74	433A	12750	175C	FORMAT	07103	1790	459A	13032
1810	465A	13044	189C	489A	13210	1650	476A	13126
57	502A	13362	71	516A	13542	219C	537A	14016
2310	551A	14345	73	564A	14462	2340	FORMAT	07121
2440	FORMAT	07125	1506	FORMAT	07344	18	591A	15443
72	601A	15502	2441	FORMAT	07347	3	FORMAT	07370
4	640A	15717	32767	FORMAT	07371	5	687A	16041

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 16216.

```
          LINBO3    CCNWAY      PHASE           N4                    12/31/69    000109      PAGE  52
       ROOT..                                STORAGE  MAP

                                       SUBROUTINE  ROOT

                                           COMMON   VARIABLES

               COMMON BLOCK       GET            ORIGIN      00001          LENGTH      00001

SYMBOL    LOCATION     TYPE           SYMBOL     LOCATION      TYPE          SYMBOL     LOCATION      TYPE
GETOUT     00000        L

               COMMON BLOCK       PLOTS          ORIGIN      00002          LENGTH      00001
ICHECK     00000        L

               COMMON BLOCK       FRCCT          ORIGIN      00003          LENGTH      00003
FVSMAG     00000        R                NT        00001         I          ICASE       00002          I

               COMMON BLOCK       ALESS          ORIGIN      00006          LENGTH      00001
IALF       00000        L

                                      DIMENSIONED PROGRAM VARIABLES

SYMBOL    LOCATION     TYPE           SYMBOL     LOCATION      TYPE          SYMBOL     LOCATION      TYPE
SUB        000C7        R

                                     UNDIMENSIONED PROGRAM VARIABLES

SYMBOL    LOCATION     TYPE           SYMBOL     LOCATION      TYPE          SYMBOL     LOCATION      TYPE
PREV       00037        L             FX0         00040         C            FX1         00042         C
FX2        00044        C             FX3         00046         C            G2          00050         C
LAMDA3     00052        C             T           00054         C            T1          00056         C
T2         00060        C             EL          00062         C            LAMDA2      00064         R
KKKKK      00065        I             FACT1       00066         R            FACT2       00067         R
X2         00070        R             H2          00071         R            DELTA2      00072         R
H3         00073        R             KICK        00074         I            TEN10       00075         R

                                             ENTRY POINTS

     ROOT        SECTION      6

                                           SUBROUTINES CALLED

     F           SECTION      7            ROUNC       SECTION      8          .XP2.       SECTION     9
     .CFMP.      SECTION     10            CSQRT       SECTION     11          CABS        SECTION    12
     .CFCP.      SECTION     13            .FWRD.      SECTION     14          .UN06.      SECTION    15
     .FFIL.      SECTION     16            .FCNV.      SECTION     17          E.1         SECTION    18
     E.2         SECTION     19            E.3         SECTION     20          E.4         SECTION    21
     SYSLCC      SECTION     22

                                           EFN   IFN   CORRESPONDENCE

EFN       IFN        LOCATION         EFN       IFN        LOCATION         EFN       IFN        LOCATION
```

```
                 ROOT.. LINB03   CONWAY     PHASE          N4
                                                      STORAGE  MAP            12/31/69   000109     PAGE  53
600          70A         01016             530
300          49A         0C653             10CC        65A         01007        230        39A        00353
999          83A         01060                        FORPAT       00132        998        82A        01055
THE FIRST LOCATIGN NOT USED BY THIS PROGRAM IS 01167.
```

```
        LINB03    CONWAY       PHASE          N4                     12/31/69    000109      PAGE  54
     SETCT.                                STORAGE  MAP

                                        SUBROUTINE  SETCTE

                                            COMMCN   VARIABLES

                 CCMMON BLOCK      ROTAT           ORIGIN       00001           LENGTH       00001

SYMBOL     LOCATICN     TYPE            SYMBOL     LOCATION     TYPE           SYMBOL     LOCATION      TYPE
ROTATE      00000        L

                 CCMMON BLOCK      ZZTZ            ORIGIN       00002           LENGTH       00052
C           00000        R              E           00024        R             T           00045         R

                 CCMMON BLOCK      FRT             ORIGIN       00054           LENGTH       00352
GAMMA       00000        D              D           00022        D             Q           00264         D

                 COMMON BLCCK      GPEPS           ORIGIN       00426           LENGTH       00060
G           00000        R              P           00025        R             EPSLON      00047         R

                 CCMMON BLOCK      CSET            CRIGIN       00506           LENGTH       00003
CLIM        00000        R              ELIM        00001        R             TLIM        00002         R

                 CCMMON BLOCK      CSET1           ORIGIN       00511           LENGTH       00005
AC12        00000        L              AC23        00001        L             AC24        00002         L
AC14        00003        L              AC34        00004        L

                 CCMMON BLOCK      BETAN           ORIGIN       00516           LENGTH       00001
NBETA       00000        I

                                            DIMENSIONED PROGRAM VARIABLES

SYMBOL     LOCATION     TYPE            SYMBOL     LOCATION     TYPE           SYMBOL     LOCATION      TYPE
CPR         00517        R              EPR         00563        R             EPSPR       00605         R
LABE        00616        I              LABC        00624        I             LABT        00631         I

                                          UNDIMENSIONED PROGRAM VARIABLES

SYMBOL     LOCATION     TYPE            SYMBOL     LOCATION     TYPE           SYMBOL     LOCATION      TYPE
DR          00637        D              RN          00641        D             RN          00643         D
RL          00645        D              CN          00647        D             SN          00651         D
CN          00653        D              SN          00655        D             CL          00657         D
SL          00661        D              I           00663        I             K           00664         I

                                              ENTRY POINTS

   SETCTE        SECTION      9

                                              SUBRCUTINES CALLED

   FF            SECTION      10          R             SECTICN      11          TIJ         SECTION      12
   CSFUN         SECTION      13          .FWRC.        SECTICN      14          .FSLO.      SECTION      15
   .UNC6.        SECTION      16          .FFIL.        SECTION      17          .FCNV.      SECTION      18
```

```
              LINBO3   CONWAY     PHASE          N4                      12/31/69   000109     PAGE  55
        SETCT.                                STORAGE  MAP

     SYSLCC      SFCTION     19

                                        EFN   IFN   CORRESPONDENCE

  EFN          IFN         LOCATION         EFN          IFN         LOCATION          EFN          IFN         LOCATION
  1601          52A         02326           1C           60A          02337            1602         101A         02731
  20           109A         02742           1603         129A         03101            30           137A         03112
  470          148A         03125           490          151A         03131            1595         FORMAT       00715
  1604         224A         03430           160C         FORMAT       00702
   THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 03464.
```

```
         LINBO3    CONWAY      PHASE          N4                       12/31/69    000109      PAGE  5b
      ROUND.                                 STORAGE  MAP

                                    FUNCTION    ROUND        TYPE     C

                                        UNDIMENSIONED PROGRAM VARIABLES

SYMBOL      LOCATICN      TYPE           SYMBOL       LOCATION      TYPE           SYMBOL       LOCATION      TYPE
F.000C       000C1          C              I             00003         R              R            00004          R

                                                ENTRY POINTS

     ROUND       SECTION     2

                                                SUBROUTINES CALLED

     SYSLOC      SECTION     3

                                          EFN    IFN    CORRESPONDENCE

EFN          IFN          LOCATION        EFN         IFN          LOCATION          EFN          IFN          LOCATION
100           12A          0C052           50          9A            00041
 THE FIRST LOCATICN NOT USED BY THIS PROGRAM IS 00C77.
```

```
CSFUN. LINB03   CONWAY     PHASE          :4                        12/31/69    000109     PAGE  57
                                     STORAGE  MAP

                              SUBROUTINE  CSFUN

                                 UNDIMENSIONED PROGRAM VARIABLES

SYMBOL      LOCATION     TYPE        SYMBOL      LOCATION     TYPE         SYMBOL      LOCATION     TYPE
X            00001        R

                                      ENTRY POINTS

     CSFUN       SECTION     2

                                      SUBROUTINES CALLED

     CSIN        SECTION     3             DCOS        SECTION     4              SYSLOC      SECTION     5

                                   EFN   IFN   CORRESPONDENCE

EFN          IFN         LOCATION        EFN        IFN        LOCATION         EFN        IFN        LOCATION
150          13A          00057          200        17A         00073           100        12A         00056
THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00137.
```

```
          LINB03   CONWAY      PHASE             N4                         12/31/69    000109      PAGE  58
       FF....                                STORAGE  MAP

                                      FUNCTION     FF           TYPE     D

                                              COMMCN   VARIABLES

            CCMMON BLOCK       FRT               ORIGIN       00001             LENGTH       00352

SYMBOL      LOCATION      TYPE           SYMBOL      LOCATION     TYPE           SYMBOL      LOCATION      TYPE
GAMMA        0000C          D            D            00022         D            Q            00264          D

                                           UNDIMENSIONED PROGRAM VARIABLES

SYMBOL      LOCATICN      TYPE           SYMBOL      LOCATION     TYPE           SYMBOL      LOCATION      TYPE
F.0000       00353          D            R            00355         I            S            00356          I
T            00357          I

                                                  ENTRY POINTS

      FF          SECTION      3

                                                  SUBROUTINES CALLED

      SYSLOC      SECTION      4

                                              EFN   IFN   CORRESPONDENCE

EFN         IFN         LOCATION         EFN         IFN         LOCATION         EFN         IFN         LOCATION
50           11A          0C465
 THE FIRST LOCATICN NOT USED BY THIS PRCGRAM IS 00546.
```

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       R.    LINBO3   CONWAY      PHASE            N4                         12/31/69    000109     PAGE  59
                                                STORAGE  MAP

                                         FUNCTION    R          TYPE    D

                                               COMMON   VARIABLES

              COMMON BLOCK      FRT                 ORIGIN      00001           LENGTH      00352

SYMBOL     LOCATION     TYPE          SYMBOL     LOCATION     TYPE           SYMBOL     LOCATION     TYPE
GAMMA       000C0        D            D           00022        D             G           00264        D

                                          UNDIMENSIONED PROGRAM VARIABLES

SYMBOL     LOCATION     TYPE          SYMBOL     LOCATION     TYPE           SYMBOL     LOCATION     TYPE
F.000C      00353        D            S           00355        I             T           00356        I

                                                  ENTRY POINTS

     R          SECTION     3

                                                  SUBROUTINES CALLED

     SYSLOC     SECTION     4

                                             EFN   IFN   CORRESPONDENCE

EFN         IFN         LOCATION        EFN        IFN         LOCATION        EFN        IFN         LOCATION
50           9A          00443
 THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00515.
```

```
TIJ.   LINB03   CONWAY      PHASE           N4                            12/31/69    000109      PAGE  60
                                          STORAGE  MAP
                                FUNCTION    TIJ         TYPE     D

                                        COMMCN   VARIABLES

              CCMMON BLOCK       FRT              ORIGIN        00001            LENGTH       00352
SYMBOL     LOCATION    TYPE            SYMBOL     LOCATION      TYPE             SYMBOL       LOCATION      TYPE
GAMMA      0C000       D               JUNK       00022         D

              COMMON BLOCK       GPEPS            ORIGIN        00353            LENGTH       00060
G          000C0       R               P          00025         R                EPSLON       00047         R

                                     UNDIMENSIONED PROGRAM VARIABLES
SYMBOL     LOCATICN    TYPE            SYMBOL     LOCATION      TYPE             SYMBOL       LOCATION      TYPE
F.0000     00433       D               R          00435         I

                                          ENTRY PCINTS
      TIJ         SECTION      4

                                          SUBROUTINES CALLED
      SYSLOC      SECTION      5

                                      EFN   IFN   CORRESPONDENCE
EFN        IFN         LOCATION        EFN        IFN           LOCATION         EFN          IFN           LOCATION
50         7A          0C500
THE FIRST LOCATICN NOT USED BY THIS PROGRAM IS 00545.
```

```
         LINBO3    CONWAY      PHASE          N4                    12/31/69     000109      PAGE  61
     F.....                                 STORAGE  MAP

                                   FUNCTION    F          TYPE     C

                                         COMMON   VARIABLES

               COMMON BLOCK      ZZTZ           ORIGIN      00001          LENGTH      00052

SYMBOL       LOCATION      TYPE         SYMBOL        LOCATION      TYPE          SYMBOL       LOCATION      TYPE
CC            00000         R            CE            00024         R             CT           00045         R
C11           00000         R            C13           00001         R             C14          00002         R
C15           00003         R            C33           00004         R             C34          00005         R
C35           00006         R            C36           00007         R             C44          00010         R
C45           00011         R            C46           00012         R             C55          00013         R
C56           00014         R            C66           00015         R             C16          00016         R
E11           00024         R            E13           00025         R             E14          00026         R
E15           00027         R            E16           00030         R             E31          00031         R
E33           00032         R            E34           00033         R             E35          00034         R
E36           00035         R            T11           00045         R             T13          00046         R
T33           00047         R

               COMMON BLOCK      FROOT          ORIGIN      00053          LENGTH      00003
FVSMAG        00000         R            NT            00001         I             ICASE        00002         I

               COMMON BLOCK      BETAN          ORIGIN      00056          LENGTH      00001
NBETA         00000         I

               COMMON BLOCK      CSETI          ORIGIN      00057          LENGTH      00005
AC12          00000         L            AC23          00001         L             AC24         00002         L
AC14          00003         L            AC34          00004         L

               COMMON BLOCK      LINK           ORIGIN      00064          LENGTH      00532
ALFA          00000         C            ALFAI         00020         C             EL           00040         C
ALFAA         00350         C            ALFAB         00364         C             BETAA        00374         C
BETAB         00440         C            EPSC          00500         R             NUA          00501         R
LAMCAA        00502         R            RHOA          00503         R             NUB          00504         R
LAMCAB        00505         R            NUB           00506         R             RHOB         00507         R
VSX           00510         R            KS            00511         I             EPSLON       00512         R
DIGIT         00513         R            WH            00514         R             WXA          00515         R
WXB           00516         R            KL            00517         I             KM           00520         I
ALL           00521         L            ROCIS         00522         L             ITER         00523         L
COEFF         00524         L            DETERM        00525         L             POLYN        00526         L
ALPHA         00527         L            BETA          00530         L             MAX          00531         I
ELL           00040         C

               COMMON BLOCK      CIA            ORIGIN      00616          LENGTH      00004
IA            00000         I

               COMMON BLOCK      COM            ORIGIN      00622          LENGTH      00002
ACAP          00000         L            EPSR          00001         R

               COMMON BLOCK      GPEPS          ORIGIN      00624          LENGTH      00060
XG            00000         R            XP            00025         R             XEPS         00047         R

               COMMON BLOCK      ALESS          ORIGIN      00704          LENGTH      00001
IALF          00000         L
```

F.....

STORAGE MAP

HXAGNL 00000 COMMON BLOCK L CIFL ORIGIN 00705 LENGTH 00001

DIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
EA	00746	C	EB	00762	C	UA	00772	C
UB	01000	C	POLY	01006	C	B1	00706	C
ALF	01030	C	AGLD	01C50	C	XEL	01072	C
XXEL	014C2	C	BB	01424	C	BB1	01454	C
BETAB1	01476	C	B1A	01506	C	BETAX	01522	C
B1B	01530	C	CA	01570	R	CB	01610	R
CD	01630	R	B1D	00706	R	ALAB	01650	R
IA1	01661	I	IA2	01664	I	LAB	01667	I
B3X	01672	R	NB	01676	I			

UNDIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
F.0000	01702	C	FVS	01704	C	JIMAG	01706	C
DA	01710	C	DB	01712	C	DC	01714	C
ALF1	01716	C	ALF2	01720	C	CXO	01722	C
CX1	01724	C	RVS	01726	R	ICB	01727	I
K1	01730	I	K2	01731	I	L1	01732	I
L2	01733	I	IB1	01734	I	IB2	01735	I
I	01736	I	K	01737	I	RA	01740	R
IC	01741	I	K4	01742	I	J	01743	I
TEN	01744	R	TEN10	01745	R	KK	01746	I
KEXP	01747	I	BMIN	01750	R	JA	01751	I
J1	01752	I	J2	01753	I	N1	01754	I
JB	01755	I	I1	01756	I	I2	01757	I
TERM	01760	R	CA22	01761	R	CAMIN	01762	R
II1	01763	I	II2	01764	I	II3	01765	I
II4	01766	I	L	01767	I	ST	01770	R
ICB1	01771	I	DD	01772	R			

ENTRY POINTS

F	SECTION	12

SUBROUTINES CALLED

.CFMP.	SECTION	13	STRIP	SECTION	14	CRCOT	SECTION	15
CSORT	SECTION	16	.CFDP.	SECTION	17	CABS	SECTION	18
.FWRD.	SECTION	19	CMATS	SECTION	20	CDET	SECTION	21
.CXPI.	SECTION	22	COSH	SECTION	23	SINH	SECTION	24
TANH	SECTION	25	CEXP	SECTION	26	.FXEM.	SECTION	27
.UN06.	SECTION	28	.FFIL.	SECTION	29	.FCNV.	SECTION	30
E.1	SECTION	31	E.2	SECTION	32	E.3	SECTION	33
E.4	SECTION	34	SYSLOC	SECTION	35			

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F.....

STORAGE MAP

EFN IFN CORRESPONDENCE

EFN	IFN	LOCATION	EFN	IFN	LOCATION	EFN	IFN	LOCATION
490	29A	03324	520	54A	03525	510	51A	03522
525	58A	03527	528	FORMAT	02063	789	74A	03563
630	94A	03624	610	87A	03606	640	132A	03732
637	130A	03721	631	107A	03651	634	124A	03710
400	560A	06325	636	126A	03713	140	373A	05340
632	117A	03666	633	FORMAT	02075	1357	739A	10262
280	501A	06042	638	FORMAT	02106	788	FORMAT	02131
810	143A	03766	980	303A	04734	940	286A	04635
830	184A	04170	820	174A	04141	100	322A	05003
200	408A	05451	975	FORMAT	02156	850	204A	04265
840	198A	04246	900	243A	04446	860	212A	04314
870	217A	04333	875	FORMAT	02142	1960	935A	12524
880	233A	04410	890	238A	04430	910	248A	04465
930	270A	04553	920	264A	04537	974	FORMAT	02150
983	622A	06545	982	315A	04762	110	335A	05042
120	346A	05070	125	350A	05107	130	365A	05326
270	605A	06450	150	383A	05370	160	FORMAT	02164
210	428A	05600	220	445A	05633	230	448A	05635
240	461A	05674	235	459A	05664	245	464A	05676
250	475A	05755	255	482A	05766	260	493A	06030
310	522A	06157	300	519A	06147	340	530A	06212
320	532A	06173	360	543A	06230	350	539A	06220
370	FORMAT	02210	430	581A	06407	410	574A	06364
420	578A	06376	450	600A	06441	440	596A	06436
460	603A	06445	984	FORMAT	02263	985	FORMAT	02275
1390	761A	10415	1330	708A	10115	1190	677A	07536
1170	673A	07526	1220	684A	07555	1290	696A	07635
1250	688A	07566	1280	694A	07623	1340	715A	10145
1335	FORMAT	02304	1343	725A	10212	1360	740A	10264
1370	742A	10320	1380	744A	10354	1345	729A	10214
1350	735A	10230	1355	FORMAT	02322	1385	755A	10377
1505	FORMAT	02334	1550	782A	11037	1595	FORMAT	02344
1620	811A	11156	1650	822A	11204	1740	850A	11444
1910	889A	12363	1915	899A	12404	1930	FORMAT	02355
1940	928A	12507	1970	FORMAT	02364			

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 12600.

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     LINBO3   CONWAY      PHASE          N4                        12/31/69    000109      PAGE  64
S RIP.                                STORAGE  MAP

                               SUBROUTINE  STRIP

                                   COMMON   VARIABLES

             COMMON BLOCK    ZZTZ            ORIGIN      00001          LENGTH      00052

SYMBOL     LOCATION     TYPE        SYMBOL     LOCATION     TYPE        SYMBOL     LOCATION     TYPE
CC          00000        R          CE          00024        R          CT          00045        R
C11         00000        R          C13         00001        R          C14         00002        R
C15         00003        R          C33         00004        R          C34         00005        R
C35         00006        R          C36         00007        R          C44         00010        R
C45         00011        R          C46         00012        R          C55         00013        R
C56         00014        R          C66         00015        R          C16         00016        R
E11         00024        R          E13         00025        R          E14         00026        R
E15         00027        R          E16         00030        R          E31         00031        R
E33         00032        R          E34         00033        R          E35         00034        R
E36         00035        R          T11         00045        R          T13         00046        R
T33         00047        R

             COMMON BLOCK    FLAG            ORIGIN      00053          LENGTH      00001
ONCE        00000        L

                                DIMENSIONED PROGRAM VARIABLES

SYMBOL     LOCATION     TYPE        SYMBOL     LOCATION     TYPE        SYMBOL     LOCATION     TYPE
ONES        00054        R          CA          00061        R          CB          00101        R
CD          00121        R

                               UNDIMENSIONED PROGRAM VARIABLES

SYMBOL     LOCATION     TYPE        SYMBOL     LOCATION     TYPE        SYMBOL     LOCATION     TYPE
ZERO        00141        C          ONE         00143        C          JIMAG       00145        C
P           00147        I          RGAM        00150        R          J           00151        I
K           00152        I          L           00153        I          M           00154        I
T1          00155        R          T2          00156        R          T3          00157        R
ZH          00160        R          ZI          00161        R          ZJ          00162        R
ZK          00163        R          ZL          00164        R          ZM          00165        R
ZN          00166        R          ZS          00167        R          ZU          00170        R
ZV          00171        R          ANORM       00172        R          TPI         00173        R

                                      ENTRY POINTS

    STRIP        SECTION      4

                                    SUBROUTINES CALLED

    .CFMP.       SECTION      5          .FWRC.      SECTION      6          CABS        SECTION     7
    .CFCP.       SECTION      8          .UN06.      SECTION      9          .FFIL.      SECTION     10
    .FCAV.       SECTION      11         E.1         SECTION      12         E.2         SECTION     13
    E.3          SECTION      14         E.4         SECTION      15         SYSLOC      SECTION     16
```

LINB03 CCNWAY PHASE N4 12/31/65 000109 PAGE 65
STRIP. STORAGE MAP

EFN IFN CORRESPONDENCE

EFN	IFN	LOCATION	EFN	IFN	LOCATION	EFN	IFN	LOCATION
670	5A	0C376	680	9A	00400	1030	88A	01245
1020	86A	01243	101C	84A	01240	1035	94A	01251
1050	FORMAT	0C216	1155	111A	01321	1157	126A	01371

THE FIRST LOCATICN NOT USED BY THIS PROGRAM IS 01526.

```
          LINB03    CONWAY      PHASE             N4                         12/31/69     000109      PAGE  66
    CRUOT.                                     STORAGE  MAP

                                         SUBROUTINE  CROCT

                                                COMMCN   VARIABLES

             CCMMON BLCCK      ORDER              ORIGIN       00001           LENGTH        00024

SYMBCL        LOCATICN      TYPE            SYMBOL       LOCATION      TYPE             SYMBOL        LOCATION      TYPE
C              00000         C              K             00022         I               M              00023         I

                                               UNDIMENSIONED PROGRAM VARIABLES

SYMBOL        LOCATION      TYPE            SYMBOL       LOCATION      TYPE             SYMBOL        LOCATION      TYPE
X1             00025         C              CXC           00027         C               JUNK           00031         I
N              00032         I

                                                      ENTRY POINTS

     CROCT         SECTION      3

                                                      SUBROUTINES CALLED

     RCUND         SECTION      4                 POL         SECTION      5                 CABS         SECTION      6
     PULLER        SECTION      7                 .CFMP.      SECTION      8                 .CFDP.       SECTION      9
     E.1           SECTION      10                E.2         SECTION      11                E.3          SECTION      12
     E.4           SECTION      13                SYSLOC      SECTION      14

                                                EFN   IFN   CORRESPONDENCE

EFN          IFN          LOCATION          EFN          IFN          LOCATION          EFN          IFN          LOCATION
100           6A           0C063            110           11A          00070            130           30A          00132
200           69A          0C337            120           24A          00124            240           52A          00252
250           67A          00314            290           73A          00341
 THE FIRST LUCATICN NOT USED BY THIS PROGRAM IS 0042Z.
```

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         LINB03   CONWAY     PHASE          A4                    12/31/69    000109     PAGE  67
    MULER.                               STORAGE  MAP

                                  SUBROUTINE  MULLER

                                     UNDIMENSIONED PROGRAM VARIABLES

SYMBOL      LOCATION     TYPE         SYMBOL     LOCATION     TYPE         SYMBOL     LOCATION     TYPE
X1           00001         C          X2          00003         C          X3          00005         C
H2           00007         C          L2          00C11         C          D2          00013         C
G            00015         C          FX          00017         C          FX1         00021         C
FX2          00023         C          D           00025         C          TWO         00027         C
CNE          00031         C          GPD         00033         C          GPD         00035         C
L3           00037         C          FX3         00041         C          H1          00043         C
K            00045         I          CGPD        00046         R          CGMC        00047         R

                                        ENTRY POINTS

     MULLER       SECTION     2

                                        SUBROUTINES CALLED

     CABS         SECTION     3            .CFMP.     SECTION     4            CVERCK     SECTION     5
     CSQRT        SECTION     6            .CFDP.     SECTION     7            E.1        SECTION     8
     E.2          SECTION     9            E.3        SECTICN    10            E.4        SECTION    11
     SYSLOC       SECTION    12

                                   EFN   IFN    CORRESPONDENCE

EFN          IFN         LOCATION      EFN          IFN        LOCATION      EFN          IFN         LOCATION
190          11A          0C173        220          14A          00210       540          52A          01001
360          31A          0C572        580          57A          01023       370          34A          00634
420          47A          0C730        450          51A          00740       390          45A          00725
570          56A          01022
 THE FIRST LOCATICN NOT USED BY THIS PRCGRAM IS 01076.
```

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POL...LINBO3   CONWAY      PHASE          N4                           12/31/65    000109     PAGE 68
                                          STORAGE MAP

                              FUNCTICN    POL        TYPE    C

                                      COMMCN    VARIABLES

              CCMMON BLOCK.  .  ORDER           ORIGIN       00001            LENGTH       00024
SYMBOL      LOCATICN     TYPE           SYMBOL      LOCATION     TYPE         SYMBOL      LOCATION      TYPE
C            00000        C             K            00022        I           M            00023         I

                                    UNDIMENSIONED PROGRAM VARIABLES
SYMBOL      LOCATION     TYPE           SYMBOL      LOCATION     TYPE         SYMBOL      LOCATION   .  TYPE
F.0000       00025        C             T            00027        C           J            00031         I

                                          ENTRY PCINTS
     PCL          SECTION      3

                                          SUBROUTINES CALLED
     .CFMP.       SECTION      4
     E.2          SECTION      7        OVERCK      SECTION     5          E.1         SECTION     6
     SYSLCC       SECTION     10        E.3         SECTION     8          E.4         SECTION     9

                                    EFN   IFN   CORRESPONDENCE
EFN        IFN        LOCATION        EFN        IFN        LOCATION        EFN        IFN        LOCATION
100         9A         0C124
THE FIRST LOCATICN NOT USED BY THIS PROGRAM IS 00150.
```

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                 LINBO3    CONWAY      PHASE          N4                        12/31/69    000109      PAGE 69
          TFUN..                                   STORAGE  MAP

                                         FUNCTION   TFUN        TYPE     C

                                                 COMMCN   VARIABLES

                CCMMON BLOCK        LINK              ORIGIN        00001             LENGTH       00532

SYMBOL        LOCATICN      TYPE             SYMBOL        LOCATION      TYPE              SYMBOL        LOCATION       TYPE
JUNK1           0000C         I              ALFAB           00364         C               JUNK2           00374          I
BETAB           00440         C              JUNK3           00500         I               VS              00510          R
JUNK4           00511         I

                CCMMON BLOCK        CIA               ORIGIN        00533             LENGTH       00004
IA              00000         I

                                               UNDIMENSIONED PROGRAM VARIABLES

SYMBOL        LOCATICN      TYPE             SYMBOL        LOCATION      TYPE              SYMBOL        LOCATION       TYPE
F.000C          00537         C              I               00541         I

                                                     ENTRY POINTS

      TFUN          SECTION      4

                                                   SUBROUTINES CALLED

      CEXP          SECTION      5                  .CFMP.       SECTION      6                   .CFDP.       SECTION      7
      E.1           SECTION      8                  E.2          SECTION      9                   E.3          SECTION     10
      E.4           SECTION     11                  SYSLOC       SECTION     12

                                                  EFN    IFN    CORRESPONDENCE

EFN          IFN          LOCATION           EFN          IFN          LOCATION            EFN          IFN          LOCATION
80           7A           0C[illegible]
 THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 01232.
```

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        LINBO3   CONWAY     PHASE           N4                          12/31/65   000109     PAGE  79
PIFUN.                                STORAGE  MAP

                                 FUNCTION    PIFUN       TYPE    C

                                         COMMCN   VARIABLES

            CCMMON BLOCK      LINK                ORIGIN       00001          LENGTH        00532

SYMBOL    LOCATION    TYPE            SYMBOL     LOCATION     TYPE           SYMBOL     LOCATION     TYPE
JUNK1      00000       I              ALFAB       00364        C             JUNK2       00374        I
BETAB      00440       C              JUNK3       00500        I

            CCMMON BLOCK      BETAN               ORIGIN       00533          LENGTH        00001
NBETA      0000C       I

            CCMMON BLOCK      CIA                 ORIGIN       00534          LENGTH        00004
IA         000C0       I

                                   UNDIMENSIONED PROGRAM VARIABLES

SYMBCL    LOCATICN    TYPE            SYMBOL     LOCATION     TYPE           SYMBOL     LOCATION     TYPE
F.0000     0054U       C              J           00542        C             NK4         00544        I
I          00545       I              L           00546        I             M           00547        I

                                          ENTRY POINTS

    PIFUN        SECTION     5

                                        SUBROUTINES CALLED

    .CFMP.       SECTION     6          .CFDP.       SECTION    7           E.1        SECTION    8
    E.2          SECTION     9          E.3          SECTION    10          E.4        SECTION    11
    SYSLCC       SECTION     12

                                      EFN   IFN   CORRESPONDENCE

EFN        IFN       LOCATION      EFN        IFN        LOCATION         EFN        IFN        LOCATION
230        33A        02102
THE FIRST LOCATICN NOT USED BY THIS PROGRAM IS 02233.
```

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              LINBO3   CONWAY      PHASE         N4                    12/31/65   000109    PAGE  71
         CDET..                                STORAGE  MAP
                                     SUBROUTINE  CDET

                                   DIMENSIONED PROGRAM VARIABLES

   SYMBOL      LOCATION     TYPE         SYMBOL     LOCATION     TYPE         SYMBOL     LOCATION     TYPE
   S            00001        R

                                  UNDIMENSIONED PROGRAM VARIABLES

   SYMBOL      LOCATION     TYPE         SYMBOL     LOCATION     TYPE         SYMBOL     LOCATION     TYPE
   B            00001        C           PHI         00001        R           SUM         00002        R
   SGN          000C3        R           I           00004        I           II          00005        I
   J            00006        I           JJ          00057        I           PI          00010        R

                                           ENTRY POINTS

        CDET        SECTION     2

                                          SUBROUTINES CALLED

        ALOG        SECTION     3             CABS       SECTION     4            .CFDP.     SECTION     5
        .CFMP.      SECTION     6             ATAN2      SECTION     7            CCS        SECTION     8
        SIN         SECTION     9             .XP3.      SECTION    10            E.1        SECTION    11
        E.2         SECTION    12             E.3        SECTION    13            E.4        SECTION    14
        CC.1        SECTION    15             CC.2       SECTION    16            CC.3       SECTION    17
        CC.4        SECTION    18             SYSLOC     SECTION    19

                                       EFN   IFN   CORRESPONDENCE

   EFN         IFN         LOCATION      EFN         IFN         LOCATION      EFN         IFN         LOCATION
   6            6A          00073        15           49A         00335        8            13A         00150
   9            18A         00163        11           24A         00212        10           38A         00253
   12           30A         00231        14           44A         00302        16           53A         00343
   20           64A         00410        18           84A         00563        100          78A         00530
   200          81A         00547
    THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00703.
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        LINBO3    CCNWAY      PHASE           N4                       12/31/69     000109      PAGE  72
  CMATS.                                    STORAGE  MAP

                                    SUBROUTINE  CMATS

                                       UNDIMENSIONED PROGRAM VARIABLES

SYMBOL        LOCATICN       TYPE        SYMBOL        LOCATION       TYPE        SYMBOL        LOCATION       TYPE
R              00001          C          M              00003          I          N              00004          I
AM1            00005          I          MM             00006          I          II             00007          I
I              00010          I          J              00011          I          JJ             00012          I
KK             00013          I

                                              ENTRY POINTS

      CMATS        SECTION      2

                                              SUBROUTINES CALLED

      CABS         SECTION      3            .CFDP.       SECTICN      4             .CFMP.       SECTION      5
      E.1          SECTION      6            E.2          SECTION      7             E.3          SECTION      8
      E.4          SECTION      9            SYSLOC       SECTION      10

                                          EFN   IFN   CORRESPONDENCE

EFN          IFN         LOCATION        EFN          IFN         LOCATION        EFN          IFN         LOCATION
199          98A          0C610          5            5A           00050          160          58A          00327
6            8A           0CC56          155          54A          00323          15           52A          00317
11           26A          0C174          10           40A          00233          12           32A          00213
14           46A          00266          166          63A          00336          28           95A          00604
287          78A          0C443          289          90A          00527          25           86A          00502

THE FIRST LCCATICN NOT USED BY THIS PROGRAM IS CC733.
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         LINBO3   CONWAY      PHASE            N4                        12/31/65    000109      PAGE  73
      OVRDK                                STORAGE  MAP

                                  SUBROUTINE  OVERCK

                                        COMMCN   VARIABLES

              CCMMON BLOCK      OVR            CRIGIN      00001            LENGTH       00002

SYMBCL      LOCATICN      TYPE           SYMBOL      LOCATION      TYPE           SYMBOL      LOCATION      TYPE
KO           000C0         I             NI           00001         I

              COMMON BLOCK      //             ORIGIN      00003            LENGTH       00001
KCUNT       0000G          I

                                     UNDIMENSIONED PROGRAM VARIABLES

SYMBOL      LOCATICN      TYPE           SYMBOL      LOCATION      TYPE           SYMBOL      LOCATION      TYPE
IAND         000C4         I             XAND         00004         R             MASK         00005         I
KNTFPT       00006         I             KNTSAV       00007         I             KNT1         00010         I
KNT2         00011         I

                                          ENTRY POINTS

     CVERCK       SECTION      4

                                          SUBRCUTINES CALLED

     SYSLCC       SECTION      5

                                    EFN    IFN   CORRESPONDENCE

EFN          IFN          LOCATION          EFN          IFN          LOCATION          EFN          IFN          LOCATION
 THE FIRST LOCATICN NOT USED BY THIS PROGRAM IS 00103.
                                    COMPILATION       LINB03            DIAGNOSTIC    MESSAGES

1      SCURCE ERRCR   13   LEVEL 1 - WARNING CNLY
THIS STATEMENT CANNOT BE REACHED.

2      SCURCE ERROR   20   LEVEL 1 - WARNING CNLY
THE PROGRAM SHOULD END WITH A TRANSFER. A RETURN HAS BEEN GENERATED.
                                    COMPILATION       ROOT..            DIAGNOSTIC    MESSAGES

1      SCURCE ERRGR   13   LEVEL 1 - WARNING CNLY
THIS STATEMENT CANNCT BE REACHED.

                                  PHASE 8 DIAGNOSTIC MESSAGES
```

```
2    SOURCE ERROR   290  LEVEL 1 - WARNING ONLY
     DATA STATEMENT    5     GROUP 1
     VARIABLE NAME -   TEN10  APPEARS ONLY IN DATA STATEMENT.
                                    COMPILATION      STRIP.          DIAGNOSTIC   MESSAGES

                                PHASE 8 DIAGNOSTIC MESSAGES

1    SOURCE ERROR   290  LEVEL 1 - WARNING ONLY
     DATA STATEMENT    1     GROUP 1
     VARIABLE NAME -   TPI    APPEARS ONLY IN DATA STATEMENT.
```

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IBLDR   CONWAY      PHASE          N4                         12/31/69    000109      PAGE  5

                          * MEMORY MAP *

SYSTEM                                      00000 THRU  027.7
FILE BLCCK ORIGIN                           G2720
FILES          1.       UNIT06              (NO BUFF POOL ATTACHED)
               2.       UNIT01              (NO BUFF POOL ATTACHED)
               3.       UNIT02              (NO BUFF POOL ATTACHED)
               4.       UNIT03              (NO BUFF POOL ATTACHED)
               5.       UNIT04              (NO BUFF POOL ATTACHED)
               6.       UNIT05              (NO BUFF POOL ATTACHED)
               7.       UNIT07              (NO BUFF POOL ATTACHED)
               8.       UNIT08              (NO BUFF POOL ATTACHED)
               9.       UNIT09              (NO BUFF POOL ATTACHED)
              10.       UNIT10              (NO BUFF POOL ATTACHED)
              11.       UNIT39              (NO BUFF POOL ATTACHED)
PRE-EXECUTION INITIALIZATION                03124
CALL CN CBJECT PRCGRAM                      C3133
OBJECT PRCGRAM                              03140 THRU  73565

     CECK  ORIGIN        CCNTROL SECTIONS  (/NAME/=NCN 0 LENGTH, (LOC)=DELETED, *=NOT REFERENCED)

 1. LINB03  03140        /ROTAT / 03141    /GET   / 03142    /PLOTS / 03143    /OVR   / C3144    /ZZTZ  / 03146
                         /LINK  / 03220    /GPEPS / 03752    /FLAG  / 04032    /BETAN / 04033    /CSET  / 04034
                         /FROOT / 04037    /COM   / 04042    /CIA   / 04044    /ALESS / C4050    /CIFL  / 04051
                          ......  21334 *
 2. ROOT..  21356        /GET   /(03142)   /PLOTS /(03143)   /FROOT /(04037)   /ALESS /(04050)    ROOT     22444
 3. SETCT.  22537        /RCTAT /(03141)   /ZZTZ  /(03146)   /FRT   / 22540    /GPEPS /(03752)   /CSET  /(04034)
                         /CSET1 / 23112    /BETAN /(04033)    SETCTE  26033
 4. RCUNC.  26064         RCUND   26145
 5. CSFUN.  26163         CSFUN   26274
 6. FF....  26322        /FRT   /(22540)    FF       26462
 7. R.      26516        /FRT   /(2254G)    R        26631
 8. TIJ.    26661        /FRT   /(22540)   /GPEPS /(03752)    TIJ      26750
 9. F.....  26774        /ZZTZ  /(03146)   /FRCOT /(04037)   /BETAN /(04033)   /CSET1 /(23112)   /LINK  /(03220)
                         /CIA   /(04044)   /COM   /(04042)   /GPEPS /(03752)   /ALESS /(C4050)   /CIFL  /(04051)
                          F       40631
10. STRIP.  40667        /ZZTZ  /(03146)   /FLAG  /(04032)    STRIP    42237
11. CROOT.  42342        /ORDER / 42343     CROOT    42715
12. MULER.  42764         MULLER  44014
13. PCL...  44062        /ORDER /(42343)    POL      44173
14. TFUN..  44206        /LINK  /(03220)   /CIA   /(04044)    TFUN     44636
15. PIFUN.  44702        /LINK  /(03220)   /BETAN /(04033)   /CIA   /(04044)    PIFUN    46271
16. CDET..  46376         CDET    47246
17. CPATS.  47301         CPATS   50215
18. CVRDK   50234        /OVR   /(03144)   ///    /(77776)    OVERCK   50323
19. .LXCCN  50334         .LXSTR  50334     .LXSIP   50337    .LXOUT   50361 *   .LXERR   50370     .LXCAL   50373
                          .LXRTN  50373 *   IGEXIT   50373    .DBCLS   50553 *   ..PLOL   50557 *   .LXARG   50643 *
                          .LO     50652 *   .CLSE    50660 *  .LFBL    50661 *   .LUNB    50662 *   .DFOUT   50663
20. .LXSL   50667         .LXSEL  50667     .LXSL1   5C670    .LXTST   50673     .LXOVL   50733     .LXMOD   50775
                          .LXIND  51021     .LXDIS   51024 *  .LXFLG   51025 *   .LTCH    51026
21. .FPTRP  51034         .FFPT.  51034 *   .FXEM.   51C36    .FXEM    51036 *   .ETAG    51265 *   .FMCRT   51401 *
                          .CGOTO  51414 *   .FXOUT   51420 *  .FXARG   51432 *   .EXIT.   51434 *   EXIT     51434
                          .CVCHK  51441 *   OVFLOW   51442 *  SYSCNE   51467     .NOP     51470     .BLANK   51503
22. .CRAS.  51520         E.1     51520     E.2      51521    E.3      51522     E.4      51523
23. .XCC.   51524         CC.1    51524     CC.2     51525    CC.3     51526     CC.4     51527
24. FCNV    51530         .FCON.  51530     .DCPRE   5154G *  .FCNV.   51546     .ENDFS   51557     .CNVSW   51561
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IBLUR   CONWAY      PHASE            N4                          12/31/69    000109      PAGE  76

                      .FDX1    51565       .FDX2    51566       .DBC     51570       .DBC14   51633 *     .STOP4   51636 *
                      .DBC20   51652       .DBC10   51666       .DBC99   51701 *     .DDSW    51724 *     .FIXSW   51733
                      .CXPSE   51747 *     .DDZET   52054 *     .IC10    52115 *     .STOPJ   52132       .FCOUT   52146 *
                      .FCARG   52154 *     .ALCOD   52156       .ANPT    52167       .ONPT    52204       .LNTP    52260
                      .AOUT    52317       .OCUT    52331       .LOUT    52360       .GOUT    52370       .TGOUT   52377 *
                      .FLT     52477       .FXFL1   52634       .FXD     52640       .FXFL2   52644       .FXFL3   52650
                      .INTG    52654       .TCPAC   52672       .WIDTH   52676       .FPACK   52703       .TEST    52704
                      .DDSF    52735 *     .KCUN    52736       .LIST    52741       .DCNE    52750       .CHAR    53310
                      .DEXP    53317       .TEN     53323 *     .FBDBF   53325       .DATUM   53333 *     .WORD    53346
                      .MQD     53347       .PEX     53350       .FEXP    53351       .DIG     53352       /DDDDDD/ 53365
25. FICB     53442    .FIOB.   53442       .FCNT    53545 *     .FBLT.   53641       .FBDT.   53661 *     .FRLR.   53705 *
                      .FWLR.   53746       .FRITE   54C27       /EEEEEE/ 54035
26. FIOH     54036    .FIOH.   54036       .DCIO    54C63 *     .DDLET   54221 *     .DCPT1   54424 *     .FXPT    54522 *
                      .FFIL.   54653       .DDFIN   54671 *     .FRTN.   54700       .DDRTN   54704 *     .FILL2   54740
                      .FILT    54746       .FILOD   54772       /DDDDDD/(53365)
27. FIOS.    550C6    ..FIOS   55006       .FCLS    55030       ..FIOC   55065       ..FSEL   55106       ..FBCK   55125
                      ..FTCK   55127       .ICEF    55217 *     ..XEM    55225 *     SETECF   55242 *     ..BBUF   55270
                      .FB18F   55271       .OUTEF   55272       ..FCHK   55273       /EEEEEE/(54035)     /DDDDDD/(53365)
28. FBCD.    55376    ..FBCD   55376       ..FBCW   55400       .UN99.   55411       ..RERD   55412       .RRDBF   55413
                      SETDBF   55414 *     /DDDDDD/(53365)     /RRRRRR/ 55443
29. FRDD.    55471    ..FRDD   55471       REREAD   55550 *     /RRRRRR/(55443)     /DDDDDD/(53365)     /EEEEEE/(54035)
30. FWRD.    55564    ..FWRD   55564       /RRRRRR/(55443)     /DDDDDD/(53365)
31. FBIN.    55634    ..FBID   55634 *     ..FBIW   55636       ..FWPR   55651       ..FWSC   55652       SETBBF   55653
                      /BBBBBB/ 55670       ..FB18   55671 *
32. FWRB.    56671    ..FWRB   56671       /BBBBBB/(55670)
33. FEFT.    56737    ..FEFT   56737
34. FVIO.    5676C    ..FVIO   56760
35. UNITC6   57024    ..UN06   57024
36. FRDU.    57025    ..FRDU   57025       /DDDDDD/(53365)
37. .CUMCB   57072    .VD      57072       .WRODB   57074 *     .WRODR   57076 *
38. UNO1     57100    .UNO1.   5710C
39. UNC2     571C1    .UNO2.   57101
40. UNO3     571C2    .UNO3.   57102
41. UNC4     57103    .UNO4.   57103
42. UNC5     571C4    .UNO5.   57104
43. UNC6     571C5    .UNO6.   57105
44. UNC7     571C6    .UNO7.   57106
45. UNC8     57107    .UNO8.   571C7
46. UNC9     57110    .UNO9.   5711C
47. UN10     57111    .UN10.   57111
48. FICU     57112    .FIOU.   57112       .CTUIC   57551       .NMLST   57574       .NAME.   60615       .INTAP   60616
49. FPUN     60737    .FPUN.   6C737       /DDDDDD/(53365)
50. FLCG     60774    ALOG10   60774       ALGG     6C775
51. FXPF     61151    EXP      61151
52. FSCN     61254    COS      61254       SIN      61255
53. FSQR     6143C    ASQRT    61430 *     SQRT     61433
54. FTNH     61474    TANH     61474
55. FATN     61566    ATAN2    61566       ATAN     61567
56. FXP1     620C6    .XP1.    62006
57. .ICE.    62112
58. .ICE56   62124
59. .RWBOE   62132
60. .RWCCE   62153
61. FSLOC    62173    .FSLO.   62211       .FSDO.   62217 *
62. FSLC     62230    .SLO.    62230       .SLO2.   62236       .SDO.    62243       .SDC2.   62252
63. PP1.     62264    PLTID.   62264       AGAIN    62345       /XCPIDX/ 62357 *     DOT.     62372       NAME.    62425
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            IBLDK    CONWAY      PHASE           N4                      12/31/69    000109     PAGE  77

64. FP2.    62460       QD.     62460    PIDOK.  62462    PINOK.  62466 *  DRWPLT  625C2 *  LINE    62510
                        NUMBER  62512    SCALE   62514    AXIS    62516    SYMBOL  62520    PLOT    62522
                        WHERE   62524    LNE222  62575    LNE222 (62575)
65. PM3.    62733       PLT1.   62733    GETMX.  63072    GETMY.  63074    NOR.    63076    ALLOW.  63104
                        FCT1.   63112    YORG    63126    PL3.    63137    FCT2.   63166
66. WRTEFE  63205
67. PP4.    63361       SYMBL.  63361
68. PM5.    64203       WTE.    64203
69. PP7.    64333       ENDPLT  64333
70. PM8.    64455       CPLOT.  64455
71. FUDGE   64641       FRAME   64641 *  CHEAT   64650 *  SCALXY  64656 *  PLOTS.  64661    PLT2.   64676
                        BEAMST  64763    WHR.    65030    FCTR.   65052    ENTER   65070    NEWFRM  65071
                        FRMCNT  65076
72. PP2.1   65121       GTBEAM  65121 *  STBEAM  65124 *  QPLOT   65144 *  CRTPLT  65154    PLOTID  65173
73. CALCOP  65246       .UN35.  65246 *  CALCMP  65247    /EEEEEE/(54035)  /888888/(55670)
74. PF1.    67426       NBER    67750
75. PF2.    70014       AXS     71072
76. PF3.    71153       LNE     71517
77. PF4.    71605       SCLE    72133
78. FCAB    72212       CABS    72213
79. FCXP    72253       CEXP    72254
80. FCAS    72342       .CFMP.  72343    .CFDP.  72344
81. FCSQ    72455       CSQRT   72456
82. FCSC    72532       DCOS    72533    DSIN    72540
83. FCX1    72724       .CXP1.  72724    .DXP1.  72726 *  DEXP2   72727 *
84. .BSF.   73112       .BSF.  (73112)
85. SKFILE  73130       SKFILE (73130)
86. FXP2    73213       .XP2.   73213
87. FXP3    73306       .XP3.   73306
88. CCSH    73375       CCSH   (73375)
89. SINH    73443       SINH   (73443)
90. .CZDIV  73512       /ACZDIV/ 73512 *
91. .CSNT.  73514       .CSNTL  73514
92. .XPER.  73520       .EXITL  73520    .EXPTL  73524
93. .LOGE.  73530       .LOGER  73530
94. .SNTL.  73536       .SNTL   73536
95. .SQRTN  73542       .SQRTN (73542)
96. .TAZ    73545       .TAZ   (73545)
97. .TLEX.  73550       .TLEXP  73550
98. .XEXP.  73554       .NGDEF  73554    .XEXP   73560    .ZUDEF  73563
99. //      77776

UNUSED CORE                        73566 THRU  77775

  BEGIN EXECUTION.

    TRANSFORMED ELASTIC CONSTANTS    ( C(I), I=1,20 )

      2.45C0000E 11
      7.5CC0000E 1C
      0.
      C.
      2.03C0000E 11
      0.
      9.CC00000E C9
      0.
      7.5CCC000E 10
      0.
     -9.COC0000E CS
      6.CCC0000E 1C
```

```
 0.
 6.COC0000E 10
 0.
 7.5CC0000E 10
-9.CGCC000E C9
 0.
 C.
 5.30CC000E 10

TRANSFCRMED PIEZOELECTRIC CONSTANTS   ( E(I), I=1,17 )

 1.3CC000OE C0
 2.COC0000E-C1
 0.
 C.
 0.
 C.
-2.5CCC000E 00
 0.
 3.70CC000E CC
 0.
 2.00C0000E-01
 2.50C00C0E 00
 0.
 0.
 2.5000000E CC
 C.
 3.7000000E 00

TRANSFCRMED CIELECTRIC CONSTANTS      ( T(I), I=1,5 )

 2.570C000E-10
 0.
 3.90C0000E-1C
 C.
 0.

COEFFICIENTS OF FCLYNOMIAL
    -0.6029385E 24      0.
     0.                 0.8562924E 23
     C.8513205E 24      0.
    -C.                -0.3294582E 24
```

```
    -0.6107596E 24      0.
     0.                 0.6860760E 23
     0.1774096E 23      0.
     0.                 0.7146826E 12
    -0.4749200E 21      0.

COEFFICIENTS OF POLYNOMIAL
    -0.7082391E 00      0.
     0.                 0.1005840E 00
     0.1000000E 01     -0.
    -0.                -0.3869567E 00
    -0.7174261E 00      0.
     0.                 0.8058563E-01
     0.2083934E-01     -0.
     0.                 0.
    -0.5578627E-03      0.

INTERMEDIATE ROOTS OF POLYNOMIAL

  -1.63345E-01  1.20000E-01  1.63345E-01  1.20000E-01  7.74633E-01  3.96277E-01  1.28413E-01 -6.48500E-02
  -7.74633E-01  3.96277E-01 -1.28413E-01 -6.48500E-02  1.03875E 00 -3.80417E-01 -1.03875E 00 -3.80417E-01

INTERMEDIATE POSITIVE ROOTS

   1.63345E-01  1.20000E-01  7.74633E-01  3.96277E-01  1.28413E-01 -6.48500E-02  1.03875E 00 -3.80417E-01

RE-ORDERED ALPHAS ( 1 ROW, 3 ZERO CASE )

   1.63345E-01  1.20000E-01  7.74633E-01  3.96277E-01  1.28413E-01 -6.48500E-02  1.03875E 00 -3.80417E-01

INTERMEDIATE BETA B

   0.           0.           2.07710E-11  5.01161E-11 -6.54416E-11  4.25594E-12  2.27313E-11 -6.97380E-11
   1.00000E-10  0.           0.           0.           0.           0.           0.           0.
   0.           0.           2.16897E-11 -5.39077E-11 -2.24545E-10  5.24802E-10 -2.08899E-11 -4.05626E-11
   0.           0.           1.00000E 00  0.           1.00000E 00  0.           1.00000E 00  0.

INTERMEDIATE L MATRIX BY COLUMNS

   0.           0.           1.22509E 00 -2.56000E-08  0.           0.           0.           0.
   6.21789E 00  5.29229E 00  0.           0.           2.50323E 00 -5.34639E 00 -2.20857E 00  1.82708E 00
  -3.14539E 01 -1.26876E 01 -0.          -0.          -4.38118E 00  1.01127E 01 -2.04377E 01 -9.92681E 00
   5.76791E 00 -7.83388E 00  0.           0.          -4.56512E 00 -5.00197E 00 -1.81849E 00 -1.43485E 00

VS =  0.3480000E 04     F(VS) = -0.3593011E 02 -0.1660051E 02     MAG =  0.3957967E 02

COEFFICIENTS OF POLYNOMIAL
    -0.6025385E 24      0.
     0.                 0.8562924E 23
     0.8708338E 24      0.
    -0.                -0.3121240E 24
    -0.6365181E 24      0.
     0.                 0.7365250E 23
     0.3101833E 23      0.
     0.                 0.4026611E 14
    -0.8934538E 21      0.

COEFFICIENTS OF POLYNOMIAL
    -0.6923692E 00      0.
     0.                 0.5833018E-01
```

```
   0.1000000E 01    -0.
  -0.               -0.3613862E 00
  -0.7322261E 00     0.
   0.                0.8457700E-01
   0.3561912E-01    -0.
   0.                0.
  -0.1025975E-02     0.

INTERMEDIATE ROOTS OF POLYNOMIAL

 -2.04412E-01  1.20000E-01  2.04412E-01  1.20000E-01  7.77745E-01  3.98383E-01  1.53931E-01 -6.58000E-02
 -7.77745E-01  3.98383E-01 -1.53931E-01 -6.58002E-02  1.04255E 00 -3.81573E-01 -1.04255E 00 -3.81573E-01

INTERMEDIATE POSITIVE ROOTS

  2.04412E-01  1.20000E-01  7.77745E-01  3.98383E-01  1.53931E-01 -6.58000E-02  1.04255E 00 -3.81573E-01

RE-ORDERED ALPHAS ( 1 ROW, 3 ZERO CASE )

  2.04412E-01  1.20000E-01  7.77745E-01  3.98383E-01  1.53931E-01 -6.58000E-02  1.04255E 00 -3.81573E-01

INTERMEDIATE BETA B

  0.           0.           2.11136E-11  4.98697E-11 -6.37559E-11  4.84614E-12  2.30816E-11 -6.97654E-11
  1.00000E-10  0.           0.           0.           0.           0.           0.           0.
  0.           0.           2.11008E-11 -5.41177E-11 -1.55631E-10  4.54277E-10 -2.07061E-11 -4.08236E-11
  0.           0.           1.00000E 00  0.           1.00000E 00  0.           1.00000E 00  0.

INTERMEDIATE L MATRIX BY COLUMNS

  0.           0.           1.53309E 00 -2.56000E-08  0.           0.           0.           0.
  6.25920E 00  5.26568E 00  0.           0.           2.47791E 00 -5.43538E 00 -2.19642E 00  1.81600E 00
 -2.72034E 01 -8.56332E 00 -0.          -0.          -3.71741E 00  1.05007E 01 -1.79968E 01 -7.32309E 00
  5.81894E 00 -7.85857E 00  0.           0.          -4.57388E 00 -5.07106E 00 -1.80946E 00 -1.42852E 00

VS =  0.3445200E 04     F(VS) = -0.1975967E 03 -0.7503187E 02     MAG =  0.2113628E 03

COEFFICIENTS OF POLYNOMIAL
    -0.6029385E 24     0.
     0.                0.8562924E 23
     0.8611016E 24     0.
    -0.               -0.3307944E 24
    -0.6246250E 24     0.
     0.                0.7113014E 23
     0.2433599E 23     0.
    -0.                0.
    -0.6680190E 21     0.

COEFFICIENTS OF POLYNOMIAL
    -0.7001943E 00     0.
     0.                0.9944150E-01
     0.1000000E 01    -0.
    -0.               -0.3841526E 00
    -0.7253790E 00     0.
     0.                0.8260365E-01
     0.2926146E-01    -0.
     0.                0.
    -0.7757725E-03     0.

INTERMEDIATE ROOTS OF POLYNOMIAL
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```
 -1.85C72E-01  1.20000E-01  1.85C72E-01  1.20000E-01  7.76190E-01  3.97339E-01  1.41843E-01 -6.53300E-02
 -7.76190E-01  3.97339E-01 -1.41843E-01 -6.53301E-02  1.04065E 00 -3.80999E-01 -1.04065E 00 -3.80999E-01

INTERMEDIATE POSITIVE ROOTS

  1.85C72E-01  1.20000E-01  7.76190E-01  3.97339E-01  1.41843E-01 -6.53300E-02  1.04065E 00 -3.80999E-01

RE-ORDERED ALPHAS ( 1 ROW, 3 ZERO CASE )

  1.85072E-01  1.20000E-01  7.76190E-01  3.97339E-01  1.41843E-01 -6.53300E-02  1.04065E 00 -3.80999E-01

INTERMEDIATE BETA B

  0.           0.           2.09434E-11  5.00031E-11 -6.45868E-11  4.58129E-12  2.29070E-11 -6.97519E-11
  1.00000E-10  0.           0.           0.           0.           0.           0.           0.
  0.           0.           2.13941E-11 -5.40149E-11 -1.84521E-10  4.86400E-10 -2.07979E-11 -4.06939E-11
  0.           0.           1.00000E 00  0.           1.00000E 00  0.           1.00000E 00  0.

INTERMEDIATE L MATRIX BY COLUMNS

  0.           0.           1.38804E 00  3.84000E-08  0.           0.           0.           0.
  6.23669E 00  5.28099E 00  0.           0.           2.49075E 00 -5.39115E 00 -2.20244E 00  1.82153E 00
 -2.91404E 01 -1.02914E 01 -0.          -0.          -4.01803E 00  1.03550E 01 -1.91064E 01 -8.41849E 00
  5.79348E 00 -7.84628E 00  0.           0.          -4.56960E 00 -5.03660E 00 -1.81395E 00 -1.43168E 00

VS =  0.3462600E 04     F(VS) = -0.1176182E 03 -0.4881579E 02     MAG =  0.1273461E 03

COEFFICIENTS OF POLYNOMIAL
     -0.6029385E 24      0.
      0.                 0.8562924E 23
      0.8470322E 24      0.
     -0.                -0.3288724E 24
     -0.6047114E 24      0.
      0.                 0.6750561E 23
      0.1468763E 23      0.
      0.                 0.7146826E 12
     -0.4007685E 21      0.

COEFFICIENTS OF POLYNOMIAL
     -0.7118248E 00      0.
      0.                 0.1010933E 00
      0.1000000E 01     -0.
     -0.                -0.3682643E 00
     -0.7139179E 00      0.
      0.                 0.7969664E-01
      0.1757623E-01     -0.
      0.                 0.
     -0.4731444E-03      0.

INTERMEDIATE ROOTS OF POLYNOMIAL

 -1.52848E-01  1.20000E-01  1.52848E-01  1.20000E-01  7.73953E-01  3.95807E-01  1.22012E-01 -6.46371E-02
 -7.73953E-01  3.95807E-01 -1.22012E-01 -6.46372E-02  1.03791E 00 -3.80160E-01 -1.03791E 00 -3.80160E-01

INTERMEDIATE POSITIVE ROOTS

  1.52848E-01  1.20000E-01  7.73953E-01  3.95807E-01  1.22012E-01 -6.46371E-02  1.03791E 00 -3.80160E-01

RE-ORDERED ALPHAS ( 1 ROW, 3 ZERO CASE )
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 1.52848E-01  1.20000E-01  7.73953E-01  3.95807E-01  1.22012E-01 -6.4637.5-02  1.03791E CC -3.80160E-01

INTERMECIATE BETA B

 0.           0.           2.06950E-11  5.01652E-11 -6.58226E-11  4.09002E-12  2.26542E-11 -6.97317E-11
 1.00000E-10  0.           0.           0.           0.           0.           0.           0.
 0.           0.           2.18195E-11 -5.38595E-11 -2.47263E-10  5.44002E-10 -2.09302E-11 -4.05048E-11
 0.           0.           1.00000E 00  0.           1.00000E 00  0.           1.00000E 00  0.

INTERMECIATE L MATRIX BY CCLUMNS

 0.           0.           1.14636E CO  0.           0.           0.           0.           0.
 6.20671E 00  5.29723F 00  0.           0.           2.50857E 00 -5.32668E 00 -2.21128E 00  1.82950E 00
-3.26097E 01 -1.40485E 01 -0.          -0.          -4.56358E 00  9.96079E 00 -2.11046E 01 -1.07797E 01
 5.75671E 00 -7.82842E 00  0.           0.          -4.56311E 00 -4.98678E 00 -1.82049E 00 -1.43625E 00

VS =  0.3487601E 04     F(VS) = -0.4064251E CC -0.1963597E CO     MAG =  0.4522477E 00

CCEFFICIENTS OF PCLYNCMIAL
    -0.6029385E 24      0.
     0.                 0.8562924E 23
     0.E465833E 24      0.
    -C.                -0.3288657E 24
    -0.6046426E 24      0.
     0.                 0.6749308E 23
     0.1455528E 23      0.
    -0.                 0.
    -0.3599609E 21      0.

COEFFICIENTS OF PCLYNOMIAL
    -0.7118658E CO      0.
     C.                 0.1C10591E 00
     0.10C0000E C1     -0.
    -0.                -0.3E82788E 00
    -C.7138779E 00      0.
     0.                 0.7568644E-01
     0.1753904E-C1     -0.
     0.                 0.
    -0.4722181E-C3      0.

INTERMECIATE ROOTS CF POLYNCMIAL

-1.52725F-01  1.20000E-01  1.52725E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21937E-01 -6.46346E-02
-7.73945E-01  3.95802E-01 -1.21937E-01 -6.46348E-02  1.03790E 00 -3.80157E-01 -1.03790E 00 -3.80157E-01

INTERMECIATE POSITIVE ROOTS

 1.52725E-01  1.20000E-01  7.73945E-01  3.95E02E-01  1.21937E-01 -6.46346E-02  1.C3790E 00 -3.80157E-01

RE-ORCERED ALPHAS ( 1 ROW, 3 ZERO CASE )

 1.52725E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21937E-01 -6.46346E-02  1.03790E 0C -3.80157E-01

INTERMECIATE BETA B

 0.           0.           2.C6942E-11  5.01658E-11 -6.58268E-11  4.08805E-12  2.26534E-11 -6.97316E-11
 1.00000E-10  0.           0.           0.           0.           0.           0.           0.
 0.           0.           2.18210E-11 -5.38589E-11 -2.47544E-10  5.44229E-10 -2.09306E-11 -4.05041E-11
 0.           0.           1.00000E CO  0.           1.000C0E 00  0.           1.000C0E 00  0.
```

```
    0.1753852E-01    -0.
    0.                0.
   -0.4722051E-03     0.

INTERMEDIATE ROOTS OF POLYNOMIAL

 -1.52723E-01  1.20000E-01  1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21936E-01 -6.46346E-02
 -7.73945E-01  3.95802E-01 -1.21936E-01 -6.46347E-02  1.03790E 00 -3.80157E-01 -1.03790E 00 -3.80157E-01

INTERMEDIATE POSITIVE ROOTS

  1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21936E-01 -6.46346E-02  1.03790E 00 -3.80157E-01

RE-ORDERED ALPHAS ( 1 ROW, 3 ZERO CASE )

  1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21936E-01 -6.46346E-02  1.03790E 00 -3.80157E-01

INTERMEDIATE BETA B

  0.           0.           2.06942E-11  5.01658E-11 -6.58269E-11  4.08808E-12  2.26533E-11 -6.97316E-11
  1.00000E-10  0.           0.           0.           0.           0.           0.           0.
  0.           0.           2.18210E-11 -5.38589E-11 -2.47548E-10  5.44232E-10 -2.09307E-11 -4.05041E-11
  0.           0.           1.00000E 00  0.           1.00000E 00  0.           1.00000E 00  0.

INTERMEDIATE L MATRIX BY COLUMNS

  0.           0.           1.14542E 00  1.28000E-08  0.           0.           0.           0.
  6.20861E 00  5.29728E 00  0.           0.           2.50863E 00 -5.32645E 00 -2.21131E 00  1.82953E 00
 -3.26236E 01 -1.40656E 01 -0.          -0.          -4.56617E 00  9.95881E 00 -2.11126E 01 -1.07904E 01
  5.75658E 00 -7.82836E 00  0.           0.          -4.56308E 00 -4.98660E 00 -1.82051E 00 -1.43626E 00

VS =  0.3487689E 04     F(VS) =  0.8723089E-04  0.6554125E-04     MAG =  0.1093503E-03
```

```
COEFFICIENTS OF POLYNOMIAL
    -0.6029385E 24      0.
     0.                 0.8562924E 23
     0.8469826E 24      0.
    -0.                -0.3288656E 24
    -0.6046417E 24      0.
     0.                 0.6749291E 23
     0.1465483E 23      0.
    -0.                -0.1870369E 14
    -0.3959498E 21      0.

COEFFICIENTS OF POLYNOMIAL
    -0.7118664E 00      0.
     0.                 0.1010992E 00
     0.1000000E 01     -0.
    -0.                -0.3882790E 00
    -0.7138773E 00      0.
     0.                 0.7568629E-01
     0.1753853E-01     -0.
     0.                 0.
    -0.4722054E-03      0.

INTERMEDIATE ROOTS OF POLYNOMIAL

 -1.52723E-01  1.20000E-01  1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21936E-01 -6.46346E-02
 -7.73945E-01  3.95802E-01 -1.21936E-01 -6.46347E-02  1.03790E 00 -3.80157E-01 -1.03790E 00 -3.80157E-01

INTERMEDIATE POSITIVE ROOTS

  1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21936E-01 -6.46346E-02  1.03790E 00 -3.80157E-01

RE-ORDERED ALPHAS ( 1 ROW, 3 ZERO CASE )

  1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21936E-01 -6.46346E-02  1.03790E 00 -3.90157E-01

INTERMEDIATE BETA B

  0.           0.           2.06942E-11  5.01658E-11 -6.58268E-11  4.08817E-12  2.26533E-11 -6.97316E-11
  1.00000E-10  0.           0.           0.           0.           0.           0.           0.
  0.           0.           2.18210E-11 -5.38589E-11 -2.47547E-10  5.44232E-10 -2.09307E-11 -4.05041E-11
  0.           0.           1.00000E 00  0.           1.00000E 00  0.           1.00000E 00  0.

INTERMEDIATE L MATRIX BY COLUMNS

  0.           0.           1.14542E 00  1.28000E-08  0.           0.           0.           0.
  6.20861E 00  5.29728E 00  0.           0.           2.50863E 00 -5.32645E 00 -2.21131E 00  1.82953E 00
 -3.26236E 01 -1.40655E 01 -0.          -0.          -4.56614E 00  9.95880E 00 -2.11126E 01 -1.07903E 01
  5.75658E 00 -7.82836E 00  0.           0.          -4.56308E 00 -4.98660E 00 -1.82051E 00 -1.43626E 00

VS =  0.3487689E 04     F(VS) =  0.2110807E-03  0.1900636E-04     MAG =  0.2119346E-03

COEFFICIENTS OF POLYNOMIAL
    -0.6029385E 24      0.
     0.                 0.8562924E 23
     0.8469826E 24      0.
    -0.                -0.3288656E 24
    -0.6046417E 24      0.
     0.                 0.6749291E 23
     0.1465482E 23      0.
    -0.                 0.
```

```
   -C.3999455E 21      0.

 COEFFICIENTS OF POLYNOMIAL
   -C.7118664E C0      0.
    C.                 0.1010992E 00
    0.10C0000E C1     -0.
   -0.                -0.3E82790E 00
   -0.7138773E 00      0.
    0.                 0.7568629E-01
    C.1753852E-C1     -0.
    0.                 0.
   -0.4722051E-C3      0.

 INTERMEDIATE ROOTS OF POLYNOMIAL

  -1.52723E-01  1.20000E-01  1.52723E-01  1.20C00E-01  7.73945E-01  3.95802E-01  1.21936E-01 -6.46346E-02
  -7.73945E-01  3.95802E-01 -1.21936E-01 -6.46347E-02  1.03790E 00 -3.80157E-01 -1.03790E 00 -3.80157E-01

 INTERMEDIATE POSITIVE ROOTS

   1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21936E-01 -6.46346E-02  1.03790E 00 -3.80157E-01

 RE-ORDERED ALPHAS ( 1 ROW, 3 ZERO CASE )

   1.52723E-01  1.2C000E-01  7.73945E-01  3.95802E-01  1.21936E-01 -6.46346E-02  1.C3790E 00 -3.80157E-01

 INTERMEDIATE BETA B

   0.           0.           2.06942E-11  5.01658E-11 -6.58269E-11  4.08808E-12  2.26533E-11 -6.97316E-11
   1.0C000E-10  0.           0.           0.           0.           0.           0.           0.
   0.           0.           2.18210E-11 -5.38589E-11 -2.47548E-10  5.44232E-10 -2.09307E-11 -4.05041E-11
   0.           0.           1.00000E 00  0.           1.00000E 00  0.           1.00000E 00  0.

 INTERMEDIATE L MATRIX BY COLUMNS

   0.           0.           1.14542E 00  1.28000E-08  0.           0.           0.           0.
   6.20861E 00  5.29728E 00  0.           0.           2.5C863E 00 -5.32645E 00 -2.21131E 00  1.82953E 00
  -3.26236E 01 -1.40656E 01 -0.          -0.          -4.56617E 00  9.95881E 00 -2.11126E 01 -1.07904E 01
   5.75658E 00 -7.82836E 00  0.           0.          -4.56308E 00 -4.58660E 00 -1.82051E 00 -1.43626E 00

 VS =  0.3487689E 04     F(VS) =  0.8576C01E-04  0.4355446E-04     MAG =  0.9636790E-04

 COEFFICIENTS OF POLYNOMIAL
   -C.6025385E 24      0.
    0.                 0.8562924E 23
    0.8465826E 24      0.
   -0.                -0.3288656E 24
   -0.6046417E 24      0.
    0.                 0.6749291E 23
    0.1485482E 23      0.
   -C.                 0.
   -0.3999495E 21      0.

 COEFFICIENTS OF POLYNOMIAL
   -C.7118664E 00      0.
    0.                 0.1C10992E 00
    0.10C0000E C1     -0.
   -0.                -0.3E8279CE 00
   -0.7138773E CC      0.
    0.                 0.7568629E-01
```

```
INTERMEDIATE L MATRIX BY COLUMNS

 0.-          0.          1.14543E 00  1.28000E-08  0.           0.           0.           0.
 6.20861E 00  5.29728E 00  0.          0.           2.50863E 00 -5.32645E 00 -2.21131E 00  1.82953E 00
-3.26234E 01 -1.40654E 01 -0.         -0.          -4.56615E 00  9.95883E 00 -2.11125E 01 -1.07902E 01
 5.75658E 00 -7.82836E 00  0.          0.          -4.56308E 00 -4.98661E 00 -1.82051E 00 -1.43626E 00

VS =  0.3487688E 04     F(VS) = -0.5621333E-02 -0.2856658E-02     MAG =  0.6305544E-02

COEFFICIENTS OF POLYNOMIAL
   -0.6029385E 24     0.
    0.                0.8562924E 23
    0.8465826E 24     0.
   -0.               -0.3288656E 24
   -0.6046417E 24     0.
    0.                0.6749291E 23
    0.1465482E 23     0.
   -0.                0.
   -0.3599495E 21     0.

COEFFICIENTS OF POLYNOMIAL
   -0.7118664E 00     0.
    0.                0.1010992E 00
    0.1000000E 01    -0.
   -0.               -0.3882790E 00
   -0.7138773E 00     0.
    0.                0.7568629E-01
    0.1753852E-01    -0.
    0.                0.
   -0.4722051E-03     0.

INTERMEDIATE ROOTS OF POLYNOMIAL

-1.52723E-01  1.20000E-01  1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21936E-01 -6.46346E-02
-7.73945E-01  3.95802E-01 -1.21936E-01 -6.46347E-02  1.03790E 00 -3.80157E-01 -1.03790E 00 -3.80157E-01

INTERMEDIATE POSITIVE ROOTS

 1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21936E-01 -6.46346E-02  1.03790E 00 -3.80157E-01

RE-ORDERED ALPHAS ( 1 ROW, 3 ZERO CASE )

 1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21936E-01 -6.46346E-02  1.03790E 00 -3.80157E-01

INTERMEDIATE BETA B

 0.           0.           2.06942E-11  5.01658E-11 -6.58269E-11  4.08808E-12  2.26533E-11 -6.97316E-11
 1.00000E-10  0.           0.           0.           0.           0.           0.           0.
 0.           0.           2.18210E-11 -5.38589E-11 -2.47548E-10  5.44232E-10 -2.05307E-11 -4.05041E-11
 0.           0.           1.00000E 00  0.           1.00000E 00  0.           1.00000E 00  0.

INTERMEDIATE L MATRIX BY COLUMNS

 0.           0.           1.14542E 00  1.28000E-08  0.           0.           0.           0.
 6.20861E 00  5.29728E 00  0.           0.           2.50863E 00 -5.32645E 00 -2.21131E 00  1.82953E 00
-3.26236E 01 -1.40656E 01 -0.          -0.          -4.56617E 00  9.95881E 00 -2.11126E 01 -1.07904E 01
 5.75658E 00 -7.82836E 00  0.           0.          -4.56308E 00 -4.98660E 00 -1.82051E 00 -1.43626E 00

VS =  0.3487689E 04     F(VS) =  0.1979960E-04  0.4836713E-04     MAG =  0.5226283E-04
```

```
                KS = 0     LITHIUM NIOBATE

       EPSLON =  0.1C00000E-10   CLOSENESS OF DETERMINANT TO ZERO
       KL     =  0               0 = COMPUTE FOURTH RCW OF L MATRIX
                                 1 = SET FOURTH ROW = 1
       KM     =  0               0 = ELECTRIC FIELD (COTH)
                                 1 = MAGNETIC FIELD (TANH)
       MAX    =  25              MAXIMUM NUMBER OF ITERATIGNS
       N      =  6               NUMBER OF ITERATIONS ACTUALLY USED
       MU B   =  C.9000000E 02       LAMDA B =  0.
       NU B   =  C.9000000E 02       LAMDA A =  0.1500000E 12
       MU A   =  0.2850000E 11
       RHO A  =  C.1888000E 05       RHO B   =  0.47000C0E 04
       WH     =  0.1C00000E 11
       VS 0   =  0.3480000E 04   INITIAL VELOCITY

       VS     =  0.3487689E 04   FINAL VELOCITY SUCH THAT F(VS) .LT. EPSLON
       1/VS   =  0.2867228E-03   INVERSE OF VS

          ..... DETERMINANT = ( 0.8723C89E-04, 0.6594125E-04) .....

       FINAL ROOTS OF POLYNOMIAL              DIVIDED BY VS

     (-0.1527228E 00, C.12000C0E 00)      (-0.4378912E-C4, 0.3440674E-04)
     ( 0.1527228E 00, C.1200000E 00)      ( 0.4378912E-04, 0.3440674E-04)
     ( 0.7739447E 00, 0.3958016E 00)      ( 0.2219076E-03, 0.1134854E-03)
     ( 0.1219358E 0C,-0.6463462E-01)      ( 0.3496179E-C4,-0.1853222E-04)
     (-0.7739447E 00, 0.3958016E 00)      (-0.2219076E-C3, 0.1134854E-03)
     (-0.1219358E 00,-0.6463470E-01)      (-0.3496178E-04,-0.1853224E-04)
     ( 0.1C37905E 01,-C.3801570E 00)      ( 0.2975909E-03,-0.1089997E-03)
     (-0.1037905E 01,-0.3801570E 00)      (-0.2975909E-03,-0.1089997E-03)
```

```
*** F I N A L   A N S W E R S ***

          PARTIAL FIELD
       RELATIVE AMPLITUDES

1  ( O.            , O.            )
2  (-0.7201401E CO, 0.8085244E 00)
3  (-0.1512902E 00,-0.1377944E 00)
4  ( 0.1C00C00E 01, O.            )

.....WX = O.

          STRESS COMPONENTS

T31 = (-5.8207661E-11, O.            )
T32 = (-C.           , O.            )
T33 = ( C.           ,-1.4551915E-11)
T11 = (-4.8295741E-C3, 1.35732C7E-C3)
T12 = (-O.           , O.            )
T22 = (-1.3872618E-C3, 3.9637460E-04)

          STRAIN COMPONENTS

S11 = (-2.3131124E-14, 6.3502930E-15)
S22 = ( C.           , O.            )
S33 = ( 7.8339108E-15,-2.1363742E-15)
S12 = (-C.           , O.            )
S13 = ( 5.8991011E-16,-2.5035702E-16)
S23 = (-C.           , O.            )

        TIME AVERAGE POWER FLOW

 P1M = ( 2.2182838E-C9,-2.5693692E-11)
 P2M = (-O.           , O.            )

        ELECTRIC DISPLACEMENT

 C1 = (-7.7528271E-14, 1.7353723E-14)
 C2 = (-C.           , O.            )
 C3 = (-3.2623770E-16,-1.7C19480E-15)

        MECHANICAL DISPLACEMENT
       MAGNITUDE          PHASE
 U1 =  8.3656159E-11   -105.444
 U2 =  O.                 C.
 U3 =  1.2363921E-10    -15.145

ELECTRIC POTENTIAL MAGNITUDE =  6.8294138E-01  PHASE =   79.149

          ELECTRIC FIELD

 E1 = (-1.9231359E-C4, 3.6663880E-05)
 E3 = ( 3.8187697E-C5,-1.3321136E-C5)
       MECHANICAL DISPLACEMENT
         -C.22282974E-10    -0.80675376E-1C
          O.                 C.
          C.11934538E-C9    -0.32300663E-10
       ELEC. POT. MAGNITUDE
          C.1285971CE 00     C.67C72476E CO
```

```
9 112245 C $ASSIGN        SYSLB4
9 112451 C $STOP

9 112451 O  PERIPHERAL FILE POSITIONS AT END CF JOB

9 112451 O  SYSPP1              REC. 00942, FILE 0000C
9 112451 C  SYSOU1              REC. 03745, FILE 00000
9 112451 O  SYSIN1              REC. 00002, FILE 00002

9 112451 C END OF JCB
```

01 UNITCS, EOF.   REC= 00000 FIL=

```
4*104318 JOB  105 REEL CRTPLT IS REQUIRED
4*1104C9 JOB  109READY CRTPLT ON UNIT C3
```

```
001094 1614 N4CONWAY      PHASE        TIME CFF WAS 112451    12/31/69
 ***7C44 SERVICE*****************
 *                              *
 *   2159 CARDS READ            *
 *                              *
 *    979 CARDS PUNCHED         *
 *                              *
 *   3716 LINES PRINTED         *
 *                              *
 ***7C9411 SERVICE***************
 *                              *
 *   2.C7 MIN. PRE-EXECUTICN    *
 *                              *
 *    .13 MIN. EXECUTICN        *
 *                              *
 ********************************
```

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13. ABSTRACT

This report describes the analyses of several piezoelectric and pure elastic surface wave propagation problems and computer programs which implement their numerical study. In addition, the formal analysis of an electric current line source located above a piezoelectric crystal half space is presented in some detail.

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Wave Propagation						
Leaky Acoustic Surface Waves						
Piezoelectric Crystal						

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